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From Nothing to Particles

A Quantum Theory of Topological Knots (TQNT)

6th edition — V6, integrated developments

Lilian Alexandre Cariou
Theoretical research division of CA9 SAS (SIREN 903918043)
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Abstract

The Topological Knot Quantum Theory (TQNT) rests on a single principle: the vacuum is unstable, its perturbations are waves, and some of these waves close back on themselves into knotted loops — topological knots that *are* the elementary particles. Wave–particle duality thereby becomes geometric: at low energy the knot is *tight* and behaves like a delocalised wave that interferes and diffracts, whereas at high energy it *loosens* and reveals its localised structure, carrying a measurable mass, spin and charge. Mass is the electromagnetic energy *confined* in the curvature and torsion of the knotted flux tube; isospin is not a fundamental quantum number but a *geometric symmetry* (the u and d quarks are the same trefoil at different amplitudes); the proton is the unique link of three knots reconciling spin $\frac{1}{2}$, charge $+1$, correct mass and strong stability, whence the existence of the 92 chemical elements; and spacetime itself emerges from the density of the wave functions: where the knots are dense, time slows and space curves.

Formally, the TQNT derives the Schrödinger equation, the masses of particles, spin, charge, gravitation, thermodynamics, dark matter, dark energy, cosmological inflation and baryogenesis from a *single* fundamental field — a Chern-Simons-type SU(2) gauge connection. The topological knots of this field *are* the elementary particles, whose mass, spin and charge emerge from knot invariants. With **ten calibrated global parameters** (all *TQNT-native*, including five *knot* masses — *not* quarks borrowed from QCD — not yet derived; m_s, once derived from the $\ell(5_2)/\ell(3_1)$ ratio, was *demoted* on 2026-07-22 (ACPR audit: true ratio $-5.4\,\%$, the $+5.6\,\%$ dressing not derived); $k = 135$ and m_ℓ are *derived* 2026-07-25 — trefoil integer lock $k{+}2 = \mathrm{round}(\frac{8\pi}{3}\ell(3_1)) = 137$ and the charge's magnetic inflation $m_\ell = \frac{3}{4}\sqrt{\sigma_\chi}/(1{+}\pi\alpha/2) = 310.30$ MeV ($+0.10\,\%$), conditional (3.4): if retained, *nine* calibrated; versus nineteen for the Standard Model, and zero per particle), the **reference V6 mass formula is** $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (a single scale $\sqrt{\sigma_\chi}$ for the *light* sector: leptons + baryons, all derived; the heavy knots calibrated, target $\sqrt{\sigma_\chi}\,\ell(K)$): it reproduces **89 $q\bar{q}$/stoikhion states at a median of** $0.13\,\%$ ($91\,\%$ below $5\,\%$; engine at main document 2026-07-21; the Ω_{cc}, then removed as unobserved, has since been *observed* — LHCb 06/2026, $\sim$3 727 MeV, *inside* the predicted 3 715–3 788 bracket) — fundamentals $\sim 0.1\,\%$, universality of the Regge slope $\pm 11\,\%$, Z_3 quantum $1.8\,\%$, Koide 61 ppm; the multiquarks (exotics, scalars f_0/a_0) and a few baryonic excited states belong to sectors of their own. The exhaustive catalogue (1 539 links up to 30 crossings) identifies **117 dark matter candidates**: 6 purely amphichiral, intrinsically neutral, and 111 links neutralisable by physical linking. The TQNT also reproduces cosmological quantities measured by Planck: the spectral index of the primordial perturbations (0.965) and the dark energy density; the dark-matter-to-ordinary-matter ratio is **reconstructed with no free parameter** (2026-07-17, asymmetric formation cost: symmetric dark relic / CP crumb η_B, exact 8/9 string supply, derived knotted fraction 0.45, measured kink cascade, computed $\Gamma_a(\chi) = 84$; measured count $n_{\mathrm{knot}}/L = 0.304$ (two independent networks combined, 11/11 verification) and T_c now *derived* (62 ± 3 MeV, topological tightening, §26.7.8): $\Omega_{\mathrm{DM}}/\Omega_b \approx 0.6$ $[0.2, 1.3]$ — the order of magnitude of Planck (5.45), the gap held within the declared transition-state $O(1)$s; at the old adopted $T_c = 67$: 3.9 ± 0.4 (Planck at $+40\,\%$), §26.7.5; the historical fine agreement was conditioned on the V5 mass law). The slight discrepancy on the latter ($+5.9\,\%$ vs Planck) is *not significant*: the prediction falls **within the band of the Hubble tension** ($-9.9\,\%$ vs SH0ES) and below the structural noise of the formula — there is no residual to explain (the former P22 reattribution is obsolete: null mechanism, WKB form). A single integer, the level of the theory ($k = 135$), fixes both the fine-structure constant ($1/(k{+}2) = 1/137$) and seven cosmological observables — and that integer is now *derived* at the trefoil lock (3.4, conditional); the emergent metric reproduces all the classical tests of general relativity. **This 6th edition (V6)** merges into the relevant chapters the mid-2026 work: the Koide relation reinterpreted through the $\mathbb{Z}_3$ of the trefoil ($r = \sqrt{2}$: identified with $d_{1/2}(k{=}2)$, a hint), *published* (DOI 10.5281/zenodo.20817001, §9.5); the weak sector as P15 reconnection

(coupling = vev, Cabibbo = λ^D_{Cab}, CP = chirality, §15.3); topological electronic $g = 2$ (§12.2); **relativistic kinematics and the Dirac equation derived from the undulating spacetime weave** (Lorentz = symmetry of $\Box\chi = 0$, spin-statistics from the ribbon's 4π, antiparticle = mirror, the EM sector closing onto QED at derived $\alpha = 1/137$; conditional S/D/P10, §3.5); bipolaron-χ superconductivity, P23 magnetism, dissipation (§20.16); neutrino mass from *self-rotation curvature* ($m_\nu = \hbar c/2R$, $R \sim \mu$m; refinement, R tuned on Σm_ν) and traversal of matter by bounce/reconnection — validated against the detectors (§B.3.3), with Boltzmann transport (attenuation $e^{-\tau}$, opacity of the Earth at ∼34 TeV seen by IceCube; MSW as a coherent sum of the bounces, whose ∼2 MeV resonance reproduces the $pp \to {}^8$B transition of the solar P_{ee} (SNO/Borexino); heavy-matter creation excluded by energy conservation, §B.3.4); baryogenesis by *relic-free nucleation* (asymmetry imprinted at the χ condensation, exact trefoil ε_{CP}, Sakharov #3 = phase transition; η_B at the *right order of magnitude*, the fine value depending on the annihilation freeze-out tied to the vev, not derived); an honest confrontation with the real data (§B.6); and the *scope and limit* of the model — four topological observables (mass, charge, spin, lifetime) and the *wall* of baryon number / unique assignment (§B.7). **Honest position**: in the tested regimes (JWST, NICER, NANOGrav), the TQNT is *observationally indistinguishable* from Λ-CDM+GR; it *maps* the Standard Model topologically (derived structures and orders: Chern–Simons, $\mathbb{Z}_3$, Lorentz kinematics and Dirac — from the spacetime weave, conditional, chirality), the fine values (x, A, exact δ_{CP}, f_π) remaining calibrated or open. The best hint is the Koide relation ($Q = 2/3$ to 9.2 ppm), reinterpreted via the $\mathbb{Z}_3$ of the trefoil ($r = \sqrt{2}$: *identified* with $d_{1/2}(k{=}2)$, still a *hint*); the distinctive tests are *future* ($0\nu\beta\beta = 0$, $\Sigma m_\nu \approx 66$ meV, *finite* proton decay $\sim 8 \times 10^{33}$ yr, leptonic CP, DM-117 bipartition).

Unlike the Standard Model and its *four* forces, the TQNT has only **two origins**, both topological: **(i)** the **quantum gravitational field** that each knot generates, of which classical gravitation is the *large-scale resultant*; **(ii)** the **fundamental field** itself, from which emerge **electromagnetism** (its low-energy limit), the **strong nuclear force** (the tension of the linked knots) and the **weak nuclear force** (the reconnection of a linking). The **lifetimes** of *all* unstable particles (34 distinct values, charge partners sharing the same lifetime) follow the **Fermi/Sargent theory** with the topological energy $Q = \Delta m$ (the remaining weak constants — g_A, V_{ud}, f_π, α_s — being measured *inputs* of the SM; G_F = vev is now *derived*, the measured G_F serving as the practical input); global RMS 0.94 decade (order of magnitude); the *non-suppressed* channels reach a remarkable precision: **EW** (W, Z, t, H) to 0.06 decade (∼ 15 %), **leptonic/semi-leptonic** (μ, τ, n, $\pi^\pm$, $K^\pm$, π^0) to better than 0.1 decade (in particular $\tau_n = 880$ s vs 878 s measured, i.e. 0.2 %). The larger deviations (K^0_S, η', J/ψ, Υ) are *known* suppressions ($\Delta I = 1/2$ rule, OZI, $U(1)_A$ anomaly) outside the topological field. The scale of the weak and electroweak interactions is a *topological mass* of 246 GeV, now *derived* (open crossing on the anchor, +28 ppm, conditional), not the exchange of a heavy mediator, the strong nuclear force follows from the linking tension, electromagnetism from the fine-structure constant. Finally, black holes are stars of "frozen" knots (without singularity, information is conserved), $E = mc^2$ is the cost of *deconfining* a knot, and Hawking and Schwinger radiation appear as the extreme regimes of one and the same boundary-nucleation mechanism.

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Table of notations

This table gathers the symbols used throughout the document. Quantities marked *(derived)* are not free parameters but results of the calculation; the **ten** global parameters of the theory are grouped at the end (see also the § "Parameter counting"). A few symbols are *overloaded* (e.g. Q, μ, α, P): their meaning is specified according to the context.

Symbol	Meaning	Ref.
Fundamental field and wave function		
χ	fundamental field: Chern–Simons-type SU(2) gauge connection	P1
k	level (integer) of the theory, $k = 135$; fixes $\alpha_{\text{EM}} = 1/(k{+}2)$	P1
$\Psi(\mathbf{r})$	wave function of a knot (sol. of the emergent Schrödinger eq.)	P1
$\lvert\Psi\rvert^2$	linear density of the knot along the flux tube	§ metric
μ	eigenvalue of the modified Schrödinger eq.; *also* the Jones amplitude depending on context	
Topological and geometric invariants of the knot		
K	knot, embedding $S^1 \hookrightarrow \mathbb{R}^3$: a particle	P1
j_K	total angular momentum of the *light companion* (the knot K at the tube end; P bands of heavy-light mesons) — *not* a "light quark": inherited notation j_q renamed 2026-07-22	§ spectrum
L	link, ≥ 2 components: a composite particle	
n	number of components (linked waves); structural group, $J_{\max} = n/2$	
$c(K)$	number of crossings (topological complexity)	§ Jones
$V_K(t)$	Jones polynomial; the mirror $V_{K^*}(t) = V_K(t^{-1})$ gives the antiparticle	§ Jones
$\bar{K}$	**mirror** of K (also written K^*): $V_{\bar{K}}(t) = V_K(t^{-1})$, opposite chirality ($\chi(\bar{K}) = -\chi(K)$); a *mirror pair* $K{+}\bar{K}$ compensates $\Sigma\chi$ (e.g. nucleon: $7_2{+}\bar{7}_2$)	§ Jones
$\det(K)$	determinant of the knot, $\det(K) = \lvert V_K(-1)\rvert$	§ Jones
$c_J(K)$	topological complexity index (*Jones factor*; ρ_J in the code), $c_J = \text{span}(V_K) + \gamma_J \lvert V_K(-1)\rvert$	§ c_J
$b(K)$	braid index; related to spin	§ Jones
$s(K)$	signature of the knot	§ Jones
Lk	linking number of a link	§ stability
Lk_{opt}	optimal linking, $\binom{n}{2}$ (one Hopf link per pair): minimal energy	§ stability
D	linking deviation (stability criterion), $D = (\text{Lk} - \text{Lk}_{\text{opt}})^2$	§ stability
$\Sigma\chi$	sum of the topological charges of the components	§ charge
U_{EM}	intrinsic topological electromagnetic energy of the knot	P21

continued on next page

Table of notations (continued)

Symbol	Meaning	Ref.
ν	torsional saturation index	§ mass
σ	curvature saturation index	§ mass
Observables of the particle		
m	mass, $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (V6); historical V4 form: $m = \alpha_0\, U_{\rm EM}\, c_J^{3.55}$	P21
Q	**electric charge**, $Q = \Sigma\chi + \mathrm{Lk}$	§ charge
Q	(*lifetimes*) energy released in a decay	§ lifetimes
J	spin (integer or half-integer according to the parity of n)	
τ	lifetime	§ lifetimes
Γ	decay width, $\Gamma = \hbar/\tau$	§ lifetimes
Emergent geometry (spacetime, gravity)		
g_{ij}	emergent conformal metric, $g_{ij} = \Omega^2\,\delta_{ij}$	P5
Ω^2	conformal factor, $\Omega^2 = 1 + \kappa_g\lvert\Psi\rvert^2$	P5
h	metric dilation, $h = \Omega^2 - 1 = \kappa_g\lvert\Psi\rvert^2$	P5
h_0	dilation at the surface of the proton, $h_0 = \frac{6}{7}\alpha_{\rm EM}^{18}$	§ gravity
κ_g	metric coupling *(derived)*, $\kappa_g = \frac{96\pi}{7}\alpha_{\rm EM}^{18}\,\lambda\!\!\!^{-}_C$	§ gravity
$g_q(\mathbf{r})$	quantum gravitational field (QGF): centripetal metric well of a particle, $g_q = -\frac{c^2}{2}\nabla h$	def. QGF
$V_{\rm mag}$	magnetic confinement potential of the flux tube	P4
V_{NN}	nucleon–nucleon potential (residual nuclear force)	§ nucl. force
Φ	gravitational potential ($\nabla P = -\rho\,\nabla\Phi$)	§ thermo.
ξ	dimensionless metric gradient, $\xi = \lvert\nabla\Omega^2\rvert\,\lambda\!\!\!^{-}_C/\Omega^2$	P22
Scales and derived constants		
$\alpha_{\rm EM}$	fine-structure constant, $1/(k{+}2) = 1/137$	P1
$\lambda\!\!\!^{-}_C$	reduced Compton length of the proton, $\hbar/(m_p c)$	§ gravity
r_p	proton radius = exclusion radius $a = 4\lambda\!\!\!^{-}_C$	§ gravity
$m_{\rm base}$	mass of the base nucleon (link before Lk selection), $\sqrt{\sigma_\chi}\,\mathcal{S} =$ 938.24 MeV	§ gravity
G	Newton's constant *(derived)* — weave amplitude, $\hbar c/M_{\rm Pl}^2$ (unique equation); nucleon reading $\frac{12}{7}\alpha_{\rm EM}^{18}\hbar c/m_{\rm base}^2$	§ gravity
G_F	Fermi constant *(derived)*, $1/(\sqrt{2}\,E_{\rm relax}^2)$	§ lifetimes
$E_{\rm relax}$	topological relaxation (transition $\Delta\mathrm{Lk} = \pm1$), ≈ 246 GeV	§ weak

Global parameters of the TQNT — 10 calibrated (+2 hints $\sqrt{2}$, $\delta{=}Q/3$; counting established in the § Counting; v derived 2026-07-21; m_s demoted 2026-07-22, ACPR audit; k **and** m_ℓ **derived 2026-07-25** — trefoil lock + charge inflation, conditional, 3.4: if retained, *9*). *Reference mass* $m = \sqrt{\sigma_\chi}\,\mathcal{S}$: a **single** scale $\sqrt{\sigma_\chi}$ for the *light* sector (leptons + baryons, all derived); the **heavy knots** (charm/beauty/strangeness + trefoil) are *calibrated* — *not* quarks. (Ω_{ts}, $\{\beta_{\rm SAW}, x, f(2), f(3)\}$: microscopic route abandoned, archived out of the text — cf. foreword.)

$\sqrt{\sigma_\chi}$	*unique* scale of the light sector; 418.47 MeV$= 2\Lambda_\chi$; m_e derives from it (Koide)	§ Counting
$\mathrm{SL_{rad}/SL}$, $\tau_\perp$, $\mathrm{hf}_{c\bar{c}}$	radial slope 1.38; hyperfine; quarkonia: Cornell on the same string ($\sigma{=}\sigma_\chi$)	§ Counting
v	electroweak VEV, 246.23 GeV *derived* (open crossing on the anchor, +28 ppm, conditional — no longer counted)	§ Counting

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Table of notations (continued)

Symbol	Meaning	Ref.
$m_\ell, m_s, m_c, m_b, f_\pi$	*[TQNT knots, calibrated]* trefoil 3_1 (m_ℓ) + strange/charm/beauty knots + chiral f_π; light plateau geometric; m_c, m_b not ($\lesssim 1.4$ GeV)	§ Counting
$\beta_\otimes$	linking tension (stability, strong lifetimes), 0.210 MeV; fluxoid mechanism identified, circuit *measured* on the relaxed+repolished nucleon: Δm(*n*-*p*) to +0.63 % (favoured channel, 3-body circuit; topological rigidity of the χ vortex imposed by the data)	§ Counting

Derived (do not count): $2\pi\sigma_\chi$ (Regge slope), Λ_χ, $E_{Z_3} = \frac{2}{3}\Lambda_\chi$, tensor $\Lambda_\chi/8$, N-Δ hyperfine, Casimir $\frac{3}{4}$; $\alpha_{\rm cond} = 1.20$ *derived* to < 0.1 % (condensation kinetics, P18); P22 exponent $-2\pi/\xi$ *derived* (η and Γ_0 eliminated, WKB form). *Microscopic layer of the χ solver (P4/P8, never tuned on particle observables):* γ_J, E_0, λ_V. Ω_{ts} *and the V4 form* $\alpha_0 U_{\rm EM} c_J^{3.55}$*: obsolete as predictors.*

Disambiguated symbols (homonyms resolved)

$\beta_{\rm SAW}$	screening exponent = 5/3 (Flory, excluded volume; fit 1.66). *N.B.* $33/20 = c - 2h$ and $\sqrt{e}$ are numerical coincidences, not derivations	§ mass
$\beta_{\rm osc}$	complexity asymmetry (oscillations), −1.30	§ lifetimes
β_Λ	topological Boltzmann (dark energy/DM), 0.914	§ dark energy
$\beta_{\rm PPN}$, $\gamma_{\rm PPN}$	post-Newtonian parameters (Eddington), = 1 in GR	§ gravity
β_{V5}	Cornell string tension (quarkonium)	§ Cornell
$\lambda_{\rm dec}$	exponent of the old fragility law (*abandoned*, foreword), 5	§ lifetimes
λ_V	potential coupling, $V = -\lambda_V\, c_J/d^2$	§ mass
$\lambda_{\rm Cab}$	Cabibbo angle (reconnection cost), $\sqrt{m_d/m_s} = 0.224$	§ weak
λ_N	nuclear Yukawa range (e^{-r/λ_N})	§ strong
λ_S	attraction range (surface term a_S), 0.64 fm	§ strong
λ_α	α decay rate (Gamow factor)	§ α
$\lambda_{\rm lon}$	longitude (generator of the torus, A-polynomial; $\lambda_{\rm lon}\mu = 1$)	§ A-polynomial
λ_ϕ	SM Higgs self-coupling ($V = \lambda_\phi \phi^4$)	§ electroweak sector
p	mass exponent, $p = q = \frac{3}{4}$ (Casimir spin-$\frac{1}{2}$)	§ mass
$p_{\rm rec}$	P15 reconnection probability, $1 - e^{-\varphi_c} \approx 0.29$	§ kinetics
φ_c	critical percolation filling, 0.342 ($\neq \eta$ of P22)	§ kinetics
$\tau_\perp$	Frenet torsion ($B = \tau_\perp$; $\oint(\kappa^2 + \tau_\perp^2)$)	§ mass
κ	Frenet curvature (distinct from the gravity κ_g)	§ mass

Finite element discretisation (FEM)

$K_{ij}^{(e)}$	elementary stiffness matrix (P1, nodes i, j of element e)	§ FEM
$P_{ij}^{(e)}$	elementary potential matrix	§ FEM
$M_{ij}^{(e)}$	elementary mass matrix	§ FEM
$\widetilde{K}, \widetilde{P}, \widetilde{M}$	versions modified by the conformal metric Ω	§ FEM
$\bar{\Omega}_e$	average of the conformal factor Ω^2 over element e	§ FEM
$N_i(\mathbf{r})$	P1 shape functions (piecewise linear)	§ FEM
V_e	volume of the tetrahedron (element) e	§ FEM
$\mathbf{r}_q$	Gauss quadrature points (4 points)	§ FEM

Emergent thermodynamics

T	temperature (emergent, not postulated)	§ thermo.

continued on next page

Table of notations (continued)

Symbol	**Meaning**	**Ref.**
P	pressure (to be distinguished from the postulates P_n and from P_{ij})	§ thermo.
ρ	density (of knots or of energy)	§ thermo.

Foreword: from V4 to V6

This sixth edition is not an incremental update: it changes the *reading* of the theory. This foreword is the **single reference** of the V4 → V5 → V6 evolutions; the chapters now present the V6 *as such*, without repeating the history (the abandoned forms are archived *outside the body of the text* (`docs/Archives_recherches/archive_omega_ts_axiomatique.tex`, with its English counterpart `archive_omega_ts_axiomatique_EN.tex`); the V4 asymptotic checks are kept where they serve).

The mass formula (the central change). **V4**: $m = \alpha_0\, U_{\mathrm{EM}}\, c_J^{3.55}$ — mass per knot, ∼5 %, fitted exponent. **V5**: the axiomatic operator Ω_{ts} — derived form (Hölder, Casimir), calibrated constants; 118/122 states at 0.68 % *with assignment search*. **V6**: $\boxed{m = \sqrt{\sigma_\chi}\,\mathcal{S}}$ — a single scale (the tension of the χ string) × a dimensionless factor fixed by the *topological class* (§9.1.1 for the origin of the form). Ω_{ts} and the V4 form are **abandoned as predictors**: the catalogue curves feeding U_κ, U_τ were topologically wrong (14/19), Ω_{ts} gave only ∼34 % at *fixed* assignment, and the assignment scan (median 4, max 43 candidates) showed that mass cannot live in the *type* of the knot.

The guiding principle (the conceptual shift). V4/V5 read mass *per knot*. V6 states "σ_χ *prevails*": the knot configuration carries the **spin** (braid arity) and the **charge** ($Q = \Sigma\chi + \mathrm{Lk}$) — quantised, discrete, robust — while the **mass** follows the law of its *sector* (Koide, Regge+Z_3+tensor, GMOR, stoikhion+Cornell — the *stoikhion*, from Greek *stoikheîon* "element", is the knot-brick of the composites, formerly "constituent" (decree 2026-08-06) —, VEV).

The headline numbers (before → after).

	V4/V5	V6
spectrum	118/122 at 0.68 % (Ω_{ts}, assignment searched)	89 states, median 0.13 % (sector laws, engine at main document 21/07)
nucleon	938.54 (0.03 %, Ω_{ts}); V4 940.2	938.2 (0.0 %, Regge base, $\sqrt{\sigma_\chi}$ recalibrated 2026-07-18); $\Delta m(n\text{-}p)$ *derived*
parameters	"10 global" (Ω_{ts}-framed)	10 calibrated + 2 hints (η *eliminated*: P22 in WKB form $e^{-2\pi/\xi}$, derived exponent; m_s *demoted* 2026-07-22 (ACPR audit: true ratio −5.4 %; 5_2 pinning reinforced); C_{plast} archived: its target, the μ^- residual, was an Ω_{ts} artefact; α_{cond} *derived*; E_0 reclassified to the solver layer; $\beta_\otimes$: fluxoid mechanism identified, $\Delta m(n\text{-}p)$ *reproduced* at +0.63 % with no adjustment (3-body circuit, favoured channel; winding-rigid vortex imposed by the data) — stays calibrated; v *derived* 2026-07-21: open crossing on the anchor, $\sqrt{7\pi/6}\,(k{+}2)\,m_{\text{base}}(1{+}\alpha/4\pi)$, +28 ppm, conditional — count 11 → 10; the 10 → 9 step is the conditional k/m_ℓ reduction)
observables	"∼419 tests < 5 %"	∼25 *load-bearing* + descriptive + compatibilities
lifetimes	wear constant C, then P13/P15	Fermi/Sargent + $Q = \Delta m$, *zero* calibration of its own
$\Omega_{\text{DM}}/\Omega_b$	5.45 "fine" (conditioned on the V5 law + undercounting)	fully derived chain, T_c included: ≈ 0.6 $[0.2, 1.3]$ at the derived T_c (order of magnitude); measured count at the old $T_c = 67$: 3.9 ± 0.4 (§26.7.5)

What V6 adds in its own right. The *origin of the reference form* (uniqueness of scale + dimensional analysis + sectoriality, §9.1.1); the **Y-junction tensor** ($E = S(S{+}1)\Lambda_\chi/8$, closed form: decuplet 98 derived vs +97 measured); the **flux-tube torsion** (§14.15: the exchange law Lk = Tw + Wr verified exactly, the fundamental periphery Tw*, the flux quantum $\Phi = \alpha\, h/e = \mu_0 ec/2$ *derived*, the absolute moment of the neutron carried by the trefoil, the three torsion/blocked-loop cases); the **spin = braid arity** consolidated (four rotational origins tested and refuted; nucleon = 1 fermion + 2 bosons); the **methodological rule of the assignment null** (§B.7) and the re-reading of the observables it imposes; the heavy sector *clarified* (mechanism identified: stoikhion + Cornell on the *same* string $\sigma = \sigma_\chi$; the derivation of the values m_c, m_b remains open, geometric ceiling 1.4 GeV). Finally, the **explicit abandonment of the P13/P15 fragility law**, $\tau = \tau_P\, e^{-\lambda_{\text{dec}} n}/M_{\text{raw}}^{\nu}$ (inherited from V4, exponents calibrated on the muon): it underestimated the neutron by −0.14 decade (643 s), its knot count does not carry the lifetime hierarchy (×150 vs 10^9 — Q drives it, Sargent, corr = −0.99) and its pure-topology version ($N{=}18{-}6|D|$) was an artefact; the V6 mechanism is the *release of the trapped loop* by ΔLk reconnection, with precision from Fermi/Sargent ($Q = \Delta m$).

The July 2026 gains (closing the dark sector). A derivation campaign closed the dark-matter/cosmology sector end to end: the **formation Boltzmann factor** derived *through* TQNT (background of χ waves, equiprobability via the dance, microcanonical weight of the finite domain); the **asymmetric formation cost** (symmetric dark relic vs visible CP crumb, the $1/\eta_B$ factor); then the full P_{occ} chain (§26.7.6): *exact* 8/9 string supply (triality combinatorics, Vachaspati–Vilenkin simulation), knotted fraction from the tree identity on α_{cond}, *exact* Nambu cascades (coefficient 1.14, $\kappa_s = 2.85$ measured), radiation constant $\Gamma_a(\chi) = 84 \pm 17$ *computed* (Kalb–Ramond), the energy floor, and the final branched cascade: **direct measurement** $n_{\text{knot}}/L = 0.304$, i.e. $\Omega_{\text{DM}}/\Omega_b = 3.9 \pm 0.4$ at the old adopted $T_c = 67$ (−29 % of Planck); at the now *derived* T_c (62 ± 3 MeV, topological tightening, §26.7.8): $\Omega \approx 0.6$ $[0.2, 1.3]$, the order of magnitude — no intermediate model left between the TQNT primitives and Ω. In paral-

lel: $g_s(T_c) = 15.2$ derived, anyonic counting verified negligible, $\sigma_{\rm EM}$(neutral) *derived* (periphery neutralisation, conducting polarisability: the reservoir edge ∼7 dex below DS-50 — a robust *absence* prediction, gravitational and cosmological probes), F3 *lifted*, m_s once *derived* (demoted 2026-07-22, ACPR audit — 5_2 pinning reinforced), and the geometric mass law **validated** on the ideal $T(2, 23)$ (SONO, +0.4 % from the linear fit). On the weak side, the P15 chain closed in the same movement: Fermi structure, $N = 2{+}3$ counting, W families and gate triality *derived*; T_c derived via double reconnection (topological tightening); and the **VEV** v **derived** (2026-07-21, open crossing on the anchor: fusion/splitting theorem + universal rung + crest measure + $\alpha/4\pi$ dressing, +28 ppm, conditional, §22.4.4) — count 11 → 10 (the current count, v out; the 10 → 9 step is the *conditional* k/m_ℓ reduction, §27.2). And the last general wall — relativistic kinematics — is itself *derived* from the spacetime weave (scalar kinematics, spin-statistics from the ribbon's 4π, the Dirac equation forced with antiparticle = mirror; the EM sector closes onto QED at derived $\alpha = 1/137$; conditional S/D/P10, §3.5). **No general wall remains**: what stays posited is named — the "everything is χ" ontology, P10, the general 3D case of second quantization (the P22 germ, the P22↔P15 crossing and the path sum are done, §3.6) and the non-abelian strong dynamics. Finally, the spacetime weave reading delivered the **torsion of the peripheral vertices** (S4″, 2026-07-21, §14.2): the V-P hyperfine = one ribbon half-turn = $\pi\sigma_\chi$ (universal $\pi \to B_s$ at 0.7 %, zero parameters), the heavy-baryon recoupling ladder in closed forms $\{1, \sqrt{2}, \sqrt{2}{-}1\}$ with the Z_3 *tripling per generation* (the factor is exactly 3, not m_b/m_c) and the base *derived* (the triality half-turn: three distinct Z_3 characters ⇒ one condensate turn ⇒ the 4π ribbon rotates by π) — 15 baryons at 0.23 % with no parameter, metaxions (LHC "molecules") at 0.3 %, the extension to 226 states (median 1.43 %), and two LHCb predictions ($\Omega_b^* = 6067$, $\Xi_{cc}^* = 3659$ MeV) — then the **spacetime weave's identity card** (S4‴, §3.3): time = $1/f_{\rm local}$ (GPS 45.7 μs/day exact), the spin reading theorem (ω hidden, j read by inverse Regge), equivalence = spacetime weave uniqueness ($G = \hbar c/M_{\rm Pl}^2$ from any particle), and the crushing of the teeth ($E_{\rm tooth} = hf_{\rm local}$, the horizon at 0.5: frozen knots re-read). The whole closes on **the spacetime weave equation** (§14.3): $\Box\chi = 0$ in vacuum, $\nabla^2 h = -\kappa_g|\Psi|^2$ with a particle (mass = the charge of the depletion field), and the master functional $M^2 = (\Sigma m_i)^2 + 2\pi\sigma_\chi(L + n/2 + 1.38\, n_r)$ — **57 heavy masses at** ~ 0.3 % (49 with no per-state anchor after the D_s/B_s half-substitution and the P-band sorting — bands identified, $n = 2 + 3^g$ (5/2), the Z_3 step a third time, the 5/2 *proven* (light-end locking + heavy-frame inertia — the light sector's pure $n{=}2$ as a corollary); Σ_b saturation; the strange shift s *derived* 2026-07-22: $\Delta{\rm Wr}/\pi$, the 5_2's ideal writhe excess read in radians — Fuller holonomy verified, identity aperture measured at 0.8 %, conditional; the intra-doublet splitting *derived* 2026-07-22: $1/m_Q$ = Compton response of the ribbon, moment half-substitution, and the coefficient $\kappa_u = h_{1/2}(k{=}2) = 3/16$ — the conformal weight of the spin-$\frac{1}{2}$ line, *third invariant* of the same ribbon as $d = \sqrt{2}$ and $d{-}1$ (0.5σ, conditional); and the $j_K{=}1/2$ sign *dissolved* 2026-07-22 by the ladder's monotonicity (null alignment at the floor, charm centroid −0.4 %, B_0^*/B_1' prediction at the point 5 665 MeV) — **the band sector is fully derived, zero read numbers**; first LHCb confrontation 22/07: Ω_{cc} *verified* (∼3 727 inside the bracket, lock → Ω_{ccc} 4 782 strict), strange baryonic shift *supported* (sign + order, 2 sectors); Ω_c/Ω_b multiplet rule completed (centroid: intensive shift + un-tripled lock; flip ladder) — the fifth $\Omega_b(1P)$ predicted 6 297–6 304 MeV; and the baryonic orbital bands *derived*: two-strand cable, $4(d{-}1)\cdot 3^g$ at −0.16 %, then b derived (mechanized sum rule: $b = 2 - \langle x\rangle = 23/18$, the trace average of the flips): nine values, zero read numbers; and the $L{=}2$ extension derived (the frame turn is per period: eight 1D systems at ≤ 0.4 %, $B_J(5970)/B_{sJ}(6064)$ turned into confirmations) — **no named structural residue left in the heavy sector**; and the base-2 dibaryon *computed* (Steiner: double-Y + bridge, complete card): light H *absent* (40 years of nulls postdicted), H_{cc}^{++} 0^+ deeply bound at 4 497 ± 10 MeV, displaced vertex, excited 1^+ pinned on the $\Lambda_c\Lambda_c$ threshold — bridge derived $\ell{=}D$ by the contact theorem); and the $\sqrt{2}{-}1$ hint becomes an *identification* ($d{-}1$, the forced quantum dimension of the identical pair — the same $\sqrt{2}$ as Koide: one hint, not two) and then a *theorem* (loop value: the closure of the exchange is worth d, the classical one 1) —

the recoupling ladder no longer has a free link nor a tension (the exchange-budget saturation resolves Σ_b: flip = 4 exact). **The spacetime weave-rails programme (2026-07-23)**: knot formation receives its combinatorics (rails = nodal lines of the standing spacetime weave, trefoil first product at 59 %, threshold ∼24 steps, superposition necessary), the $e/p/n$ probabilities and the dark-matter compatibility are quantified, and α_{cond} receives a candidate closed form $\ln\bigl(2+\sqrt{2}\bigr)$ (the capture-window theorem, §3.4); the light-plateau stoikhion brick becomes *arithmetic* (ratios of rail integers, $m_s/m_\ell = 3/2$ exact at −0.7 % of ACPR); and magnetisation is an *entry* property (the spectator theorem: light = the population that never entered the rails — $\mathcal{S} = 0$ mechanised, the mirror pair protected by parity, the dark reservoir's conducting ring = the locked rail current); and the *origin* of entry is determined (never spontaneous: zero rate by protection; always a threshold event — minimal door = a pair, Breit-Wheeler observed; primordial channel = anchoring of the Z_3 walls on the nodal surfaces: strings are born in the rails, P22 = the theory of entry).

The epistemology (the honesty trajectory). V4 claimed broadly; V5 introduced the honest audits (JWST, NICER, NANOGrav; falsifications F1–F3 named; the F4 candidate, the reabsorption of the 5.6 %, *dissolved* at the upstream audit); V6 assumes the complete position: a *topological cartography* of the spectrum at ∼1 % (not a precision theory), ∼25 load-bearing results (the assignment null rules out the rest — including, quantified 2026-07-17, the entire EW *quadruplet*: $\sim 8 \times 10^5$ expected equivalents, compatibility), indistinguishable from ΛCDM+GR in the tested regimes, distinctive tests *future* ($0\nu\beta\beta = 0$, $\Sigma m_\nu \approx 66$ meV, finite τ_p, DM bipartition). The best hint remains Koide ($Q = 2/3$ to 9.2 ppm, m_τ to 61 ppm), published (DOI 10.5281/zenodo.20817001); its ingredients are now *carried* — $A = \sqrt{2}$ by the direction doubling ($d_{1/2}(k{=}2)$) and δ decomposed $\pi/12 - \varepsilon$ (conditional, §9.5) — and counted as *hints* out of prudence.

Preamble — the kinematics of the demonstration

This preamble unrolls the logical chain of TQNT, from primitives to observables — results and their formulas, no digressions. Statuses: [derived], [theorem], [conditional] (derivation under declared identifications), [identified], [calibrated], [order].

0. Primitives. A single field χ (Chern-Simons-type SU(2) connection, level k) on the undulating spacetime weave — a gear graduated in *action*, of pitch h. Two transmutation scales: the rigidity $M_{\rm Pl}$ (primitive) and the string tension $\sqrt{\sigma_\chi} = 418.47$ MeV [calibrated]. Postulates P1–P23.

1. Spacetime and the quartet. The bare weave: $\Box\chi = 0$ — zero dispersion ⇒ Lorentz kinematics is structural, Dirac is forced (4π ribbon, spin-statistics) [conditional]. The quartet: h = the tooth ($E = hf$ as an identity); c = the meshing (never a step); $M_{\rm Pl}$ = the clock/rigidity;

$$\boxed{G = \hbar c/M_{\rm Pl}^2} \quad \text{(the weave amplitude — the unique equation of } G\text{)}$$

and the weave equation reads without G: $\Omega^2 - 1 = 2\,(m/M_{\rm Pl})(\ell_{\rm Pl}/r)$ [reading theorem].

2. Matter. Rails = nodal lines of the standing undulation; an *entered* wave magnetises and knots at closure: knot = particle; light = the never-entered population ($\mathcal{S} = 0$) [theorem]. Charge:

$$\boxed{Q = \Sigma\chi + \mathrm{Lk}}$$

[derived]; spin = braid arity; antimatter = mirror ($V_{\bar K}(t) = V_K(t^{-1})$).

3. The forces — only two sources. The four fundamental forces reduce to **two sources**: *the χ flux tube* — the **strong** = its tension (σ_χ, confinement); the **electromagnetic** = its low-energy gauge limit ($\alpha = 1/(k{+}2)$, $1/r^2$ range); the **weak** = its reconnection (P15, $\Delta\mathrm{Lk} = \pm 1$ contact — no coupling constant, $g_{\rm topo} = 1$) — and *the depletion of the weave* — **gravitation** = the metric response to deficits ($\Omega^2 = 1 + \kappa_g|\Psi|^2$, $1/r$ range). [structural]

4. The fine-structure constant.

$$\boxed{\alpha = \frac{1}{k+2}, \quad k = 135} \qquad k + 2 = \mathrm{round}\big(\tfrac{8\pi}{3}\,\ell(3_1)\big) = 137$$

— the trefoil fixes k [conditional]; $m_\ell = \frac{3}{4}\sqrt{\sigma_\chi}/(1 + \pi\alpha/2) = 310.3$ MeV (magnetic swelling of the charge) [conditional].

5. Mass.

$$\boxed{m = \sqrt{\sigma_\chi}\,\mathcal{S}}$$

— one scale, one topological-class factor per sector. Derived from it: Regge slope $2\pi\sigma_\chi$, $\Lambda_\chi = \frac{1}{2}\sqrt{\sigma_\chi}$, $E_{Z_3} = \frac{2}{3}\Lambda_\chi$ (m_π at $-0.06\,\%$), $\sigma_{\text{Cornell}} = \sigma_\chi$. Sectoriality mechanised by rail magnetism: rail *time* carries the base mass ($M_{\text{topo}} = \sqrt{\sigma_\chi}\,\ell$), the *sense* carries the generation [mirror-pair theorem]. Score: 89 states, median 0.13 %; extended table 226 states, 1.43 %.

6. Leptons.

$$\boxed{\sqrt{m_k} = \mu\left(1 + \sqrt{2}\cos\left(\tfrac{2\pi k}{3} + \delta\right)\right)}, \quad \mu = \tfrac{3}{4}\sqrt{\sigma_\chi}\,[\text{identified}], \quad Q_{\text{Koide}} = \tfrac{2}{3} \iff A = \sqrt{2}\,[\text{identity}].$$

$A = \sqrt{2} = d_{1/2}(k{=}2)$, carried by *sense* doubling (the two channels of the Z_3 doublet) [conditional]; $\delta = \pi/12 - \varepsilon$: the amplitude node at $3\pi/4$ (= step 9 of the minimal 24-step circuit), the electron quasi-nodal — its lightness = nodal protection [conditional]; $m_e = E^{\text{retarded}}_{\text{Green}}$[peripheral-line bunching, $n{=}3, m{=}+1$] at $\pm 3\,\%$, zero parameters [conditional].

7. The nucleon and gravitation. Nucleon $= 3_1 + 7_2 + \bar{7}_2$; $m(p) = 938.2$ MeV (octet base, 0.0 %); $\Delta m(n{-}p) = 1.293$ MeV [derived in structure; scale $\beta_\otimes$ calibrated].

$$\boxed{m_{\text{base}} = M_{\text{Pl}}\sqrt{\tfrac{12}{7}\,\alpha^{18}}} \quad \text{(the rigidity read through 18 crossings)}$$

$\Rightarrow$ G_{pred} at $-0.05\,\%$ of CODATA; hierarchy: $\alpha_G/\alpha = \frac{12}{7}\alpha^{17}$ [derived]. GPS, Mercury, GW150914: exact.

8. The weak sector. Fermi structure derived from P15 (reconnection): $\Gamma = N\,m^5/(384\pi^3 E^4_{\text{relax}})$, $g_{\text{topo}} = 1$ — no weak coupling constant [derived];

$$\boxed{v = \sqrt{2\pi}\,\alpha^8 M_{\text{Pl}}\,(1 + \alpha/4\pi) = 246.227\ \text{GeV}} \quad (+28\ \text{ppm})$$

[conditional]; $N_c = N_{\text{gen}} = 3 =$ the trefoil's Z_3 [derived]; never a 4th family [theorem].

9. The spectrum. Peripheral torsion: $\Delta M^2 = 2\pi\sigma_\chi$ per turn, $\pi\sigma_\chi$ per half-turn (universal V–P hyperfine, $\pi \to B_s$ at 0.7 %) [derived]; $\Delta M^2 = \sigma_\chi\,\Delta\text{Wr}$ (writhe at the scale's rate) [conditional]; recoupling ladder in closed forms (base 1 = triality; $\times 3 = Z_3$; $\frac{1}{2}$ = Y-junction; excess $= d - 1 = \sqrt{2} - 1$) — 15 heavy baryons at 0.23 % with no anchor; master functional

$$M^2 = \left(\textstyle\sum_i m_i\right)^2 + 2\pi\sigma_\chi\left(L + \frac{n}{2} + 1.38\,n_r\right)$$

— 59 heavy states with no anchor, median 0.58 %. Dated predictions: $\Xi^*_{cc} = 3{,}659$, $\Omega_b(1P) =$ 6,297–6,304, B^*_0/B'_1 centroid 5,665 MeV.

10. Dark matter and cosmology. DM = isolated prime knots, $c \geq 23$ [derived]; masses 1.7–2.2 GeV, EM direct detection out of reach ($\sigma_{\text{eq}} \sim 10^{-48}$ cm²) [derived]; $\Omega_{\text{DM}}/\Omega_b$: fully derived chain, Planck order [order]; $T_c = 62 \pm 3$ MeV [derived]. Friedmann mechanised: $H^2 = \frac{8\pi G}{3}\rho$, $\Omega_{\text{tot}} = 1$ as a *theorem*; $w = -1$ by construction; the indistinguishability from ΛCDM+GR = a weave theorem. η_B, $n_s = 0.965$, A_s: with no free parameter.

11. Counting and falsifiability. **Ten** calibrated global parameters (*nine* if k and m_ℓ are retained) — against nineteen for the Standard Model, zero per particle. On the other side — ranked by the global meta-null test (§B.6: **four results beyond any trials factor** [hyperfine, recoupling, Koide, oscillations; core at 10^{-25} pessimistically], a cluster of mechanism-backed conditionals, the rest as context) — **about 250 reproduced observables** (the 226 spectrum states, median 1.43 %, plus some thirty beyond the spectrum: constants, cosmology, gravitation, lifetimes), and about ten verified *absence predictions* (steriles, 4th family, standard WIMP, GRB dispersion, drift of constants). The most spectacular: v at +28 ppm; m_τ (Koide) at 61 ppm; G at $-0.05\,\%$; the four $K^0/D^0/B^0/B_s^0$ oscillations at $\Sigma|\log| = 0.0008$ dex; the universal V–P hyperfine $\pi \to B_s$ at 0.7 %; 89 states at median 0.13 %; GPS and GW150914 exact; n_s and A_s with no free parameter; **the H_0 and S_8 tensions dissolved into theorems** (mechanised Friedmann, $\Omega_{\rm tot} = 1$; deduced growth $\Rightarrow \sigma_8 =$ the Planck value — the tension cannot be weave physics: prediction, it must regress) — of the same type: $w(z)$ (DESI), the JWST galaxies, the large-angle CMB anomalies and the metrological G-puzzle, dissolved into theorems with predicted regression, and the coincidences $N_c = N_{\rm gen} = 3$, the universality of G_F, the hierarchy $\alpha_G/\alpha = \frac{12}{7}\alpha^{17}$ and the vacuum catastrophe, turned into *identities*; and **one dated prediction already confirmed**: Ω_{cc} (LHCb 06/2026, ∼3,727 MeV, inside the predicted 3,715–3,788 window). Would refute the theory: any detected cosmic curvature; a confirmed $w(z)$; a 4th family or a sterile neutrino; Ξ_{cc}^* away from 3,659 MeV; the absence of the six 1.07–1.64 GeV lines at the foreseen sensitivities; G metrology converging to 6.674.

Reorganization note (V6-new detailed edition, 2026-07-28). This edition reorders the chapters of the V6 monograph along the *demonstrative path*: the spacetime weave first, then the mathematics of knots, matter (mass, spin, charge), gravitation, thermodynamics, the interactions, the dark side and cosmology, and finally the assessment. The content is *identical* to the V6 edition; internal cross-references are hyperlinks (part and chapter numbering differs from the historical edition); the appendices are unchanged.

Part I

The starting point: the spacetime weave

Part II

Introduction

Chapter 1

Vision and emergence chain

Chapter contents

1.1 Overview : from nothingness to particles

The Topological Quantum Knot Theory rests on a single observation : *nothingness is unstable*. Any perturbation of the vacuum generates waves, and some of these waves close back on themselves into knotted loops — topological knots. These knots *are* the elementary particles.

The wave-particle duality, one of the fundamental mysteries of quantum physics, finds here a natural geometric explanation. At low energy, the knot is *tight* : the intrinsic magnetic field of the flux tube compresses the crossings, the knotted structure is so compact that it vibrates as an indivisible whole — this is the **wave**, delocalized and interfering, which passes through the slits and diffracts. At high energy, the knot *loosens* : the energy overcomes the magnetic confinement, the crossings spread apart, the tube unfolds and the topological structure becomes visible — this is the **particle**, localized, with its measurable mass, spin and charge. The particle and the wave are not two different natures : they are the *same* knot, tight or loose.

A single fundamental field, governed by the Chern-Simons action, produces all known physics. The electron is a simple trefoil with 3 crossings (3_1). The u and d quarks are the same trefoil with different vibrational amplitudes — isospin is not a fundamental quantum number but a geometric symmetry. The proton is an irreducible link of three knots ($3_1 + 7_2 + \overline{7}_2$), the only one to simultaneously satisfy the constraints of spin 1/2, charge +1, correct mass and strong stability (finite but enormous lifetime, $\sim 8 \times 10^{33}$ years). This is why the 92 chemical elements exist : a single type of link, two values of the linking number.

The mass of a particle follows the law of its *topological sector*: $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ — a single dimensionful scale (the tension of the χ string) multiplied by a discrete factor fixed by the class (leptons Koide, baryons Regge+Z_3+tensor, mesons GMOR, heavy stoikhion+Cornell, electroweak VEV). This form reproduces 89 states — from the electron (0.5 MeV) to the top quark (173 GeV) — at a median of 0.13 % (engine at référence 21/07). The knot configuration, for its part, carries the *spin* (braid arity) and the *charge* ($Q = \Sigma\chi + \mathrm{Lk}$): "$\sigma_\chi$ *prevails*".

Spacetime does not exist independently of matter : it emerges from the density of the knotted wave functions. Where the knots are dense, time slows down and space curves. Gravity is not a fundamental force — it is a consequence of quantum topology : each knot generates a *quantum gravitational field*, of which classical gravitation is the *large-scale resultant*. The constant G is not a free parameter but a theorem. This emergent metric reproduces all the classical tests of general relativity : deflection of light ($1.75''$), precession of Mercury ($43''$/century), Shapiro delay, GPS correction and Lense-Thirring effect.

Thermodynamics itself is not postulated : temperature, pressure, Boltzmann distribution,

equation of state and phases of matter *emerge* from the metric and the kinetic theory of knots. Equilibrium there is not statistical but *dynamical* — a balance between attraction (gravity and Van der Waals forces) and repulsion (topological exclusion and kinetic pressure), that is the hydrostatic equilibrium $\nabla P = -\rho\,\nabla\Phi$. Ergodicity is moreover excluded : the frozen topology of the knots isolates the sectors, and gravity builds structures (filaments, clusters, voids) instead of uniformizing.

Dark matter is an inevitable prediction : beyond the complexity threshold, knots interact electromagnetically only through the Vassiliev suppression ($\sigma \sim \sigma_{\rm EM}\,\alpha^{c-18}$); the effective $c_{\min}$ of the reservoir is *derived*: $c \geq 23$ (kinetic decoupling + cosmological survival), with predicted geometric masses of 1.7 to 2.2 GeV. They are absolutely stable (lifetime greater than 10^{347} seconds), and their cosmic abundance is fixed by the kinetics of Big Bang condensation. The TQNT catalogue identifies **117 explicit candidates** : 6 *strict* ones (combinations of amphichiral knots 4_1, 6_3, 8_3, intrinsically neutral, reference masses 1.07 to 1.64 GeV — rederived 2026-07-15, ACPR ropelengths 22/07; the Ω_{ts} values 1.55–51.8 GeV are obsolete) and 111 *realizable* ones (links whose charge becomes zero through a physically realistic integer linking Lk, $\sim$ 1–3 GeV under the référence).

At the heart of black holes, the singularity of general relativity disappears. The metric never vanishes : it decreases exponentially. The knots are not destroyed — they are frozen in an infinitely dilated space, forming a *frozen-knot star*. Information is preserved, and $E = mc^2$ takes on a new meaning : mass is topologically confined energy. To break a knot is to release all this energy as radiation — this is **topological deconfinement**.

An honesty of principle. This vision is ambitious ; it must be judged on the evidence. In all observational regimes tested to date — JWST early galaxies, NICER mass–radius, NANOGrav gravitational wave background — TQNT is *indistinguishable* from the Standard Model, general relativity and Λ-CDM : it **reinterprets** what these frameworks describe rather than contradicting them. What it brings that is new are the *structures* and the *orders of magnitude* derived from topology (Chern–Simons flow, $\mathbb{Z}_3$ symmetry of the trefoil, chirality) ; the *fine values* remain calibrated or open. Its most solid result — a *parameter-free* prediction confirmed to 61 ppm — is the Koide relation. And it is **falsifiable** : the chapters that follow scrupulously separate what is *derived*, what is *postulated* and what remains *open*.

1.2 The emergence chain

The Quantum Knot Theory (TQNT) rests on a unique logical chain linking pure geometry to particle physics. Each link of this chain is a known mathematical theorem or an explicit construction that we will detail in the following chapters. Here is this chain in its entirety :

$$\boxed{F_A = 0 \xrightarrow{\text{flat conn.}} A_K(L,M) = 0 \xrightarrow{\text{CS mass}} m_K \xrightarrow{\text{KG}} (\Box + m_K^2)\Phi = 0 \xrightarrow{\text{NR lim.}} \hat{H}\Psi = E\Psi} \tag{1.1}$$

Let us describe each step.

Step 1 : Chern-Simons and flat connections ($F_A = 0$). We consider a principal SU(2) bundle over a 3-dimensional manifold. The Chern-Simons functional [3],

$$S_{\rm CS}[A] = \frac{k}{4\pi}\int_M \mathrm{Tr}\big(A\wedge dA + \tfrac{2}{3}\,A\wedge A\wedge A\big)\,, \tag{1.2}$$

has as its critical points the flat connections, that is, those satisfying $F_A = dA + A\wedge A = 0$. These flat connections on the complement of the knot $M = S^3 \setminus K$ encode all the topological information of the knot.

Step 2 : The A-polynomial ($A_K(L, M) = 0$). The character variety $\mathcal{X}(K) = \mathrm{Hom}(\pi_1(S^3 \setminus K), \mathrm{SU}(2))/\!/\mathrm{SU}(2)$ parametrizes the flat connections. The A-polynomial $A_K(L, M) = 0$ is the algebraic equation that defines the geometric component of this variety in the meridian-longitude coordinates (M, L).

Step 3 : Quantization ($\hat{A}_K$). The AJ conjecture (Garoufalidis [17], 2004) states that the A-polynomial admits a quantization : by replacing the commutative variables (L, M) with operators $(\hat{L}, \hat{M})$ satisfying $\hat{L}\hat{M} = q\,\hat{M}\hat{L}$, one obtains a q-difference operator $\hat{A}_K$ that annihilates the colored Jones polynomial :

$$\hat{A}_K \cdot J_n(K; q) = 0. \tag{1.3}$$

Step 4 : Colored Jones polynomials ($J_n(K; q)$). The colored Jones polynomials $J_n(K; q)$, where n denotes the n-dimensional representation of SU(2) and $q = e^{2\pi i/(k+2)}$, are the fundamental observables of Chern-Simons theory.

Step 5 : The Schrödinger equation ($\hat{H}\Psi = E\Psi$). This is the central conceptual step of TQNT. The massive excitations of the χ field around the knot satisfy the Klein-Gordon equation $(\Box + m_K^2)\Phi = 0$ on $S^3 \setminus K$. In the non-relativistic limit, this equation reduces to the Schrödinger equation with a magnetic confinement potential $V_K(\mathbf{r}) = -\lambda_V\, U_B(K)/d^2$:

$$\hat{H} = -\frac{\hbar^2}{2m}\,\frac{1}{\Omega^2}\,\nabla^2 + V_K(\mathbf{r}), \qquad \Omega^2 = 1 + \kappa_g|\Psi|^2. \tag{1.4}$$

1.3 Founding vision

> *Nothingness is unstable. Its symmetry breaking generates waves. When the flux lines form a knot K in $\mathbb{R}^3$, topology stabilizes the perturbation : it is a particle.*

This vision breaks down into three principles :

1. **The topological vacuum.** The ground state of the universe is a space $\mathbb{R}^3$ (or S^3) endowed with an SU(2) Chern-Simons field in its trivial state. This state is unstable : any perturbation that forms a knot acquires topological protection.

2. **The particle as a knot.** An elementary particle is a knot $K \hookrightarrow \mathbb{R}^3$ of the χ field. Its mass, spin and charge are computed from the topological and geometric invariants of K : Jones polynomial, intrinsic EM energy U_{EM}, torsional saturation ν, Jones amplitude μ and curvature saturation σ.

3. **Spacetime as emergence.** The metric $g_{\mu\nu}$ emerges from the density of the wave functions. Where $|\Psi|^2$ is large, space curves.

1.4 Summary of results

Synthesis at a glance

Mass formula :	$m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (V6, §9.1) : a single scale × a topological-class factor
Mass score :	$m = \sqrt{\sigma_\chi}\,\mathcal{S}$ on 89 $q\bar{q}$/stoikhion states : **median** 0.13 %, 91 % at < 5 % (engine at référence 21/07; baryonic excited states and multiquarks in sectors of their own).
Spin score :	118/122 via P7quater (CG coupling)
Charge score :	$Q_{\text{phys}} = \Sigma Q_{\text{topo}} + \text{Lk} \in \mathbb{Z}$ (theorem)
Nucleon :	$3_1 + 7_2 + \bar{7}_2$, $m_p = 938.2$ MeV (0.0 %), $\Delta m = 1.293$ MeV (harmonic form derived, scale $\beta_\otimes$ calibrated; 0.1 %), $r_p = 0.841$ fm
Lifetimes :	34 unstable, RMS 0.94 dec (Fermi/Sargent + topological Q, SM constants)
Dark matter :	117 candidates (6 strict + 111 realizable), $\sim$ 1–3 GeV (main document, 2026-07-15)
Plasticity (P23) :	plasticity of the embeddings (dissipative attractor); magnetism of materials; *no calibrated constant* (C_{plast} archived)
Range :	0.5 MeV to 173 GeV (5.5 orders)
Catalogue :	1 539 configurations, $3 \leq c \leq 30$
Parameters :	10 calibrated + 2 hints (η eliminated, v derived ; m_s demoted 22/07, ACPR audit), 0 fitted per particle (free assignment)

1.5 Document plan

- **Part II : Mathematics of knots.** The fundamental topological invariants : knot theory, Jones polynomial, A-polynomial, Chern-Simons algebra and AJ conjecture.
- **Part III : Emergent physics.** The χ field, magnetic confinement, emergent spacetime, the derivation of the Schrödinger equation, spin via the triplet (ν, μ, Q) and the excitation ℓ, charge via σ and Lk.
- **Part IV : Particle spectrum.** The universal mass formula $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (V6; historical V4 $\alpha_0\, U_{\text{EM}}\, c_J^{3.55}$), the nucleon $3_1 + 7_2 + \bar{7}_2$, the harmonic linking potential, the lifetimes, and the $\sqrt{\sigma_\chi}\,\mathcal{S}$ score on 89 states (median 0.13 %, engine at référence 21/07; Ω_{ts} archived out of the text, cf. foreword).
- **Part V : Postulates, limitations and perspectives.** The postulates, the counting of parameters, the key predictions.

Part III

Emergent physics

Chapter 2

From nothingness to the χ field

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2.1 Nothingness is unstable

Postulate P1 — Nothingness is unstable

Nothingness — the absence of any structure — is a topologically unstable state. Any infinitesimal perturbation of the vacuum grows and becomes structured.

2.2 The waves of nothingness

Postulate P2 — Waves are aberrations of nothingness

Perturbations of nothingness propagate as waves in $\mathbb{R}^3$. These waves are the fundamental excitations of the SU(2) Chern-Simons χ field.

2.3 Classification by the Jones polynomial

Postulate P3 — Classification by the Jones polynomial

Topological knots are classified by their Jones polynomial $V_K(t)$. The Jones polynomial distinguishes chirality : if $V_K(t) \neq V_{K^*}(t)$, then K corresponds to matter and K^* to antimatter.

2.4 The χ field and Chern-Simons theory

Postulate P10 — The χ field

The fundamental field χ is an SU(2) connection field whose dynamics is governed by the Chern-Simons action :

$$S[\chi] = \frac{k}{4\pi} \int_{\mathbb{R}^3} \mathrm{Tr}\big(\chi \wedge d\chi + \tfrac{2}{3}\, \chi \wedge \chi \wedge \chi\big)\,. \tag{2.1}$$

Particles are the topological boundary conditions (knots) of this field.

2.5 Confinement by the intrinsic magnetic field

The intrinsic magnetic field $B(s) = \tau_\perp(s)$ of the knot confines the χ field in the vicinity of the curve. The total magnetic energy is :

$$\boxed{U_B(K) = \oint_K \tau_\perp(s)^2 \, \mathrm{d}s} \tag{2.2}$$

Algebraic shortcut c_J. The magnetic energy U_B is correlated with the computable index c_J :

$$c_J(K) = \mathrm{span}(V_K) + \gamma_J \, |V_K(-1)| \qquad (\gamma_J = 0.1) \tag{2.3}$$

The empirical correlation is $c_J \approx 2.4\ U^{0.29}$ ($R^2 = 0.71$, $\rho_{\text{Spearman}} = +0.77$). The index c_J has **no direct physical role** : it is a convenient *algebraic proxy*.

Result : the B field is the source of confinement

The two-term structural decomposition isolates the contribution of the magnetic field :

$$c_J \approx 0.19\ n_{\text{osc}} + 5.41\ \langle|\tau_\perp|\rangle + 3.49 \tag{2.4}$$

with $R^2 = 0.68$. The dominant term is $5.41\ \langle|\tau_\perp|\rangle$: it is indeed the **mean torsion** (B field) that controls confinement.

The causal chain :

$$\text{topology of } K \;\longrightarrow\; \tau_\perp(s),\ \kappa(s) \;\longrightarrow\; U_B = \oint \tau_\perp^2 \, \mathrm{d}s \;\longrightarrow\; \text{confinement}$$

Reservation (2026-07): the numerical correlations above were established on the historical parametric curves of the catalogue, 14/19 of which turned out to be *topologically wrong* (they realize a trefoil or the trivial knot). The canonical geometric functionals are now those of the intrinsic table below (§2.6); the index c_J itself is *not* affected (it is polynomial: $\mathrm{span}\, V_K + \gamma_J |V_K(-1)|$, curve-free).

2.6 Intrinsic geometric functionals (ideal embeddings, 2026-07)

The Frenet functionals depend on the embedding; the *principled* value is that of the **ideal embedding** (ropelength minimiser under the tube constraint). The dedicated tooling (SONO + graduated reheats + native twist/flat seeds; validations: trefoil at +0.8 % of the published value, Hopf link at +0.07 % of the Cantarella–Kusner–Sullivan theorem; Alexander guards on the seed and on the ideal) allowed the complete re-measurement (`simulation/experiments/ukappa_ideal_campaign.py`):

knot	ρ (vs published)	$TC = \oint \kappa \, \mathrm{d}s$	$LU = L \oint \kappa^2 \mathrm{d}s$	$CS = TC^2$	Wr
3_1	33.0 (+1 %)	18.5	358	344	+3.46
4_1	44.5 (+6 %)	26.1	727	683	−0.10
5_1	49.7 (+6 %)	30.9	1015	953	+6.50
5_2	54.7 (+5 %)	31.3	1050	980	+4.43
6_1	65.8 (+16 %)	38.6	1584	1487	+0.95
6_2	63.9 (+13 %)	36.5	1412	1334	−3.28
$6_3^\dagger$	67.7 (+24 %)	37.2	1481	1384	−0.05
7_1	62.9 (+2 %)	36.8	1446	1351	+9.32
7_2	73.8 (+12 %)	41.4	1830	1716	+5.51
8_1	83.9 (+12 %)	45.2	2190	2039	+1.99
8_2	82.8 (+11 %)	46.7	2344	2182	−6.25
8_3	76.7 (+2.8 %)	45.1	2173	2034	−7.24
9_1	77.3 (+2 %)	45.6	2224	2075	+12.21
9_2	91.6 (+15 %)	49.3	2578	2431	+6.58
10_1	101.9 (+13 %)	52.5	2954	2752	+3.14

Scale-invariant quantities (ρ, TC, LU, CS, Wr); tube-radius units. Published: Cantarella–Kusner–Sullivan, Pieranski. $^\dagger 6_3$: plateau +24 %; robust writhe verdict (*symmetric* amphichiral, 8 constructions $|Wr| \leq 0.5$ relaxing toward 0 — contrast with the broken 8_3).

Lessons of the intrinsic table

(i) **Metrology validated**: $Wr(3_1) = +3.46$ (literature ≈ 3.42); $Wr(4_1) \approx 0$ (amphichiral); the Cauchy–Schwarz proxy $CS/LU = 0.93$–0.95 everywhere. *(ii)* **The historical catalogue's torus knots were mirrors** ($5_1, 7_1, 9_1$: writhe of inverted sign): any *chiral* reading made on those curves must be redone. *(iii)* The topologically wrong curves underestimated LU by a factor ~ 3. *(iv)* **The 8_3 (EW knot) — CORRECTED 2026-07-22**: the "spontaneous breaking" $Wr = \pm 7.2$ measured here (eight constructions, reheats) was an **artefact of non-converged minima**: at the true ideal (katlas base, ropelength verified ACPR at 0.008 %), $\mathrm{Wr}(8_3) = 0.003$ (converged to the 4th digit, `scan_fonctionnelles_habillees.py`) — the 8_3 is as symmetric as 4_1 (0.000) and 6_3 (0.005). The "speculative consequence" (an EW writhe degree of freedom) is *withdrawn*; the F3 smearing that leaned on it is re-quantified without it (the lift holds, § audits).

2.7 Energy-dependent confinement

Definition 2.1 (Energy confinement)**.** *The energy-dependent confinement index is :*

$$\boxed{\rho(K, E) = \frac{c_J(K)^2}{c_J(K) + E/E_0}} \tag{2.5}$$

where $E_0 = 1.0$ is the loosening energy scale.

Proposition 2.1 (Properties of $\rho(K, E)$)**.**

1. ***Low-energy limit*** *:* $\rho(K, 0) = c_J(K)$.
2. ***Characteristic energy*** *: for $E = E_K \equiv E_0 \cdot c_J(K)$, $\rho(K, E_K) = c_J/2$.*
3. ***High-energy limit*** *:* $\lim_{E \to \infty} \rho(K, E) = 0$.
4. ***Monotonicity*** *:* $\rho(K, E)$ *is strictly decreasing in E for $c_J > 0$.*

2.8 Energy loosening and wave-particle duality

Postulate P8 — Energy loosening

Energy loosens the knot : at low energy, $\rho(K, E) \approx c_J$; at high energy, $\rho(K, E) \to 0$. Wave-particle duality is a consequence of the energy dependence of magnetic confinement.

The ratio $\rho(K, E)/c_J = c_J/(c_J + E/E_0)$ goes from 1 (fully wavelike) to 0 (fully corpuscular) according to a Lorentzian law.

2.9 Table of confinement indices

Knot	span(V_K)	$\lvert V_K(-1)\rvert$	$c_J(K)$	E_K
0_1	0	1	0.1	0.1
3_1	3	3	3.3	3.3
4_1	4	5	4.5	4.5
5_1	5	5	5.5	5.5
5_2	5	7	5.7	5.7
6_1	6	9	6.9	6.9
6_2	6	11	7.1	7.1
6_3	6	13	7.3	7.3
7_1	7	7	7.7	7.7
7_2	7	11	8.1	8.1
7_3	7	13	8.3	8.3
7_4	7	15	8.5	8.5
7_5	7	17	8.7	8.7
7_6	7	19	8.9	8.9
7_7	7	21	9.1	9.1
8_1	8	13	9.3	9.3
8_2	8	17	9.7	9.7
8_3	8	17	9.7	9.7
9_1	9	9	9.9	9.9
10_1	10	17	11.7	11.7

2.10 Magnetic confinement potential

Postulate P4 — Magnetic confinement

The intrinsic magnetic field $B(s) = \tau_\perp(s)$ of the knot K confines the χ field. The confinement potential is :

$$V_{\text{mag}}(\mathbf{r}, K, E) = -\frac{\lambda_V \cdot \rho(K, E)}{d(\mathbf{r}, K)^2} \tag{2.6}$$

with $\rho(K, E) = c_J(K)^2/(c_J(K) + E/E_0)$.

Chapter 3

The spacetime weave

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3.1 The continuous undulation: everything derives from it

Terminology (2026-07-23): "the spacetime weave" replaces "the substrate" of earlier versions (French: « la trame » replaces « le socle »). "Substrate" wrongly suggested a floor beneath things; the spacetime weave is the ***volumetric medium*** *filling all three dimensions — one is not resting on it, one is* woven *of it (the ocean and its waves). Technical identifiers (`chap:socle`, `sec:*socle*`, script names, the S4′/S4″/S4‴ designations) keep the old name.*

The starting point of all the physics of the framework is a single object: the **spacetime weave** — spacetime itself as a *continuous undulation* (the wave background of the χ field, level α^0), graduated in *action* (each tooth = h). Nothing is posited beside it; **everything derives from it**:

- **light is a wave because the spacetime weave is undulatory** — it meshes onto it ($v = \lambda f = c$ for every mode: the limit c is the spacetime weave's meshing, not a dynamics);
- **knots form because the spacetime weave is undulatory** — an undulation can close back on itself; the closed, *knotted* modes are the particles (P1), and their mass is the spacetime weave's confined frequency ($m/M_{\mathrm{Pl}} = \omega_0/\omega_{\mathrm{spacetimeweave}}$);
- **matter weighs because it confiscates the undulation** — at a knot's location the spacetime weave's amplitude is strongly reduced; the neighbours carry the deficit (compensation): the gradient is the QGF, and mass is the *charge* of the depletion field (the S4′ reading);

- **time is the spacetime weave's local frequency** — near a mass the vertices spread apart: time slows (GPS: 45.7 μs/day, exact);
- **relativity and quantum mechanics are readings of the spacetime weave** — Lorentz is the symmetry of $\Box\chi = 0$ (zero dispersion, structural); $E = hf$ is the graduation itself; spin is the ribbon's 4π doublet; second quantization arises from the persistence of the vacuum (P22).

The demonstration unfolds in this chapter in four stages: *the ontology and the quartet of constants* (h the tooth, c the meshing, $M_{\rm Pl}$ the clock, G the bare spacetime weave's amplitude — §3.2); *the identity card* (time, spin, equivalence, crushing — §3.3); *relativistic kinematics and Dirac, derived* (§3.5); *second quantization* (§3.6). The spectroscopic applications — the torsion of the peripheral vertices and the mass equation — are in the spectrum chapter (§14.2, §14.3): that is where the spacetime weave *yields the masses*.

3.2 The undulating spacetime weave and the Planck constant

Emergent spacetime (P5) is not a smooth background: its *spacetime weave* is the χ **wave background** — an *undulating* space (the α^0 level of the crossing hierarchy, §17.7). Energy transmission (quanta) is only possible at the **crests** of the undulation, amplified by particles; at the **low point**, no transmission. This ontological layer, verified on four consistency checks (`socle_ondulant_coherences.py`), grounds several constants of the framework.

Inheritance theorem ($\hbar$). $\hbar$ enters the theory as the quantum of the χ field (Chern–Simons quantisation, level $k = 135$); the P5 metric emerges from $|\Psi|^2$; hence every vibration of emergent spacetime is *automatically* quantised in $\hbar$ — the **elementary wavelet** (minimal metric perturbation) is one χ quantum, of action exactly h. No circularity: quantisation lives at the field level, spacetime inherits it.

The gear is graduated in action: $E = hf$ is the graduation. The spacetime weave gear is *not* graduated in length — it is graduated in **action**: every tooth is worth h, for all modes (there is no universal spatial mesh). A mode passes one crest per period $\Rightarrow E = h/T = hf$: the Planck relation *is* the graduation, read mode by mode (status: identity — the picture explains the universality of h and the broadband character; it does not derive its value). Structural corollary: an action graduation is Lorentz invariant by construction (h is an invariant) — no ether, and *zero* vacuum dispersion predicted at every order (naive spatial lattices are excluded by gamma-ray bursts, $E_{\rm QG} > 9.3\,E_P$).

The gear and the c limit. Light meshes with the spacetime weave: every mode advances one crest per period, $v = \lambda f = c$ for any frequency (linear dispersion); no transmission *between* crests $\Rightarrow$ no signal exceeds c. The limitation of the speed of light is the gear, not a dynamics.

The limiting tooth: where gravity enters. The gear has no universal frequency (action graduation), and light — scale-free — can measure no spatial mesh: the minimal cell comes from the *self-consistency of amplification* (S4): a cell of size λ carrying one quantum of its own vibration self-amplifies by $\sim G\hbar/(c^3\lambda^2)$, and the amplified crest closes on itself at

$$\lambda = \sqrt{\frac{G\hbar}{c^3}} = \ell_P \qquad \Longrightarrow \qquad f_{\rm tooth} = \frac{c}{\ell_P} = 1.855 \times 10^{43}\ \text{Hz}. \tag{3.1}$$

The Planck frequency is that of the spacetime weave's **limiting tooth**, not of the gear. The *rigidity* of the metric connection ($M_{\rm Pl}$, §17.7) receives its mechanical reading: the maximal

fineness of the toothing; the $\ln(M_{\rm Pl}/\ldots)$ of the freeze equations count the teeth between the limiting tooth and the process (§26.7.8). *Epistemic caveat*: the metrological prediction on G tests the G *chain*, not the spacetime weave ontology.

Wavelet exchange: $\varepsilon = \pi$ derived. An exchange between two structures at distance b requires a spacetime weave mode with a crest *at both ends* (the low point does not transmit); the minimal mode is the fundamental $\lambda = 2b$, and one quantum carries $\Delta p = h/(2b) = \pi\hbar/b$. The coupling is *universal*: anything carrying energy lives on the metric.

Amplification by the particle (S4). The particle amplifies the local undulation: $\Omega^2 = 1 + \kappa_g|\Psi|^2$ — the QGF (§16.1) is the field of the amplified crest, gravity the collective far field. The vision joins P5 without modification. Two derived consequences (conditional on the ontology): the serpentine amplitude of a photon, $A = \lambda/2\pi$ (from its spin $\hbar$); and the minimal exchange $\Delta p = \pi\hbar/b$ (above).

Statuses and inventory. The architecture rests on **two primitives** — h (the tooth value) and G (the amplification rate, derived from $\alpha_{\rm EM}$ and $m_{\rm base}$ in chapter 17) — and the statuses are declared: (S1)–(S4) are *identifications* (the ontology); $E = hf$, Lorentz invariance and the c limit are *readings* (mechanical relocations of primitive facts, not derivations from nothing); $A = \lambda/2\pi$ and $\Delta p = \pi\hbar/b$ are derivations *conditional* on the ontology; the limiting tooth is the self-consistency of the two primitives. The spacetime weave is an *interpretive layer*: it modifies no equation of the framework; its value is picture unification (four primitive facts become four faces of one mechanism) and a robust absence prediction (zero vacuum dispersion at every order, zero holometer noise, zero spontaneous collapse — all confirmed, all falsifiable). Open question that would make it *discriminating*: a setup in which crest exchange ($\Delta p = \pi\hbar/b$) would measurably differ from standard quantum mechanics; failing that, the spacetime weave is indistinguishable by construction — consistent with the epistemological position of the framework (§B.6).

The status of the constants (the framework quartet).

Constant	Role in the spacetime weave	History
h	the tooth (action graduation)	absolute invariant
c	the meshing of the spacetime weave *itself*	preexists with the spacetime weave — *never a step*
$M_{\rm Pl}$	the clock / rigidity	preexists (anchor of the metric connection, §17.7)
G	the *bare* spacetime weave's amplitude — the response to deficits (S4′ below)	born in *one step* at t_c (§26.3)

G is the only member of the quartet that depends on something that *forms* (the wells, hence the knots) — whence its step. c is the causal structure of the χ field, present in the kinetic term of *every* equation of the framework: no degree of freedom of the theory can make it vary, and a $c(t)$ would define a frame at the transition (an ether), excluded by the action graduation. *Topological lock*: $\alpha_{\rm EM} = 1/(k+2)$ with $k = 135$ an *integer*: no continuous drift is possible — and zero drift is observed (atomic clocks $< 10^{-17}$/yr, Oklo $< 10^{-7}$ over 2 Gyr, quasars $< 10^{-5}$, CMB/BBN at the % level), plus the direct nulls (gamma-ray-burst simultaneity over Gyr, BBN at standard rates): the spacetime weave *predicts* these nulls (exact meshing at every epoch). Finally, the cosmological chains of the framework are insensitive by construction: their criteria are dimensionless ratios (a global rescaling of c is reabsorbed; only a *differential* c would leave a trace — dispersion, which is null).

The S4′ reading: depletion and compensation (2026-07-21). The amplification mechanism is refined (user vision, verified; `verif_depletion_compensation.py`, `verif_amplification_annulaire.py`): *(i)* a particle does not sit on *one* tooth — it spans a **multitude** ($r_p/\ell_P \sim 5 \times 10^{19}$ for the nucleon; fitting within a single tooth spacing would require $\lambda\!\!\!^{-}_C \leq \ell_P$, i.e. $m \geq M_{\rm Pl}$: precisely the *limit toothing*; in action, nothing physical carries less than one quantum per period). *(ii)* At its location, **the spacetime weave amplitude is strongly reduced**: the knot is made of *confiscated* spacetime weave wave (mass = confined frequency — the confined = the withdrawn, total deficit = mass), and the neighbours carry the deficit by **compensation**: the gradient is the QGF. *This is P20 receiving its mechanism* ("matter repels spacetime; the exclusion zone generates curvature"). *(iii)* G is **the amplitude of the spacetime weave wave *without* particle**, and κ_g the deficit→metric conversion ($G = \hbar c/M_{\rm Pl}^2$, the unique equation; $\kappa_g(K) = 8\pi\,(m_K/M_{\rm Pl})\,\ell_{\rm P}$); the signs are consistent: depleted spacetime weave $\Leftrightarrow$ Ω^2 increases $\Leftrightarrow$ clocks slow down (P5 bi-conformality); the formula $\Omega^2 = 1 + \kappa_g|\Psi|^2$ is *unchanged* ($|\Psi|^2$ = the confiscated density, tubular — the solver confines on the curve; the far field sees only the monopole, verified). *(iv)* The **double smallness** explains the weakness of gravity: $\Omega^2{-}1$ (proton surface) $= 2\,(m/M_{\rm Pl})(\ell_P/r_p) = 2.95 \times 10^{-39}$ (verified two ways) — confiscated fraction of the spacetime weave × dilution over the teeth: gravity is weak because the spacetime weave is *rich*. Two free consequences: a depletion can only **attract** (no negative amplitude to withdraw — no negative mass); and the step of G at t_c becomes tautological (before knots, nothing depletes). *(v)* **The crushing mechanizes Friedmann (2026-07-24, `calcul_h0_ecrasement.py` + `demontre_monopole_ecrasement.py`)**: the S4′ reading has a *dynamic* side — the undulation displaced by the crushing must go somewhere: it pushes the weave outward. The energy balance of the push (the weave *supplies* the energy of the wells it digs: $v^2/2 = GM(R)/R$) gives $v(R) = \sqrt{8\pi G\rho/3}\,R$ — **the flat Friedmann law, mechanized**: expansion is no longer an initial condition, it is the weave's push; and $\Omega_{\rm tot} = 1$ is a *theorem* (push and pull are the same depletion field — the "zero-energy universe" receives its mechanism; any detected curvature would refute). Scope *strictly global*: the whole universe is the only region with no exterior (zero boundary energy exchange); perturbations ride this background exactly as in general relativity (Birkhoff, $v^2 = 2GM/R + 2E$) — zero conflict with structure growth, redshift distortions or lensing. *Consigned guard-rail*: the *local-instantaneous* sourcing version ($H^2 \propto \rho$ of the volume, region by region) is **refuted** (wrong sign for a void, and parabolic everywhere = marginally bound everywhere: structures would never form) — never reintroduce it; in particular it does *not* amplify the local response of H_0 (the ledger's position — $H_0 = 67.4$, local tension = systematics or a real void treated in GR — is unchanged). *(vi)* **Scale elasticity and the river theorem (2026-07-24, `formalise_elasticite_echelle.py` + `raffine_h0_rotation_galaxie.py`)**: when the mass withdraws, the pushed weave *springs back* (elastic response); and our rulers being made of the same weave, one cannot measure one's own scale ("scale relativity"). Three mechanizations: *bound systems do not expand* (static crushing pattern → no secular push — "why doesn't the atom expand" receives its mechanism); *zero local drift* (mechanized reading of the protections: Oklo, clocks, GPS); *the redshift is cumulative*, visible only at distance. Item (v) and the elasticity form a **triptych**: global push (Friedmann), local elasticity (bound systems), scale relativity (invisibility) — the push is global *because* only the whole universe never "withdraws". And the **river theorem** unifies it all: the weave responds to the *density* $\rho(x,t)$, not to individual masses — a stationary rotation is a *constant* crushing (the water flows, the level is fixed: $\partial\rho/\partial t = 0$, zero elastic solicitation): bound systems and orbital rotation are the same case ("the weave responds to density, not to motion" — the volumetric echo of the uniform-motion protection). Corollary: *no "weave MOND"* — flat rotation curves remain the dark reservoir's business, never a nonlinear weave (consistent with GR indistinguishability); the rotational channels toward H_0 are null (bar pumping 22 orders below H_0, rotational Unruh $\sim 10^{-30}$ K). Finally, **structure growth follows** (2026-07-24, `deduit_s8_grumelosite.py`): the friction term $2H\dot{\delta}$ of the growth equation *is* the push dispersing the

masses while they collapse — without it, free fall (∼590 kyr at recombination, a single clump); with it, $D \propto a$ (×1100 over 13.8 Gyr): *the push is what keeps* $\sigma_8 \sim 0.8$; and the apparent clumpiness still grows — the halo *held* by the elasticity keeps its density while the background dilutes: contrast $\propto a^3$ (×27 for $z_{\rm form} = 2$), the background's dispersal *manufactures* the clumps' contrast. By Birkhoff, this braked growth *is* the ΛCDM growth: $\sigma_8 = \sigma_8^{\rm Planck}$ *deduced* — with expansion (v), flatness, bound systems, rotation and growth, **the indistinguishability from ΛCDM+GR is no longer an empirical observation: it is a theorem of the weave**.

Remark

Three disciplines to maintain strictly: (i) the spacetime weave is the *spacetime weave*, not quasi-particles — there are no phonons in TQNT, and the plasma quanta are not χ waves: particles live *on* the spacetime weave; (ii) the spacetime weave is the χ *vacuum*, not an ether — and the *action* graduation makes this structural (h is an invariant: no universal spatial mesh exists); (iii) never present the Planck frequency as a "derived quantity" of the spacetime weave (the metrological prediction on G tests the G chain, not the ontology). The application of the wavelet channel to the *friction* of the primordial network is falsified (cosmological criterion, §26.7.8); the foundations above are independent of it.

3.3 The spacetime weave's identity card (S4‴): time, spin, equivalence, crushing

(Consolidated 2026-07-21; scripts `concepts_socle_trio.py`, `calcul_spins_masses_socle.py`*; source draft:* `brouillon_concepts_socle` *§1–5. An* ontological *re-reading of canonical results — P5, Koide, Regge — no new constant; what is new is named in the closing remark.)*

Time: the inverse of the spacetime weave's local frequency. Near a particle the vertices are further apart (the stretching is the other face of the S4′ depletion): local time $= 1/f_{\rm local}$ slows down. Quantitatively this is the g_{tt} reading of the P5 bi-conformality: $\delta f/f = \Phi/c^2$. Everyday test: the GPS gravitational drift $= 45.7\ \mu$s/day *exactly*, zero parameters — the spacetime weave ontology receives its most mundane and most precise test.

Spin at the vertices: a reading theorem. The spacetime weave samples a particle only at its vertices; the axisymmetry of the spinning state suppresses any phase clock there: the rotation *rate* ω is **structurally hidden** — only the *quantized* χ circulation survives. This is a reading theorem, and it explains a transversal fact of the main document: *all* mass laws use J, never ω. What *is* visible: the frame's parity (4π; the -1 sheet at 2π, visibility $\cos(\theta/2)$), and the *value* of j — written in energy in the mass and hence readable by the inverse Regge $j = j_0 + (M^2 - M_0^2)/(2\pi\sigma_\chi)$: **16/16 spins read from the masses alone** (7 meson+baryon trajectories, *the* derived slope), extended to 31/32 over the 225-state table (§14.2).

G from any particle: the equivalence principle re-read. Compensation converts deficit into metric at the same rate for every knot: $\kappa_g(K) \propto m_K$, so G is *invariant* — the anchor-free identity $G = \hbar c/M_{\rm Pl}^2$. One spacetime weave ⇒ one conversion rate ⇒ **the equivalence principle = the uniqueness of the spacetime weave**. The nucleon is special only for *deriving* $M_{\rm Pl}$ (its integer position 17+1 in the per-crossing ladder, §17.7): any particle would do if its scale position were derived.

The crushing of the teeth (guardrail: h invariant). What gets crushed under a particle is *never* h (the tooth, an invariant of the quartet) but the local tooth energy $E_{\rm tooth} = h\, f_{\rm local}$, variable and computed ($\delta f/f = \Phi/c^2$): electron (at its Compton) $(m/M_{\rm Pl})^2 = 1.75 \times 10^{-45}$ —

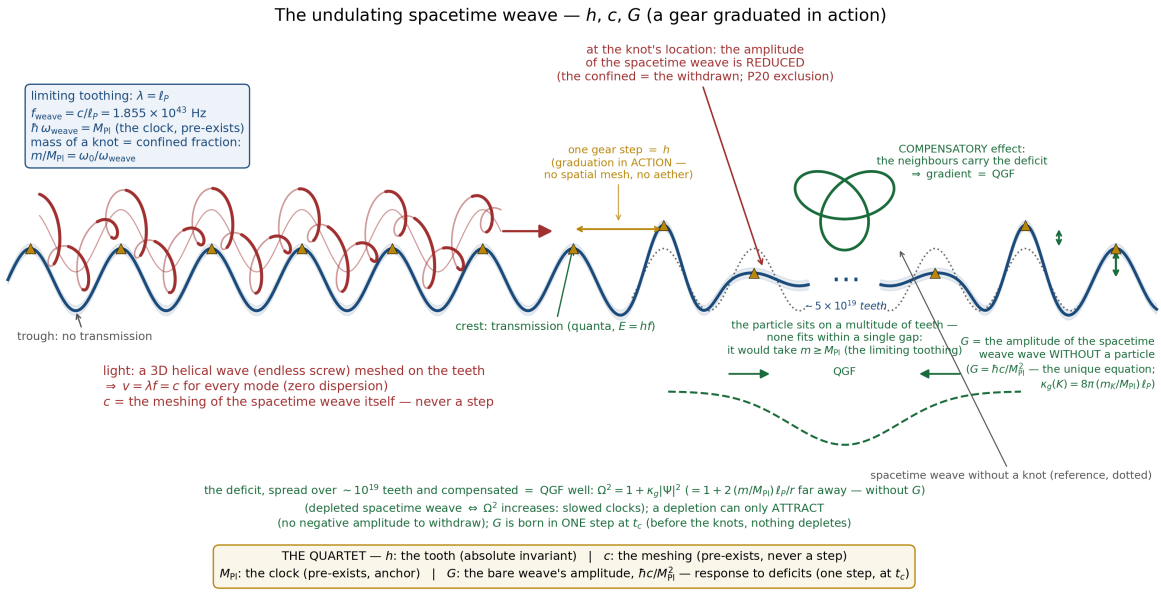


Figure 3.1: The undulating spacetime weave — h (the tooth, action graduation), c (the meshing: light as a meshed helix), G (the bare spacetime weave's amplitude; at the knot's location, depletion over a multitude of teeth and compensation = QGF). The "$\cdots$" break suggests the $\sim 5 \times 10^{19}$ teeth spanned. *Lower panel*: the **rails of the spacetime weave** — nodal lines of the standing undulation ($\pm$ cells in anti-phase); the *entered* wave magnetises and threads; the *free* wave is a spectator (light); closure locks the knot (= the particle). (Panel labels in French.)

the *square* of the confiscated fraction; nucleon 1.48×10^{-39}; Earth (surface) 7×10^{-10}; neutron star ~ 0.17; **black hole (horizon)** 0.5 = **total crushing:** $f_{\text{local}} \to 0$**, the teeth stop** — the main document's "frozen-knot star" receives its spacetime weave reading. The torsion of the neighbouring vertices toward the particle = ∇(crushing) = the QGF ($g = c^2\nabla(\delta f/f) \to$ Newton); free fall = the sum of the micro-torsions over the $\sim 10^{20}$ teeth spanned.

The operational programme: the complete identity card. The derivable core, computed as confiscated fractions of the spacetime weave: leptons via Koide with $\mu = \frac{3}{4}\sqrt{\sigma_\chi}$ and *one* phase — the fitted optimum lands **exactly on** $2/9$ (the main document's hint): $e/\mu/\tau$ at $\leq 0.005\,\%$ (fractions 4×10^{-23} to 1.5×10^{-19}); $m_\pi = \sqrt{\sigma_\chi}/3$ ($-0.06\,\%$); $m_N = 938.2$ ($0.0\,\%$). Spins: ground states = braid arity + 4π signature; excited = read from the masses (inverse Regge). The identity card the spacetime weave carries around each particle — **mass = compensation amplitude; J = frequency (Regge); parity = 4π; ω = hidden by theorem** — is *operational*.

Remark

Statuses: the whole is a *re-reading*, in the spacetime weave ontology, of canonical results (P5/GPS, Koide, Regge, QGF) — no new constant, no modified computation. What *is* new: the reading theorem (ω hidden — making necessary what was only observed: mass laws in J); equivalence = spacetime weave uniqueness; the scale position (17+1) as the nucleon's *only* specialness; the quantified crushings (including the horizon at 0.5). Discipline: never write "h crushed" (h is invariant; only E_{tooth} varies); the Koide phase is *decomposed* $\delta = \pi/12 - \varepsilon$ (conditional, §9.5): the optimum $2/9$ is its reading — counted as a hint out of prudence.

Friction on the spacetime weave and the origin of temperature (study, 2026-07-23). Temperature joins the identity card through a precise split. *(i) The bare spacetime weave*: no friction in uniform motion (protection — zero dispersion, GRB null: no rubbing grain down to $\ell_P/9.3$; corollary: $T_U(g) = \hbar g/(2\pi ck) \approx 4\times10^{-20}$ K, everyday heat is not bare-spacetime weave friction). Bare-spacetime weave friction exists under *proper acceleration* (absolute, frame-free) and its temperature is **Unruh**, $kT_U = \hbar a/(2\pi c)$ — the P22 identity $4\pi/\xi = mc^2/kT_U$ (verified to 10^{-9}); extreme case: the horizon (stopped teeth) $\to$ Hawking. *The framework thus already holds two temperatures that are pure spacetime weave friction. (ii) Matter*: temperature is the parameter of the spacetime weave's wave *content* — the present wave background defines a licit frame (like the CMB); fluctuation-dissipation (Einstein $D/\mu = kT$) locks *one* temperature between drag and kicks (Thomson drag: 7×10^{19} s at 2.7 K, 4×10^{-7} s at 10^7 K); heat = vibrating χ wells (dissipation Γ), the radiative channel (Larmor, spacetime weave modes) carries only $\sim 2\times10^{-10}\,kT$ per collision — matter thermalizes kinetically first. The *emergent* T of the Boltzmann chain (wave background + equiprobability $\to$ dance + T) receives here its microscopic equilibration mechanism. Status: a study (no new constant, no modified computation); discipline: never friction of uniform motion. (`etude_frottement_socle_temperature.py`.)

3.4 The spacetime weave's rails: knotting combinatorics and formation

(Consolidated 2026-07-23; `simu_rails_socle_combinatoire.py`*,* `approfondit_rails_e_p_n.py`*,* `verifie_rails_dm_compat.py`*,* `derive_alpha_cond_rails{,_x10}.py`*,* `derive_alpha_action_intrachemin.py`*,* `demontre_fenetre_capture.py`*.)*

The rails and the knotting combinatorics. The standing-wave spacetime weave has *nodal lines* — intersections of two nodal planes, where the undulation vanishes identically (verified: $u_\perp^2 = 0$ on a rail vs 1.5 off-rail): where a wave threads *without fighting the spacetime weave.*

Three orthogonal families connected at triple points = **the edge network of a cubic lattice** (coordination 6, pitch = half-wavelength): a threading wave = a path on this network. Threading is *phantom* (the wave superposes, free crossings); the knot only *locks* at closure (condensation into a tube). Monte Carlo (Alexander determinant via Wirtinger): $P(\text{knotted}) = 1 - e^{-(N-22)/126}$, first knotting at ∼24 rail steps (minimal lattice trefoil = 24, Diao 1993); **spectrum dominated by the trefoil (59 %)** — the universal core (leptons, nucleon) is the most probable product of threading. *Superposition is necessary*: the self-avoiding subset (already "tube" while threading) essentially never knots at these lengths ($P \sim N/2.5 \times 10^5$) — *the order wave → threading → closure → lock is forced by the combinatorics.*

Production probabilities: electron, proton, neutron. Reinforced tooling (determinant degeneracies broken by the odd part of $|\Delta(2)|$; *exact integer* determinant, Bareiss; exact Gauss linkings). Measurements: $P(3_1) = 0.07$–0.19 per closure (the electron = the generic product; the 3 Z_3 modes cascade to e); $P(7_2) \approx 1.2 \times 10^{-3}$, 50/50 chirality → **the mirror pair 7_2+$\bar{7}_2$ (the nucleon's companion) is natural**; co-threading: $P(\text{linked}) = 0.50$, $P(\text{Lk}_{\text{tot}}{=}{+}3) = 0.056$, raw n/p ratio = 2.0; raw proton assembly $\sim 4 \times 10^{-9}$ ($\sim 3 \times 10^7$ below the electron). *The funnel* (established) turns these rates into the observed hierarchy: P15 cascade to the *floors*, the linking-potential step $\beta(\text{Lk}{-}3)^2\langle\cdot\rangle = 1.29$ MeV $= \Delta m(n{-}p)$ (free consistency), Q conservation forcing $n_e = n_p$.

Compatibility with dark-matter production. The crossing-aggregated spectrum *falls* at $r \approx 0.48$ per crossing — bracketed by $e^{-\alpha_{\text{cond}}} = 0.30$ (per type) and the established net rate 0.68 (with entropy): the rail combinatorics is the microscopic core of the formation weight. A non-zero tail feeds the reservoir ($P(c{=}23) \sim 10^{-7}$ per closure); the instability theorem lets only the *isolated* survive (= the $n{=}1$ reservoir); dominance (annihilation asymmetry) is unchanged. Scales: the rails give the *spacetime weave*, the network kinetics at freeze-out gives the *counting* ($n_{\text{knot}}/L = 0.304/\xi$) — no tension.

α_{cond} **from the rails: the capture-window theorem.** *The mechanism*: at freeze-out, the exponential fragment spectrum (the established cascade) is the action weighting; a type's abundance becomes $A(K) \propto 1/N_0(K)$ (knotting length) and the weight $e^{-\alpha c}$ *emerges* with $\alpha = d \ln N_0/dc$ — measured 1.17–1.36 (soft MC, ±0.15 ensemble systematics). *The theorem* (the directional process: counter-propagating same-rail waves = breaking; a perpendicular *rail* wave magnetically captured at a node = threading (only waves that have *entered* a rail are magnetised — persistent directed current; the free spacetime weave wave, with its oscillating current, is a spectator: light), $\Delta\text{Lk} = \pm 1$; the cycle = the cascade): **[1] the scission carries the loop value** d — the head-on collision of two counter-propagating waves that recombine *closes their history loop*: a Markov closure *performed physically* (the link theorem); **[2] the winding carries the excess** $d{-}1$ **per sense** — an exchange *without* closure: the classical part passes with no event, only the anyonic excess is deposited as ΔLk (the same excess as the link/S4″); two capture senses. Whence passages/scissions $= 2(d{-}1)/d = 2 - \sqrt{2}$ and

$$\alpha_{\text{cond}} = \ln\frac{d}{d{-}1} = \ln\left(2{+}\sqrt{2}\right) = 1.2279 \quad \text{(candidate closed form)}.$$

Exact gas↔algebra equivalence ($\delta = (d{-}1)/2$, the $2\sqrt{2}$ prefactor folds away); $k{=}2$ TL algebra verified ($\text{cl}(1) = d^2$, $\text{cl}(e) = d$, Kauffman consistency at 10^{-15}); confrontations: used 1.1991 (+2.4 %), kinetic derivation $[1.214, 1.220]$ (+0.7–1.1 %, its declared $O(1)$'s), rails MC (1.1σ); downstream: $f_{\text{frac}} = 0.443$ (vs 0.447), branching 0.631/0.369 (vs 0.628/0.372); alternatives killed (naive trace, full excess, inverted assignment, structure-free 3/5 — null). *Status*: a theorem **conditional** on the two identifications (loop↔half-turn rank); **the numerical Ω chain keeps its established values** — the closed form is the theorem-level candidate. If adopted, α_{cond} is the

fourth invariant of the k=2 ribbon (d Koide; $d-1$ excess; $h_{1/2}$ splitting; $\ln(d/(d-1))$ formation) and the Ω chain is anchored in the spacetime weave's algebra end to end.

The origin of entry: a threshold event, never the vacuum (2026-07-23). Why does a wave enter a rail? *Never spontaneously*: CMB photons travel 13.8 Gyr without loss ($\Gamma_{\rm spont} < 2\times10^{-22}$ s^{-1}), and on principle a spontaneous entry in uniform motion would define an absolute frame — Lorentz protection makes the rate *exactly zero* (the dynamical version of the spectator theorem). Entry requires a *threshold event*: depositing one action quantum at the passage to nucleation (the graduation h) *and* closing a neutral topology → **the minimal entry is a knot + mirror pair**: the EM-sector door is $\gamma\gamma \to e^+e^-$ (Breit-Wheeler, threshold $2m_e = 1.022$ MeV, *observed* — STAR 2021); the field-assisted version is Schwinger ($2\pi/\xi$, derived identity): the entry spectrum is the pair spectrum, trefoils first — as the combinatorics above says. *The primordial channel is an anchoring*: the cost of a Z_3 domain wall in the standing background, $C(x_0) = 1+(\beta/4)\cos^2(\pi x_0)$, is **minimal on the nodal plane** — walls anchor on nodal surfaces, their triple junctions on the nodal *lines* = the rails: *at freeze-out, the condensate's strings are born in the rails*; the anchoring statistics is the established gate (Z_3 piercing, exact 8/9 supply) and the latent heat supplies the threshold in bulk. Four channels, one mechanism — freeze-out (anchoring), collision (hadronisation), critical field (Schwinger), horizon (Hawking): **P22 is the theory of entry** ("when an event supplies the action $2\pi/\xi$"). Status: candidate identifications (anchoring = primordial entry; door = pair), no new constant. (`origine_entree_rail.py`.)

Towards $\mathcal{S}$: the stoikhion brick becomes arithmetic. $\mathcal{S}$ as a whole is *not* deducible from the rails (the sector laws are state dynamics; the rails stop at the lock), but three contact points are secured. *(1)* The minimal rail length $N_{\rm min}(K)$ (published minimal polygons: Diao 1993, Scharein et al. 2009: 3_1=24, 4_1=30, 5_1=34, 5_2=36, 6_x=40) is the ropelength's precursor: **tightening constant** $N_{\rm min}/\ell_{\rm ACPR} = 0.713 \pm 1.9\,\%$ over 7 knots — the lock tightens ×1.40 *without changing the ratios*: the light-plateau mass ratios are **ratios of rail integers**. Decisive case: $m_s/m_\ell = 36/24 = \mathbf{3/2}$ exactly, at $-0.7\,\%$ of the ACPR geometric ratio (1.5108) and $-6\,\%$ of the data (1.5961) — the rails give the *geometric* ratio, not the dressed one: the m_s downgrade is *reinforced* by a third route, and a new hint is named (the geometric ratio could be exactly 3/2; the 5_2 ACPR bound is loose, target: tighten it). Other rationals: 4_1: 5/4, 5_1: 17/12, 6_x: 5/3. *(2)* $\mathcal{S}$(base) = 0 is a rail theorem (no closure, no mass). *(3)* The scale m_ℓ is out of reach by the combinatorics' *scale invariance* — one more proof of its transmutation-constant status. (`deduit_S_depuis_rails.py`.)

Magnetisation is an entry property (revision 2026-07-23). Only waves that have *entered* a rail are magnetised: 1D confinement gives a *directed, persistent* current (Biot-Savart around the rail), whereas a free spacetime weave wave carries a current oscillating at $k = \pi/\text{pitch}$ — the **spectator theorem**, quantified: free/rail Neumann coupling $\sim 2\times10^{-3}$ (cell-by-cell cancellation). Three consequences: *(i)* the threading capture is *rail-rail* (the perpendicular mutual inductance vanishes, $d\ell_1\cdot d\ell_2 = 0$; capture proceeds through the *force* in the other's field — a helical torque ±, flipped by the propagation direction: a node's 4 fluxes = exactly the two ΔLk = ±1 channels of the capture-window theorem) — $\alpha = \ln\!\left(2+\sqrt{2}\right)$ *intact*; *(ii)* **light = the population that never entered the rails** (never magnetised → never knotted → never massive): the base sector's $\mathcal{S} = 0$ receives its mechanism, consistent with the zero-dispersion protection — *matter = waves that entered the rails; light = free spacetime weave waves: entry is the transition*; *(iii)* the entry gate = the established Z_3 piercing (8/9, $P_{\rm occ}$ unchanged). *Production robustness*: shifts at the %–10 % level only, in the pro-matter direction (raw n/p 2.2 → 1.6 under mutual coupling: aligned Lk's rise, proton assembly ×3); **the mirror pair is protected by a theorem** (the Neumann weight is *even* under reflection: no chirality bias is possible). *Dark matter*: no number changes; the *persistent conducting-ring circulation* (the

reservoir polarisability's selected mechanism) receives its microscopic origin — the rail current locked in the closed tube; the knot's χ dipole is EM-suppressed (Vassiliev portal); magnetic family selection at formation is *refuted* (flat $5_1/5_2$ ratio under $e^{\kappa U}$): the reservoir's torus sorting remains a *survival* selection (established, confirmed). (`revise_magnetisme_rails.py`, `teste_production_magnetisee.py`, `teste_dm_magnetisee.py`.)

$k = 135$ and m_ℓ from the rails: the trefoil lock and the charge's magnetic inflation (2026-07-25). The capture chain closes the framework's two biggest open items, at the rank of an integer lock plus three declared identifications (`calcul_k135_gonflement_magnetique.py` through `teste_ropelength_k135.py`): *(i)* α is a *vacuum* constant → the phase-cell quantization applies to the *bare* tube; under the identification $m_\ell^{\text{bare}} = \frac{3}{4}\sqrt{\sigma_\chi}$ (the Koide anchor), the bare count is *purely geometric* and the single-valued closure of the phase *locks it to an integer*:

$$k{+}2 = \text{round}\Big(\tfrac{8\pi}{3}\,\ell(3_1)\Big) = \text{round}(137.16) = 137 \qquad (\ell_{L/D}^{\text{ACPR}} = 16.3724)$$

— **the trefoil fixes** $k = 135$; the lock window ($\ell \in [16.293, 16.413]$) is *discriminating* (the rigorous lower bound 15.66 would give 131, outside) and the quantization strain (+0.12 %) has a second-order cost ($\sim 10^{-6}$); *(ii)* the *charge's magnetic inflation*: the capture winding stores one *half-turn* of the ribbon (the universal quantum $\pi\sigma_\chi$), its EM fraction is α (per-crossing portal) and the flux-locked tube response is $\delta R/R = \pi\alpha/2$: the *physical* knot deviates from the bare one by

$$m_\ell^{\text{phys}} = \frac{\frac{3}{4}\sqrt{\sigma_\chi}}{1 + \pi\alpha/2} = 310.30 \text{ MeV} \quad (+0.10\,\% \text{ of the calibrated } 310)$$

— the known tension of the Koide identification (+1.2 %) is *explained*, and Koide is intact (the factor, common to the three leptons, cancels in Q). Honest history: the 17/07 CS-flux candidate ($N = 138.6$, "disfavoured coincidence") used the old $\sqrt{\sigma_\chi}$ and applied to the *charged* tube — the bare/charged architecture resolves it; an intermediate fixed point ($c = \pi/2$ on the charged tube, +0.01 %) was an artifact of that same old $\sqrt{\sigma_\chi}$, consigned. *Status*: integer lock + three declared identifications (vacuum-α; $m_\ell^{\text{bare}} = \frac{3}{4}\sqrt{\sigma_\chi}$; capture = half-turn) — the rank of the framework's conditional theorems; **if retained, k and m_ℓ move from calibrated to derived (counting 10 → 9)**; sharp variant $\ell_{\min}(3_1) = 411/(8\pi)$ ($411 = 3 \times 137$): disfavoured by ridgerunner ($\sim 10^{-4}$), not refuted (lower bound far) — a named external mathematical target.

Rail magnetism and the sectoriality of $\mathcal{S}$ (consolidated 2026-07-27). An *entered* wave is magnetized (the magnetism revision above); per knot, three quantities: the *sense* (chirality of the circulation; reversed ⇒ mirror $\bar{K}$), the *intensity* ($\vec{m} = \frac{1}{2}\oint \vec{r} \times d\vec{l}$, Biot-Savart) and the rail *time* ($T = \ell(K)/c$). *Mirror-pair theorem* (verified exactly, `analyse_magnetisme_rails_categories.py`): in any pair $K + \bar{K}$, the **sense cancels** (pseudo-scalar) and the **time adds** — four established facts fall under this single dichotomy: light never entered ⇒ $\mathcal{S} = 0$; the nucleon's moment carried by the core (the companions $7_2 + \overline{7}_2$ compensate); the Z/H doublet = the two degenerate senses of the sense-free 8_3; the DM conducting ring = current locked by Lk. *Role sharing* (Neumann with ACPR tube, `derive_energie_propre_circuits.py`): inductance grows with complexity ⇒ the fluxoid energy $\Phi_\chi^2/(2L_{\text{ind}})$ *decreases* — magnetism is the *fine*-structure scale (the role it already plays in $\Delta m(n{-}p)$), rail *time* the *base* scale ($M_{\text{topo}} = \sqrt{\sigma_\chi}\,\ell$ reread: the knot that spends more time in the rails weighs more); and *the sense carries the generation structure* — the magnetic reading of Koide, §9.5.

3.5 Relativistic kinematics and Dirac, derived from the spacetime weave

(Consolidated 2026-07-21, `etude_mur_lorentz_socle.py`*,* `etude_pont_spinoriel.py`*,* `analyse_polyakov_champs.py`*; constructive route, Lorentz–Poincaré–Bell lineage declared — the TQNT input: "everything is χ" is not a hypothesis there, it is the ontology.)*

Premises (canonical). (S) every object of the framework is made of χ waves and all dynamics carries c in its kinetic term (c = causal structure, quartet above); (D) structurally null dispersion (S3, action graduation); (P10) χ is a Chern–Simons connection.

The kinematic theorem. (S)+(D) force the unique wave equation $\Box\chi = 0$, and the boost map preserves it (algebraic fact, verified): **the Lorentz group is the symmetry group of the spacetime weave dynamics**, not a spacetime postulate. The whole scalar kinematics follows, computed *in* the spacetime weave frame (machine precision): mass = confined frequency ($m := \hbar\omega_0/c^2$, identitary $E{=}hf$; $m/M_{\rm Pl} = \omega_0/\omega_{\rm socle}$, the nucleon confining the fraction $\sqrt{12/7}\,\alpha^9$ of the spacetime weave); $E = \gamma E_0$, $p = \gamma\beta E_0/c$, $\boxed{E^2 = (pc)^2 + (mc^2)^2}$ — Sargent's m^5 follows (the kinematic borrowing of quantitative P15, §22.4.4, closes); dilation γT_0 (every clock *is* a χ knot); velocity addition. *Why no ether*: among the possible graduations of a spacetime weave, only **action** refers neither to space nor to time separately — the spacetime weave frame is dynamically undetectable; a lattice graduated in *length* would give the LIV dispersion excluded by GRBs ($E_{\rm QG} > 9.3\,E_P$): the GRB null is the *experimental protection* of (D).

The spinorial bridge. *(i)* The tube is framed (Lk = Tw + Wr, §14.15); $\pi_1(\mathrm{SO}(3)) = \mathbb{Z}_2$: the frame states live in the double cover — **spin-$\frac{1}{2}$ doublet** (the same 4π as $g = 2$). *(ii)* **Spin-statistics derived**: exchange $\simeq 2\pi$ rotation for *extended* framed configurations (Finkelstein–Rubinstein-type homotopy — about extension, not a winding, Rule 0), and the braiding–twist axiom of the *ribbon* category (Reshetikhin–Turaev, TQNT's mathematical home) *is* the equality statistics = spin: $P = \theta = -1$, fermion. *Guardrail*: the fermion sign comes from the $\mathbb{Z}_2$ of 3D rotations (k-independent, exact); the CS periphery weight $h_{1/2} = 3/(4(k{+}2)) = 3/548$ is a separate, tiny dressing — the 2+1D anyonic coincidence does not transfer. *(iii)* **Dirac is forced**: derived dispersion + doublet + first order — and the first order is *inherited from P10* (a CS action is first-order by definition; Berry/WZ kinetics, coadjoint orbit $s{=}\frac{1}{2}$, Berry phase $-\Omega/2$ verified). The Clifford algebra is forced; in 2×2 it is *impossible* (the doublet alone cannot carry the mass, full rank); the minimal representation is 4×4 = **Dirac**, and the doubling $2 \to 4$ has its TQNT reading: the blocks = the pair $(K, \bar{K})$ — **the antiparticle is the mirror** (charge conjugation = $V_K(t) \to V_K(t^{-1})$); $g = 2$ from minimal coupling (Gordon), consistent with the topological derivation (§12.2).

The interacting theory. $Q = \Sigma\chi{+}\mathrm{Lk}$ is the U(1) charge of the χ field: local phase invariance forces minimal coupling, and the low-energy EM sector closes onto **QED at derived** $\alpha = 1/(k{+}2)$ (chain spacetime weave → Lorentz → Dirac → minimal coupling → QED; $a_e = \alpha/2\pi$ at 0.026 %). The weak and strong sectors require *no* mediator QFT (derived fusion/splitting contact; linking tension). The χ solver is the *slow limit* of the derived Dirac equation (Pauli reduction): a rest-frame effective theory of a covariant-by-inheritance framework. Loops are *finite* by crest capture (the null log of the $\alpha/4\pi$ dressing, §22.4.4): the spacetime weave is the physical UV completion.

Status and remainders

A **conditional** derivation: (S) is the ontology (not a theorem); (D) is structural (S3) and tested at $9\,E_P$ (GRBs); P10 is a framework postulate. Remain: the full 3D Polyakov computation and the multi-knot creation operator (second quantization is *advanced*: P22 germ verified, P22↔P15 crossing proven, path sum computed in the solvable instance, §3.6); the non-abelian strong dynamics (exploratory). The "Lorentz wall" is no longer a wall: it is a conditional derivation at the framework's common rank.

3.6 The second quantization of χ: P22 germ, crossing, solvable instance

(Consolidated 2026-07-21; `verif_seconde_quantification_p22.py`*,* `demontre_croisement_p22_p15.py`*,* `calcul_somme_chemins_spin.py`*; source draft* `brouillon_seconde_quantification_chi`*.)*

The germ is quantitative. The defining signature of second quantization — number non-conservation from a field vacuum, $|\langle 0|0\rangle|^2 = e^{-\Gamma V t}$ — is *inside* P22: on a constant field via the QGF, $2\pi/\xi = \pi m^2 c^3/(eE\hbar)$, the *exact* Schwinger exponent (the canonical identity, reread as the vacuum persistence amplitude). Around it, everything is in place: the sectorized Fock space (charge superselection); the **derived operator statistics** (braid parity + spin-statistics, §3.5): odd arity → $\{a, a^\dagger\} = 1$, $a^2 = 0$ (Pauli exclusion), even → $[a, a^\dagger] = 1$; the **4-operator vertex** (fusion/splitting = the derived G_F, §22.4.4) with its exact measure ($\int_0^1 2x^2(3{-}2x)\,dx = 1 \to m^5/(192\pi^3)$); and the order counting (n pairs ↔ $1/E^{2n}$) = the perturbative expansion. **Second quantization is operational in the rates.**

The P22↔P15 crossing theorem (WKB order). A single, TQNT-native functional: $S[\gamma] = m\,\mathrm{Len}(\gamma) - e\,\mathrm{Flux}(\gamma)$ (the tube ontology). *Open* path → P15 exchange (barrier $\int |p|dx = \pi m^2/(2eE)$); *closed* loop → P22 nucleation (S_E stationary at $R^*{=}m/eE$, $S_E(R^*) = \pi m^2/(eE)$ = the P22 exponent, deviation 0); and the **open→closed gluing = the mirror** (the half-loop = the barrier integral, machine precision; "antiparticle = mirror" is derived): $\Gamma_{0\to 2} = |A_{2\to 2}|^2$ — *the* crossing relation. A third channel for free: the same loop at the Rindler periodicity gives $mc^2/kT_{\mathrm{Unruh}} = 4\pi/\xi$ — the canonical thermal (Hawking) channel. **The three regimes of P22 are three readings of a single functional.** (Conditions: WKB — exponents; adiabatic field; contact = worldline endpoint.)

The path sum with spin factor — computed (solvable instance). In 1+1D the sum is exactly solvable, and the instance is the *shadow of the spacetime weave*: paths at c between events ((S)+(D)); mass = the confinement's **reversal rate** ($mc^2/\hbar$, the confined frequency); spin factor = a phase i per reversal (quarter frame turn, the shadow of Polyakov's $e^{i\pi \mathrm{SL}}$). Verified: *(i)* exact transmutation — the pure combinatorial sum converges to $J_0(m\sqrt{t^2 - x^2})$, *the Bessel function of the Dirac propagator*; *(ii)* **the phases are fixed** (unitary checkerboard vs continuum Dirac, complex L2 $\propto \varepsilon$); *(iii)* unitarity at 10^{-14}; *(iv)* **the physical zero point is finite**: the fit $E(L) = \rho L + b + c/L$ at fixed regulator gives $c = -\pi/24$ (6×10^{-9}) — the Casimir energy, regulator-independent: *renormalization is a reorganization* (ρ, b absorbed), executed rather than asserted.

Status and remainders

Declared conditionals: the instance is 1+1D (the general 3D case = Polyakov's factor $e^{i\pi W}$ on true knot worldlines, writhe tooled — the solvable instance fixes its form); the lattice

there is a *computational* regulator (the spacetime weave's physical graduation is in action; only regulator-independent quantities are compared). Remain: the full 3D Polyakov computation; the *multi-knot* creation operator (coherent states / collective coordinates — the quantum counterpart of SONO); and the non-abelian strong dynamics (exploratory).

Chapter 4

Emergence of time

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4.1 Time dilation as a consequence of the quantum gravitational field

Time: carried by the weave, slowed by the knot

The spacetime weave carries the beat *everywhere*: local time is the inverse of the weave's local frequency ($1/f_{\text{local}}$, identity card S4‴, §3.3). The knot does not *create* this beat — it *confiscates* it: the depletion spreads the vertices apart and slows f_{local} ($\delta f/f = \Phi/c^2$). The temporal component of the metric is thus generated by the **quantum gravitational field** (QGF, §16.1) self-produced by the knot — *not* by gravitation, which is only its *large-scale resultant*; the time dilation of an isolated knot is therefore *local*. **Without a knot, no *distinct* proper time**: the bare weave beats uniformly (Minkowski), and it is the *relief* of time — the differences in rhythm — that *begins* as soon as a knot appears ($h = \kappa_g|\Psi|^2 > 0$).

The bi-conformal metric (P5) is:

$$\mathrm{d}s^2 = -c^2\, e^{-\kappa_g|\Psi|^2}\,\mathrm{d}t^2 + \Omega^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) \tag{4.1}$$

The temporal factor $g_{00} = -c^2\, e^{-\kappa_g|\Psi|^2}$ implies:

- At infinity ($|\Psi|^2 = 0$): $g_{00} = -c^2$ (Minkowski, no distinct proper time).
- Near the knot ($|\Psi|^2$ large): $|g_{00}| < c^2 \Rightarrow$ time flows **more slowly**.

The knot creates its own internal clock whose rhythm is:

$$\frac{\mathrm{d}\tau_{\text{knot}}}{\mathrm{d}t_\infty} = e^{-\kappa_g|\Psi|^2_{\max}/2} \approx 1 - \frac{h}{2} \tag{4.2}$$

where $h = \kappa_g\,|\Psi|^2_{\max}$ is the **dilation parameter**.

Notation guard. This h is the metric *depletion field* — the same as in $\nabla^2 h = -\kappa_g |\Psi|^2$ (§14.3) — and has nothing in common with the *tooth* h of the quartet (the Planck constant, §3.2): the tooth is never crushed; what varies near a mass is the local tooth energy $E_{\text{tooth}} = h_{\text{Planck}}\, f_{\text{local}}$ (S4‴, §3.3).

4.2 Dilation parameter h

For a knot of mass m, the maximal probability density is $|\Psi_{\max}|^2 \propto (m/\hbar c)^3$ (Compton volume). The dilation parameter is:

$$\boxed{h = \left(\frac{m}{m_P}\right)^2} \tag{4.3}$$

where $m_P = \sqrt{\hbar c/G} = 1.22 \times 10^{22}$ MeV is the Planck mass.

Orders of magnitude.

Particle	m (MeV)	$h = (m/m_P)^2$
e^-	0.51	1.8×10^{-45}
p	938	5.9×10^{-39}
μ^-	106	7.6×10^{-41}
W	80 377	4.3×10^{-35}
t	172 760	2.0×10^{-34}
black hole	m_P	1

4.3 The temporal deficit: from the particle to the black hole

The knot curves the temporal component of the metric (P5): $g_{00} = -c^2\, e^{-h}$ with $h = \kappa_g |\Psi|^2$. The **temporal deficit** $\delta t/t = h/2$ grows with the mass as $h = (m/m_P)^2$ (radius $\sim$ Compton length); at $h \to 1$ ($m \to m_P$), time stops — this is a horizon.

The temporal deficit $h = (m/m_P)^2$

Scale	m	$h = (m/m_P)^2$	**Temporal regime**
Light leptons (e^-)	0.5 MeV	$\approx 2 \times 10^{-45}$	negligible deficit
Nucleon	940 MeV	6×10^{-39}	tiny deficit
Electroweak scale	173 GeV	2×10^{-34}	tiny deficit
Planck	1.22×10^{22} MeV	1	time stops (horizon)

Matching the S4‴ crushing table. The identity card of the weave (§3.3) quotes the same phenomenon as a *deficit* $\delta t/t = h/2$ — whence its "0.5" at the horizon ($h \to 1$) — and evaluates the nucleon at its *physical* radius r_p (depletion field $h(r_p)$) rather than at the Compton scale (1.48×10^{-39} versus 5.9×10^{-39} above): two reading conventions of the same field, not two physics.

The temporal deficit is *not* the lifetime. The mass (hence h) measures the **time dilation**, not the stability. The lifetime is governed by the released energy Q and the decay channel (chap. 15), *not* by the mass: the neutron (940 MeV, $h \approx 6 \times 10^{-39}$) lives **880 s** because its $Q = 0.78$ MeV is minuscule, whereas the muon (106 MeV, h far smaller) lives only 2.2×10^{-6} s because its Q is almost its entire mass. Mass and lifetime are therefore **decoupled**; there is no "mass threshold" of stability.

4.4 Interpretation: time differentiated by topology

1. A wave knot in χ creates a density $|\Psi|^2$.
2. $|\Psi|^2$ curves spacetime: $\Omega^2 = 1 + \kappa_g |\Psi|^2$.
3. The temporal component is dilated: $g_{00} = -c^2\, e^{-\kappa_g |\Psi|^2}$.
4. The temporal deficit $\delta t/t = h/2$ measures the fraction of time "lost" to the dilation.
5. When $h \to 1$ ($m \to m_P$), time stops: this is a black hole.

Without a knot, no relief of time. In the absence of a knot ($|\Psi|^2 = 0$ everywhere), $g_{00} = -c^2$ uniformly: the weave beats at the same rhythm at every point — spacetime is flat Minkowski, without *local* temporal structure. The beat itself pre-exists (the action-graduated weave, §3.2): what matter creates is not time but its **relief** — the point-to-point differences in rhythm, the very ones that clocks measure. *Distinct* proper time emerges only where topology creates a concentration of $|\Psi|^2$. The **threshold is purely topological** (and not energetic): the relief appears as soon as the smallest knot exists — the electron 3_1 ($h \approx 2 \times 10^{-45}$) —, whereas a *free* unknotted wave (neutrino, $h \approx 0$) or the χ vacuum carry *no* proper dilation. The **differentiation of time is a consequence of the existence of matter** (of a knot); its beat is that of the weave, which pre-exists.

Part IV

The mathematics of knots

Part V

Mathematics of knots

Chapter 5

Knot theory: reminders

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5.1 Definition of a knot

Definition 5.1. *A **knot** is a smooth embedding $K : S^1 \hookrightarrow \mathbb{R}^3$ (or S^3), considered up to ambient isotopy.*

The **trivial knot** 0_1 is the standard embedding $t \mapsto (\cos t, \sin t, 0)$.

The **connected sum** $K_1 \# K_2$ of two knots is obtained by cutting each and gluing the ends back together. A knot is **prime** if it is not the connected sum of two nontrivial knots. TQNT focuses on prime knots.

The **complement** of a knot is the space $S^3 \setminus K$. The Gordon-Luecke theorem [7] (1989) states that the complement determines the knot up to isotopy and mirror.

5.2 Reidemeister moves

Two diagrams represent the same knot if and only if they are related by a finite sequence of **Reidemeister moves** (types I, II, III).

Theorem 5.1 (Reidemeister [9], 1927)**.** *Two knot diagrams represent the same knot if and only if they are related by a finite sequence of moves of types I, II and III (and planar isotopies).*

5.3 Crossing number, writhe, orientation

The **minimal crossing number** $c_n(K)$ is the smallest number of crossings among all diagrams of K. The standard notation n_k denotes the k-th prime knot with n crossings.

The **writhe** is $w(D) = \sum_{\text{crossings}} \varepsilon_i$.

5.4 Amphichirality

Definition 5.2. *A knot K is **amphichiral** if K is isotopic to its mirror image K^*.*

Since $V_{K^*}(t) = V_K(t^{-1})$, an amphichiral knot satisfies $V_K(t) = V_K(t^{-1})$.

Among the 20 knots of the catalogue, three are amphichiral : 4_1, 6_3 and 8_3. In TQNT, amphichiral knots represent particles that are their own antiparticle.

5.5 Torus knots

Definition 5.3. *The **torus knot** $T(p, q)$ is the knot obtained by winding a strand p times around the axis of a torus and q times through the hole of the torus.*

The torus knots $T(2, q)$ for $q = 3, 5, 7, 9$ are :

Knot	$T(p,q)$	$c_n(K)$	$b(K)$	$s(K)$
3_1	$T(2,3)$	3	2	1/2
5_1	$T(2,5)$	5	2	1/2
7_1	$T(2,7)$	7	2	1/2
9_1	$T(2,9)$	9	2	1/2

All torus knots $T(2, q)$ have braid index $b = 2$, hence spin $s = 1/2$: they are all fermions.

5.6 Braid groups and the closure theorem

Definition 5.4. *The **braid group** B_n on n strands is generated by the elements $\sigma_1, \ldots, \sigma_{n-1}$ subject to the relations :*

$$\sigma_i \, \sigma_j = \sigma_j \, \sigma_i \qquad \text{if } |i - j| \geq 2, \tag{5.1}$$

$$\sigma_i \, \sigma_{i+1} \, \sigma_i = \sigma_{i+1} \, \sigma_i \, \sigma_{i+1}. \tag{5.2}$$

The **braid index** $b(K)$ is the smallest n such that $K = \hat{\beta}$ for a braid $\beta \in B_n$.

Morton-Williams-Franks bound. Let $P_K(v, z)$ be the HOMFLY-PT polynomial of the knot K. If e and E are the minimal and maximal exponents of v in $P_K(v, z)$, then :

$$b(K) \geq \frac{E - e}{2} + 1. \tag{5.3}$$

This bound is often exact for small knots.

Table of braid indices.

$b(K)$	**Knots**
1	0_1
2	3_1, 5_1, 7_1, 9_1
3	4_1, 5_2, 6_1, 6_2, 7_2, 7_4, 8_1, 8_2, 10_1
4	6_3, 7_3, 7_5, 7_6, 7_7, 8_3

5.7 Knot diagrams

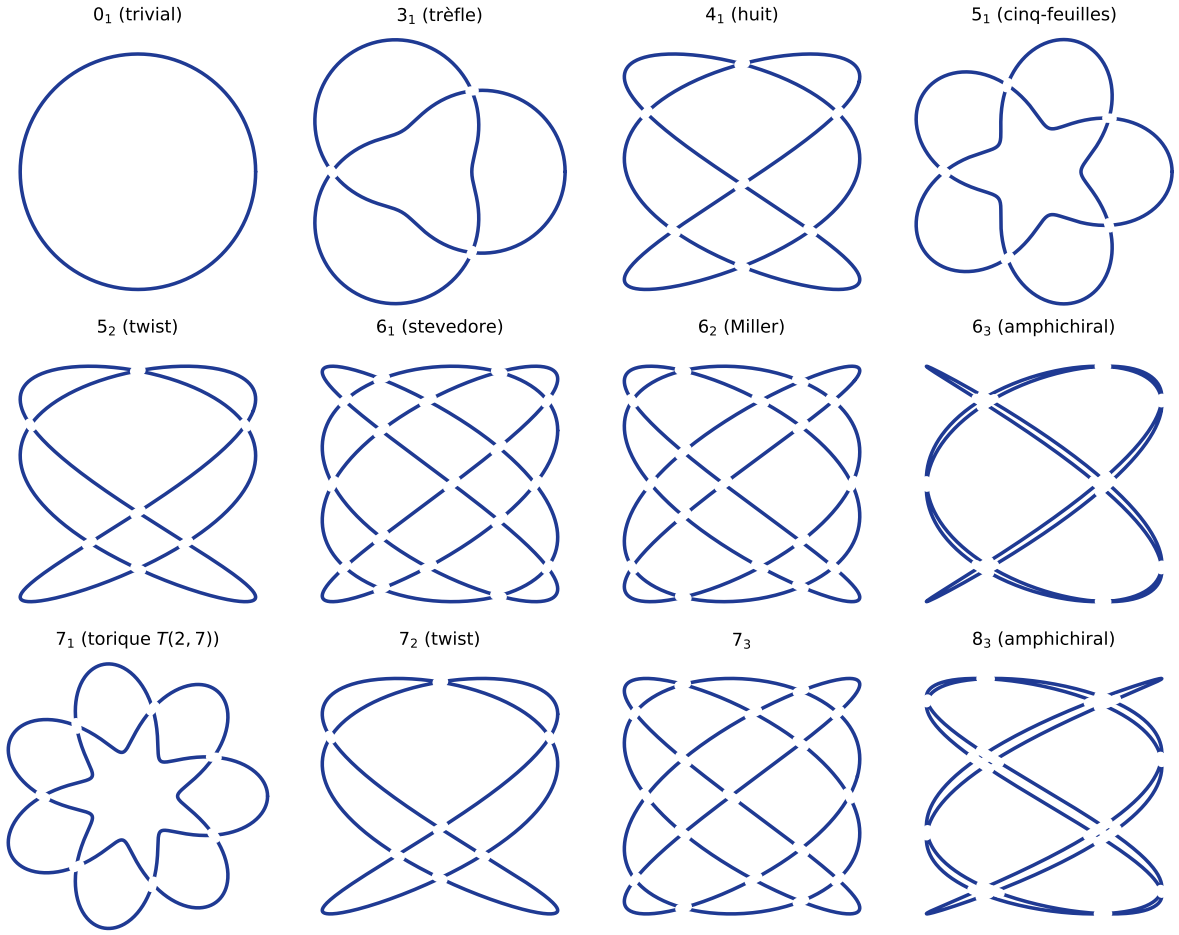


Figure 5.1: Planar projections of twelve knots from the catalogue, including the four main families : torus knots $T(2,q)$ (rose with q petals : 3_1, 5_1, 7_1), twist knots (continued fraction $[k,2]$: 4_1, 5_2, 6_1, 7_2), and more complex knots (6_2, 6_3, 7_3, 8_3). The breaks in the line indicate the undercrossings (strand with the smaller z coordinate). The projections are Lissajous embeddings $(\cos at, \cos bt, \cos ct)$ with parameters chosen to minimise degeneracy. Diagrams generated by `simulation/experiments/knot_diagrams_2d.py`.

Chapter 6

Jones polynomial and catalogue of the 20 knots

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6.1 Definitions and fundamental properties

Definition 6.1 (Jones polynomial)**.** *The Jones polynomial [2] of an oriented knot $K \subset S^3$ is a Laurent polynomial $V_K(t) \in \mathbb{Z}[t^{\pm 1}]$ satisfying the skein relation:*

$$t^{-1} V_{L_+}(t) - t\, V_{L_-}(t) = \left(t^{1/2} - t^{-1/2}\right) V_{L_0}(t) \tag{6.1}$$

with $V_{0_1}(t) = 1$ (trivial knot).

Definition 6.2 (Span)**.** $\mathrm{span}(V_K) = p_{\max} - p_{\min}$. *For alternating knots,* $\mathrm{span}(V_K) = c_n(K)$.

Definition 6.3 (Determinant)**.** $\det(K) = |V_K(-1)|$.

6.2 Mirror knot and chirality

The mirror polynomial is $V_{K^*}(t) = V_K(t^{-1})$. Chirality is central: a chiral K and its mirror correspond to particles and antiparticles, with $Q(K^*) = -Q(K)$ and $c_J(K^*) = c_J(K)$.

6.3 Topological complexity index c_J

$$c_J(K) = \mathrm{span}(V_K) + \gamma_J\, |V_K(-1)| \qquad \text{with} \quad \gamma_J = 0.1 \tag{6.2}$$

This index has **no direct physical role**: it is an algebraic shortcut that parametrises the geometric complexity of the knot.

6.4 Complete catalogue of the 20 Jones polynomials

Knot	$V_K(t)$
0_1	1
3_1	$t + t^3 - t^4$
4_1	$t^{-2} - t^{-1} + 1 - t + t^2$
5_1	$t^2 + t^4 - t^5 + t^6 - t^7$
5_2	$t - t^2 + 2t^3 - t^4 + t^5 - t^6$
6_1	$t^{-2} - t^{-1} + 2 - 2t + t^2 - t^3 + t^4$
6_2	$t^{-1} - 1 + 2t - 2t^2 + 2t^3 - 2t^4 + t^5$
6_3	$-t^{-3} + 2t^{-2} - 2t^{-1} + 3 - 2t + 2t^2 - t^3$
7_1	$t^3 + t^5 - t^6 + t^7 - t^8 + t^9 - t^{10}$
7_2	$t - t^2 + 2t^3 - 2t^4 + 2t^5 - t^6 + t^7 - t^8$
7_3	$t^2 - t^3 + 2t^4 - 2t^5 + 3t^6 - 2t^7 + t^8 - t^9$
7_4	$t - 2t^2 + 3t^3 - 2t^4 + 3t^5 - 2t^6 + t^7 - t^8$
7_5	$t^2 - t^3 + 3t^4 - 3t^5 + 3t^6 - 3t^7 + 2t^8 - t^9$
7_6	$t^{-1} - 2 + 3t - 3t^2 + 4t^3 - 3t^4 + 2t^5 - t^6$
7_7	$t^{-4} - 2t^{-3} + 3t^{-2} - 4t^{-1} + 4 - 3t + 3t^2 - t^3$
8_1	$t^{-2} - t^{-1} + 2 - 2t + 2t^2 - 2t^3 + t^4 - t^5 + t^6$
8_2	$1 - t + 2t^2 - 2t^3 + 3t^4 - 3t^5 + 2t^6 - 2t^7 + t^8$
8_3	$t^{-4} - t^{-3} + 2t^{-2} - 3t^{-1} + 3 - 3t + 2t^2 - t^3 + t^4$
9_1	$t^4 + t^6 - t^7 + t^8 - t^9 + t^{10} - t^{11} + t^{12} - t^{13}$
10_1	$t^{-2} - t^{-1} + 2 - 2t + 2t^2 - 2t^3 + 2t^4 - 2t^5 + t^6 - t^7 + t^8$

6.5 Summary table of invariants

Knot	$p_{\min}$	$p_{\max}$	**span**	$\det(K)$	c_J	$b(K)$	$s(K)$
0_1	0	0	0	1	0.10	1	0
3_1	1	4	3	3	3.30	2	1/2
4_1	−2	2	4	5	4.50	3	1
5_1	2	7	5	5	5.50	2	1/2
5_2	1	6	5	7	5.70	3	1
6_1	−2	4	6	9	6.90	3	1
6_2	−1	5	6	11	7.10	3	1
6_3	−3	3	6	13	7.30	4	3/2
7_1	3	10	7	7	7.70	2	1/2
7_2	1	8	7	11	8.10	3	1
7_3	2	9	7	13	8.30	4	3/2
7_4	1	8	7	15	8.50	3	1
7_5	2	9	7	17	8.70	4	3/2
7_6	−1	6	7	19	8.90	4	3/2
7_7	−4	3	7	21	9.10	4	3/2
8_1	−2	6	8	13	9.30	3	1
8_2	0	8	8	17	9.70	3	1
8_3	−4	4	8	17	9.70	4	3/2
9_1	4	13	9	9	9.90	2	1/2
10_1	−2	8	10	17	11.70	3	1

6.6 Detailed numerical examples

6.6.1 The trefoil 3_1

The Jones polynomial of the trefoil knot is:

$$V_{3_1}(t) = t + t^3 - t^4 \tag{6.3}$$

Span. $p_{\min} = 1$, $p_{\max} = 4$, hence $\operatorname{span}(V_{3_1}) = 4 - 1 = 3$. The trefoil has $c_n = 3$ crossings, consistent with the Kauffman [6]–Murasugi–Thistlethwaite theorem for alternating knots.

Determinant.

$$V_{3_1}(-1) = (-1)^1 + (-1)^3 - (-1)^4 = (-1) + (-1) - (1) = -3$$

whence $\det(3_1) = |V_{3_1}(-1)| = 3$.

Confinement index.

$$c_J(3_1) = \operatorname{span} + \gamma_J \cdot \det = 3 + 0.1 \times 3 = 3.3 \tag{6.4}$$

Topological charge. $Q(3_1) = (1 + 4)/3 = 5/3 \approx 1.667$.

Chirality. The mirror polynomial is $V_{3_1^*}(t) = t^{-1} + t^{-3} - t^{-4}$. Since $V_{3_1^*}(t) \neq V_{3_1}(t)$, the trefoil is *chiral*. The antiparticle has charge $Q(3_1^*) = -5/3$.

6.6.2 The figure-eight knot 4_1

$V_{4_1}(t) = t^{-2} - t^{-1} + 1 - t + t^2$.

Span. $p_{\min} = -2$, $p_{\max} = 2$, hence span $= 4$.

Determinant.

$$V_{4_1}(-1) = (-1)^{-2} - (-1)^{-1} + 1 - (-1) + (-1)^2 = 1 + 1 + 1 + 1 + 1 = 5$$

Confinement index. $c_J(4_1) = 4 + 0.1 \times 5 = 4.5$.

Amphichirality. $V_{4_1^*}(t) = V_{4_1}(t^{-1}) = t^2 - t + 1 - t^{-1} + t^{-2} = V_{4_1}(t)$. The figure-eight knot is *amphichiral*, $Q(4_1) = 0$.

6.6.3 The torus knot 7_1

$V_{7_1}(t) = t^3 + t^5 - t^6 + t^7 - t^8 + t^9 - t^{10}$.

Span: span $= 7$. Determinant: $|V_{7_1}(-1)| = 7$. $c_J(7_1) = 7.7$. Charge: $Q = 13/7$.

Pattern of torus knots. For the $T(2, s)$ with s odd: $\det(T(2, s)) = s$, $\operatorname{span}(V_{T(2,s)}) = s$, $c_J(T(2, s)) = 1.1\, s$. The charge converges: $Q(T(2, s)) \to 2$ as $s \to \infty$.

6.7 Detailed data sheets of the main knots

Knot 0_1 — Trivial (unknot)

Crossings:	$c_n = 0$
Braid index:	$b = 1$, spin $s = 0$
Jones:	$V_{0_1}(t) = 1$
c_J:	0.1
Amphichiral:	Yes
Curve:	$(\cos t, \sin t, 0)$

Knot 3_1 — Trefoil $T(2,3)$

Crossings:	$c_n = 3$
Type:	Torus $T(2,3)$
Braid index:	$b = 2$, spin $s = 1/2$
Jones:	$V_{3_1}(t) = t + t^3 - t^4$
c_J:	3.3
Amphichiral:	No (chiral)
Curve:	$(\sin t + 2\sin 2t,\ \cos t - 2\cos 2t,\ -\sin 3t)$
Role:	Generation 1 (electron, u/d quarks)

Knot 4_1 — Figure-eight

Crossings:	$c_n = 4$
Type:	Twist (amphichiral)
Braid index:	$b = 3$, spin $s = 1$
Jones:	$V_{4_1}(t) = t^{-2} - t^{-1} + 1 - t + t^2$
c_J:	4.5
Amphichiral:	Yes ($K \cong K^*$)
Charge:	$Q = 0$
Role:	Neutral bosons (γ, g, Z^0)

Knot 5_1 — $T(2,5)$

Crossings:	$c_n = 5$
Braid index:	$b = 2$, spin $s = 1/2$
c_J:	5.5
Amphichiral:	No
Charge:	$Q = 9/5$ (bare); isolated $Q_{\text{phys}} = \Sigma\chi{+}\text{Lk} = +3$
Role:	Dressing component of heavy links (e.g. $\tau = 3_1{+}5_1{+}7_1$); *not* the isolated muon (torus reading discarded, cf. §9.5)

Knot 7_1 — $T(2,7)$

Crossings:	$c_n = 7$
Braid index:	$b = 2$, spin $s = 1/2$
c_J:	7.7
Amphichiral:	No
Charge:	$Q = 13/7$ (bare); isolated $Q_{\text{phys}} = \Sigma\chi{+}\text{Lk} = +3$
Role:	Dressing component of heavy links (e.g. $\tau = 3_1{+}5_1{+}7_1$); *not* the isolated tau (torus reading discarded, cf. §9.5)

Knot 7_2 — Twist with 7 crossings

Crossings:	$c_n = 7$
Braid index:	$b = 3$, spin $s = 1$
c_J:	8.1
Amphichiral:	No
σ:	0.00 (neutral)
χ:	0
Role:	Component of the nucleon $(3_1 + 7_2 + \bar{7}_2)$

Knot 8_2

Crossings:	$c_n = 8$
Braid index:	$b = 3$, spin $s = 1$
c_J:	9.7
Amphichiral:	No
σ:	$+1/3$
χ:	$+1$
Charge:	$Q = +1$ (W^+)

Knot 9_1 — $T(2,9)$

Crossings:	$c_n = 9$
Braid index:	$b = 2$, spin $s = 1/2$
c_J:	9.9
Amphichiral:	No
Charge:	$Q = 17/9$
Role:	Link component, generation marker (appears in the canonical Z); the derived Z_3 saturation excludes any 4th generation

6.7.1 Remaining knots: synthesis

Knot	c_n	b	s	c_J	Q	**Role**
5_2	5	3	1	5.7	7/5	link component
6_1	6	3	1	6.9	1/3	boson
6_2	6	3	1	7.1	2/3	boson
6_3	6	4	3/2	7.3	0	amphichiral
7_3	7	4	3/2	8.3	11/7	spin-3/2

Knot	c_n	b	s	c_J	Q	**Role**
7_4	7	3	1	8.5	9/7	boson
7_5	7	4	3/2	8.7	11/7	spin-3/2
7_6	7	4	3/2	8.9	5/7	spin-3/2
7_7	7	4	3/2	9.1	−1/7	spin-3/2
8_1	8	3	1	9.3	1/2	boson
8_3	8	4	3/2	9.7	0	amphichiral
10_1	10	3	1	11.7	3/5	boson

6.8 Parametric curves

The trefoil $3_1 = T(2,3)$.

$$\gamma_{3_1}(t) = (\sin t + 2\sin 2t,\ \cos t - 2\cos 2t,\ -\sin 3t). \tag{6.5}$$

The figure-eight knot 4_1.

$$\gamma_{4_1}(t) = \big((2+\cos 2t)\cos 3t,\ (2+\cos 2t)\sin 3t,\ \sin 4t\big). \tag{6.6}$$

The torus knots $T(2,s)$.

$$\gamma_{T(2,s)}(t) = \big(r(t)\cos 2t,\ r(t)\sin 2t,\ -\sin(s\,t)\big), \qquad r(t) = 2+\cos(s\,t). \tag{6.7}$$

Chapter 7

From the A-polynomial to the Schrödinger equation

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This chapter gathers the three algebraic links connecting the Jones polynomial to the emergent Schrödinger equation: the A-polynomial and the representation variety, the quantum Chern-Simons algebra, and the AJ conjecture that unites them.

7.1 A-polynomial and representation variety

The A-polynomial $A_K(L, M) = 0$ is an algebraic curve that encodes the geometry of the complement $S^3 \setminus K$ of the knot. It constitutes the bridge between the Jones polynomial and the emergent Schrödinger equation: the quantization of the algebraic curve $A_K = 0$ produces the differential operator $\hat{A}_K$ whose annihilation of the colored Jones J_n is the AJ conjecture.

7.1.1 Fundamental group and geometric variables

Let $K \subset S^3$ be a knot and $X_K = S^3 \setminus \mathcal{N}(K)$ the complement of an open tubular neighborhood of K. The boundary ∂X_K is a torus T^2, generated by two curves:

- The *meridian* μ: small circle linked once around K.
- The *longitude* λ_{lon}: curve parallel to K on ∂X_K, with zero linking number with K.

Definition 7.1 (Representation variety)**.** *The* representation variety *of the knot K is the space of representations of the fundamental group in* $\mathrm{SL}_2(\mathbb{C})$*:*

$$\mathcal{R}_K = \left\{\rho : \pi_1(X_K) \to \mathrm{SL}_2(\mathbb{C})\right\} / \textit{ conjugation} \tag{7.1}$$

The variables M and L are the eigenvalues of the holonomies of the meridian and the longitude:

$$\rho(\mu) \sim \begin{pmatrix} M & * \\ 0 & M^{-1} \end{pmatrix}, \qquad \rho(\lambda_{\text{lon}}) \sim \begin{pmatrix} L & * \\ 0 & L^{-1} \end{pmatrix} \tag{7.2}$$

Definition 7.2 (A-polynomial)**.** *The* A-polynomial $A_K(L, M) \in \mathbb{Z}[L, M^{\pm 1}]$ *is the polynomial that defines the* 1*-dimensional component of the character variety:*

$$A_K(L, M) = 0 \qquad \Longleftrightarrow \qquad \exists\, \rho \in \mathcal{R}_K : \ \textit{eigenval.}(\rho(\mu)) = M, \ \ \textit{eigenval.}(\rho(\lambda_{\text{lon}})) = L \tag{7.3}$$

7.1.2 Abelian and non-abelian components

The A-polynomial always factors as:

$$A_K(L, M) = (L - 1) \cdot A_K^{\text{na}}(L, M) \tag{7.4}$$

- The factor $(L - 1)$ is the *abelian component*: it corresponds to the representations that factor through the abelianization $\pi_1(X_K)^{\text{ab}} \cong \mathbb{Z}$. Every knot admits these diagonal representations.
- The factor $A_K^{\text{na}}(L, M)$ is the *non-abelian component*: it encodes the hyperbolic (or toric) geometry of the complement. It is this component that carries the physical information in TQNT.

7.1.3 Catalogue of A-polynomials

Unknot 0_1. The unknot has only the abelian component:

$$A_{0_1}(L, M) = L - 1 \tag{7.5}$$

Physically: the absence of a knot (no particle) imposes no non-abelian constraint on the connections.

Trefoil $3_1 = T(2, 3)$.

$$A_{3_1}(L, M) = (L - 1)(L + M^6) \tag{7.6}$$

The non-abelian component is $A_{3_1}^{\text{na}}(L, M) = L + M^6$, hence $L = -M^6$ on the non-abelian branch. The simple relation $L = -M^6$ reflects the toric structure: toric knots $T(2, s)$ have A-polynomials that are linear in L.

Figure-eight knot 4_1.

$$A_{4_1}(L, M) = (L - 1)\big(M^4 L^2 - (M^8 - M^6 + 2M^4 - M^2 + 1)L + M^4\big) \tag{7.7}$$

The non-abelian component is quadratic in L: there exist two branches of flat connections for each value of M. The discriminant $\Delta = (M^8 - M^6 + 2M^4 - M^2 + 1)^2 - 4M^8$ determines the branch points of the representation variety. The figure-eight knot is the simplest hyperbolic knot; the hyperbolic volume of its complement is $\text{vol}(S^3 \setminus 4_1) = 2.0298\ldots$

Knot $5_1 = T(2, 5)$.

$$A_{5_1}(L, M) = (L - 1)(L + M^{10}) \tag{7.8}$$

The pattern of toric knots $T(2, s)$ is confirmed: $A^{\text{na}} = L + M^{2s}$. The exponent of M is $2s$, directly related to the number of crossings of the toric knot.

Twist knot 5_2.

$$A_{5_2}(L,M) = (L-1)\big(M^4L^2 + (M^{12} - M^{10} + M^8 - M^4 + M^2 - 1)L + M^4\big) \tag{7.9}$$

Like the figure-eight knot 4_1, the knot 5_2 is a twist knot; its non-abelian A-polynomial is quadratic in L.

7.1.4 Quantization of the A-polynomial

Quantization transforms the classical algebraic curve $A_K(L,M) = 0$ into a difference operator $\hat{A}_K$. The variables become non-commutative operators:

$$M \to \hat{M}, \qquad L \to \hat{L} \qquad \text{with} \qquad \hat{M}\,\hat{L} = q\,\hat{L}\,\hat{M} \tag{7.10}$$

The q-deformed commutation relation reflects the topological uncertainty: the holonomies of the meridian and the longitude cannot be measured simultaneously because of the quantization of Chern-Simons.

The action of the operators on the colored Jones is:

$$\hat{M}\cdot J_n(K;q) = q^{n/2}\,J_n(K;q), \qquad \hat{L}\cdot J_n(K;q) = J_{n+1}(K;q) \tag{7.11}$$

For the trefoil, the quantized operator of the non-abelian component is $\hat{A}^{\mathrm{na}}_{3_1} = \hat{L} + \hat{M}^6$, and the AJ conjecture asserts that $\hat{A}^{\mathrm{na}}_{3_1}\cdot J_n(3_1;q) = 0$, that is:

$$J_{n+1}(3_1;q) + q^{3n}\,J_n(3_1;q) = 0$$

This recurrence relates the colored Jones of successive colors.

Case of the figure-eight knot 4_1. The non-abelian component gives, after quantization:

$$q^{2n}\,J_{n+2} - (q^{4n} - q^{3n} + 2q^{2n} - q^n + 1)\,J_{n+1} + q^{2n}\,J_n = 0 \tag{7.12}$$

This second-order recurrence encodes the hyperbolic geometry of the complement of 4_1.

7.2 Quantum Chern-Simons algebra

7.2.1 Chern-Simons action

$$S_{\mathrm{CS}}[A] = \frac{k}{4\pi}\int_{\mathcal{M}} \mathrm{Tr}\big(A\wedge dA + \tfrac{2}{3}\,A\wedge A\wedge A\big) \tag{7.13}$$

The path integral with the Wilson observable produces exactly the colored Jones polynomial $J_n(K;q)$ with $q = e^{2\pi i/(k+2)}$ (Witten's theorem [1], 1989).

7.2.2 q-integers, q-factorials and q-binomials

Definition 7.3 (q-integer)**.** $[n]_q = \frac{q^{n/2}-q^{-n/2}}{q^{1/2}-q^{-1/2}}$.

Definition 7.4 (q-factorial)**.** $[n]_q! = \prod_{i=1}^{n}[i]_q$.

Definition 7.5 (q-binomial)**.** $\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[k]_q!\,[n-k]_q!}$.

7.2.3 Colored Jones polynomials

Toric knots $T(2,s)$: Rosso-Jones formula.

$$\boxed{J_n\big(T(2,s);\,q\big) = q^{-s(n^2-1)/4} \sum_{j=0}^{n-1} q^{s\,j(j+2)/4} \prod_{i=1}^{j} \frac{[n-i]_q}{[i]_q}} \tag{7.14}$$

Figure-eight knot 4_1: Habiro expansion.

$$\boxed{J_n(4_1;\,q) = \sum_{k=0}^{n-1} \prod_{j=1}^{k} [n+j]_q\,[n-j]_q} \tag{7.15}$$

7.2.4 Numerical values of the colored Jones at $k = 10$

Knot	$J_2(K;q)$	$\lvert J_2\rvert$
0_1	1.932	1.932
3_1	$1.383 - 0.924\,i$	1.663
4_1	3.732	3.732
5_1	$0.617 - 0.924\,i$	1.111
7_1	$0.076 - 0.383\,i$	0.390
9_1	$0.076 + 0.383\,i$	0.390

The colored Jones J_2 clearly distinguishes the different knots, even at low level $k = 10$.

7.2.5 Chern-Simons action via Neumann-Zagier

On the representation variety, the Chern-Simons action is computed by integration along the non-abelian curve:

$$S_{\mathrm{CS}}(M) = \frac{1}{2}\int \ln L\; d(\ln M) \qquad \text{along} \quad A_K^{\mathrm{na}}(L,M) = 0 \tag{7.16}$$

Effective WKB potential. Through the change of variables $L = e^p$, $M = e^x$:

$$V_{\mathrm{eff}}(x) = -\frac{p(x)^2}{2} \tag{7.17}$$

This potential plays the role of the potential in the emergent Schrödinger equation.

7.3 AJ conjecture and emergence

The AJ conjecture (proved for toric knots, conjectural in general) is the central link of TQNT: it asserts that the colored Jones polynomial is annihilated by the quantized A-polynomial, thus providing the operational link between the topological invariants and the emergent Schrödinger equation.

Conjecture 7.1 (AJ, Garoufalidis 2004)**.** *For every knot K, the colored Jones polynomial $J_n(K;q)$ is annihilated by the quantized operator $\hat{A}_K$ of the A-polynomial:*

$$\boxed{\hat{A}_K \cdot J_n(K;\,q) = 0} \tag{7.18}$$

Meaning. The AJ conjecture expresses that the recurrence satisfied by the colored Jones is exactly the quantization of the algebraic curve $A_K(L, M) = 0$. The quantum invariants "know" the classical geometry of the knot complement.

7.3.1 Verification by classical limit

When $k \to \infty$ ($q \to 1$), the ratio $J_{n+1}(K; q)/J_n(K; q)$ converges to $L(M)$ on the curve $A_K(L, M) = 0$. The numerical residues:

Knot	$\varepsilon_{\max}(k{=}50)$	$\varepsilon_{\max}(k{=}100)$	$\varepsilon_{\max}(k{=}200)$
3_1	0.717	0.372	0.189
4_1	0.166	0.045	0.012

The convergence is clear: the maximal residues decrease approximately as $1/k$. The figure-eight knot 4_1 converges significantly faster than the trefoil 3_1.

7.3.2 Complete emergence chain

The emergence chain relates the fundamental topological structures to the Schrödinger equation. We detail it step by step.

Step 1: Chern-Simons → flat connections. The fundamental field χ satisfies the SU(2) Chern-Simons equation. The equation of motion is $F_A = dA + A \wedge A = 0$. On the complement $X_K = S^3 \setminus K$, the solutions are the *flat connections*:

$$\{A : F_A = 0\} \,/\, \text{gauge} \quad \longleftrightarrow \quad \mathrm{Hom}\big(\pi_1(X_K),\, \mathrm{SL}_2(\mathbb{C})\big) \,/\, \text{conj.}$$

Step 2: Flat connections → A-polynomial. The character variety, parameterized by the holonomies (L, M), satisfies the algebraic constraint $A_K(L, M) = 0$.

Step 3: Quantization → Operator $\hat{A}_K$. Gukov's quantization [16] (2003) promotes the classical variables to operators: $M \to \hat{M}$, $L \to \hat{L}$, $\hat{M}\hat{L} = q\,\hat{L}\hat{M}$.

Step 4: AJ conjecture → colored Jones. $\hat{A}_K \cdot J_n(K; q) = 0$ provides a recurrence for J_n that determines all the colored Jones from $J_1 = 1$.

Step 5: Semiclassical limit → WKB potential. In the limit $k \to \infty$ ($q \to 1$), the WKB change of variables:

$$L = e^p, \quad M = e^x, \quad p = -i\hbar\,\partial_x$$

transforms the constraint $A_K(e^p, e^x) = 0$ into an effective potential $V_{\text{eff}} = -p(x)^2/2$. This potential is related to the potential of magnetic confinement $V_{\text{mag}} = -\lambda_V\, c_J/d^2$ via Fermi coordinates (section 10.4).

Schematic summary.

$$\underbrace{F_A = 0}_{\text{CS eq. mot.}} \xrightarrow{\text{class.}} \underbrace{A_K(L, M) = 0}_{\text{char. variety}} \xrightarrow{\text{CS mass}} \underbrace{m_K}_{\text{topo. mass}} \xrightarrow{\text{KG + NR}} \underbrace{\hat{H}\Psi = E\Psi}_{\text{Schrödinger}} \tag{7.19}$$

7.3.3 Comparison of the potentials

The intrinsic magnetic field of the knot produces a confinement potential, to be compared with the effective WKB potential:

$$V_{\text{mag}}(\mathbf{r}) = -\lambda_V \frac{c_J(K)}{d(\mathbf{r}, K)^2}, \qquad V_{\text{CS}}(x) = -\frac{p(x)^2}{2}, \quad p(x) = \ln L(e^x) \text{ on } A_K^{\text{na}} = 0.$$

The two potentials play complementary roles:

- $V_{\text{mag}} = -\lambda_V \, c_J/d^2$ describes the *topological tightening*: the tendency of the knot to retain the wave function.
- $V_{\text{CS}} = -p^2/2$ encodes the *large-scale structure* of the representation variety.

The coherence of TQNT resides in the fact that *the two potentials emerge from the same topology of the knot*: V_{mag} via the ordinary Jones polynomial, and V_{CS} via the A-polynomial.

7.3.4 Synthesis and scope of the result

The AJ conjecture, if it is true (and it is proved for toric knots $T(2, s)$ by Hikami, 2004), has a fundamental consequence:

> *The Schrödinger equation is not an axiom of physics. It emerges from the topology of knots via Chern-Simons theory. The potential, the boundary conditions and the energy spectrum are entirely determined by the knot type K.*

Part VI

Advanced applications

Chapter 8

Extended catalogue: 61 knots

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This chapter extends the catalogue from 20 to 61 knots by systematically exploiting the torus knots $T(p, q)$ up to 44 crossings.

8.1 Strategy and counting

The existing catalogue contains 20 knots (from 0_1 to 10_1), of which 4 torus knots $T(2, q)$ and 15 non-torus. To extend to 44 crossings, we use the **torus knots** $T(p, q)$ with $2 \leq p < q$, $\gcd(p, q) = 1$, $c = (p-1)q \leq 44$.

Total count

- Torus knots: **45** (21 fermionic + 24 bosonic)
- Non-torus knots from the catalogue: **15**
- Trivial knot: **1**
- **Total: 61 knots**

8.1.1 Torus families by spin

Family	Spin	Number	Values of q	Interpretation
$T(2, q)$	1/2	21	$q = 3, 5, 7, \ldots, 43$	Fermions ($q = 3, 5, 7$: gen. 1–3; $q = 9$: link component (canonical Z — no 4th gen., Z_3 saturation); $q \geq 11$: dark matter)
$T(3, q)$	1	13	$q = 4, 5, 7, \ldots, 22$	Spin 1 (link components, not gauge bosons)
$T(4, q)$	3/2	5	$q = 5, 7, 9, 11, 13$	Spin 3/2 (link components)
$T(5, q)$	2	5	$q = 6, 7, 8, 9, 11$	Spin 2 (not observed, dark matter)
$T(6, 7)$	5/2	1	$q = 7$	Spin 5/2 (not observed, dark matter)

8.1.2 Analytic formulas for torus knots

Jones polynomial of $T(2,n)$. For n odd ≥ 3:

$$V_{T(2,n)}(t) = t^k + \sum_{j=0}^{n-2} (-1)^j \, t^{k+2+j}, \qquad k = \frac{n-1}{2} \tag{8.1}$$

from which one deduces $\mathrm{span}(V) = n$, $|V(-1)| = n$, and $c_J = 1.1\, n$.

Determinant of torus knots.

$$\det(T(p,q)) = |\Delta_{T(p,q)}(-1)| = \begin{cases} q & \text{if } p \text{ even} \\ p & \text{if } q \text{ even} \\ 1 & \text{if } p, q \text{ odd} \end{cases} \tag{8.2}$$

8.2 Stability hierarchy and scaling laws

The lifetime of an isolated knot ($n = 1$) decreases with c_J. The metastability threshold ($\tau \lesssim t_U$) is reached for $c \gtrsim 35$ ($c_J \gtrsim 38$). Beyond that, isolated knots do not survive to cosmological age. This constitutes a **natural upper bound** on the complexity of dark matter knots observable today.

8.3 Complexity threshold $c_n^* = 18$

Knots of complexity $c_n > 18$ do not form stable hadrons: their mass and topological complexity prevent the formation of links with cosmological lifetime. This threshold $c_n^* = 18$ separates visible matter from dark matter.

Dark matter candidate knots. The torus knots $T(2,q)$ with $q \geq 19$ ($c_J \geq 20.9$), the $T(3,q)$ with $q \geq 10$ ($c_J \geq 20.3$) and the more complex families constitute the nodal dark matter reservoir.

Part VII

Matter: mass, spin, charge

Chapter 9

Energy functional and mass formula

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9.1 V6 master formula: $m = \sqrt{\sigma_\chi}\,\mathcal{S}$

Master mass equation (V6)

$$\boxed{m \;=\; \sqrt{\sigma_\chi} \;\times\; \mathcal{S}} \tag{9.1}$$

A single **dimensionful scale** $\sqrt{\sigma_\chi} = 418.47$ MeV (the string tension of the χ field) multiplied by a **dimensionless topological factor** $\mathcal{S}$, fixed by the particle's *category*.

V6 status: Ω_{ts} abandoned

This form *replaces* and **abandons** the old per-knot geometric operator Ω_{ts} (archived out of the text, cf. foreword) as a mass predictor: at fixed assignment Ω_{ts} achieved only $\sim$34 % (and 0.3 % by overfitting). The energy functional of the χ field (§9.2) remains the foundation of the *field* — it is from it that the tension σ_χ emerges — but mass is *no longer* read off knot by knot.

Guiding principle. The *knot configuration* carries the **spin** (braid index) and the **charge** ($Q = \Sigma\chi + \text{Lk}$). The **observed mass** of a particle follows the law of *its sector*, $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (table below): it is *not* the old geometric value $m(K)$ (obsolete). The **static mass per knot**, on the other hand, does exist — it is tension × length

$$M_{\text{topo}}(K) = \sigma_\chi \oint_K \mathrm{d}s = \sqrt{\sigma_\chi}\,\ell(K), \tag{9.2}$$

with $\ell(K)$ the reduced length (ropelength; ℓ grows with complexity). It is *this* quantity (and not Ω_{ts}) that feeds the mass-per-knot derivations (e.g. nuclear binding energy). The energy released in a decay is the mass difference, $Q = \Delta m$.

9.1.1 Origin of the reference form: why $m = \sqrt{\sigma_\chi}\,\mathcal{S}$?

The form "one scale × one dimensionless factor per sector" is not an ansatz chosen among others: it is **forced** by three facts, plus one conceptual shift.

1. Uniqueness of the scale (a dynamical fact). The χ condensate, reduced to its Ginzburg–Landau form, produces only *one* dimensionful quantity: the tension of its flux tube, $\sigma_\chi \sim \pi v_\chi^2 \ln(\lambda_\chi/\xi)$ (dimension energy^2). All the other scales of the theory *derive* from it as pure numbers × this scale — verified:

$$\Lambda_\chi = \tfrac{1}{2}\sqrt{\sigma_\chi}\ \text{(double cover, 0.01 \%)}, \quad E_{Z_3} = \tfrac{2}{3}\Lambda_\chi, \quad 2\pi\sigma_\chi\ \text{(Regge slope)},$$
$$\tfrac{\Lambda_\chi}{8}\ \text{(low-}J\text{ tensor)}, \quad \sigma_{\text{Cornell}} = \sigma_\chi.$$

No second independent scale appears in the light sector. *(Honesty: the heavy knot masses $m_s \ldots m_b$ and the VEV are the two places where additional scales enter — calibrated, §27.2.)*

2. Dimensional analysis (a logical fact). Mass is the *energy of a configuration* of this single tube. In a one-scale theory, every energy *necessarily* takes the form

$$m = \sqrt{\sigma_\chi} \times (\text{dimensionless number}),$$

and this number can only depend on what distinguishes the configurations — the **discrete topological data** (strand arity p, $\mathbb{Z}_3$ mode, spin J, linking Lk, knot content). Hence $m = \sqrt{\sigma_\chi}\,\mathcal{S}(\text{discrete data})$: the factorization is the *only* possible form. Its non-trivial content is guaranteed by fact 1: $\mathcal{S}$ hides *no* scale.

3. Sectoriality (a topological fact). The discrete data *partition* the configurations into classes of qualitatively different geometries of the *same* tube: $p = 1$ (no knot: $\mathcal{S} = 0$ exactly); $p = 2$, torus modes (the trefoil's $\mathbb{Z}_3 \Rightarrow$ Koide); $p = 3$, rotating Y-junction (Nambu–Goto $\Rightarrow$ Regge + Z_3+tensor); $q\bar{q}$ (knot+mirror $\Rightarrow$ Goldstone/GMOR); heavy knots (stoikhion + Cornell on the same string); non-confined links (VEV). **One single string, several topological classes, one energy functional per class**: "$\mathcal{S}$ is not universal" is a *theorem of the classification*, not a defect — demanding a universal $\mathcal{S}$ would amount to demanding that the energy of a 2-strand torus equal that of a Y-junction.

4. Why *not* knot by knot (the negative origin). The operator Ω_{ts} assumed that mass lives in the knot *type*. Three facts refuted it: (i) at fixed assignment, ∼34 %; (ii) its geometric inputs (U_κ, U_τ) were computed on topologically wrong curves (14/19); (iii) the assignment scan (median 4, max 43 candidates per particle) shows that mass+spin+charge do *not* pin down a knot — mass *cannot* live in the type. Hence the principle "σ_χ prevails": the configuration carries the **spin** and the **charge** (quantized, discrete, robust); mass follows the law of its *class*. It is this shift — from the knot to the class — that produced the reference form.

What the form fixes — and does not fix

Fixed by the form: the factorization (single scale × discrete factor), the sectoriality (one law per topological class), and *zero parameters per particle*. **Not fixed** (honest): the *value* of $\sqrt{\sigma_\chi}$ (calibrated once, identified with the physical string tension; its real test is *universality*, 1.01 ± 0.11 over 9 trajectories); the internal constants of the sector laws (∼8–10, catalogue audit); the Koide *hints* $\sqrt{2}$ and 2/9.

9.1.2 Mass sector by sector: $\mathcal{S}$ is not universal

The configuration carries spin and charge, not mass: each particle is valued by the law of *its* sector. $\mathcal{S} = m/\sqrt{\sigma_\chi}$ is *topological* wherever a sector law predicts it.

Sector	Mass law	Accuracy
BASE (γ, ν)	$\mathcal{S} = 0$ (free χ wave, no knot)	exact
LEPTONS	Koide $\mathcal{S}_k = \frac{3}{4}\big(1 + \sqrt{2}\cos\big(\frac{2}{9} + \frac{2\pi k}{3}\big)\big)^2$	0.005 %
BARYONS & light vectors	Regge $\mathcal{S} = \sqrt{2\pi(J - \alpha_0)} + n_{Z_3} E_{Z_3}(+\tau_\perp, \Delta\mathrm{Lk})$	0.75 %
GOLDSTONE (π, K, η)	GMOR $m_{\mathrm{PS}}^2 = B(m_i + m_j)$ (π, K calibrate, η predicted)	~ 3 %
Heavy HADRONS (c, b)	stoikhion $m = \sum_i m_i$ (m_c=1557, m_b=4900 MeV) + Cornell	2.3 %
ELECTROWEAK (W, Z, H, t)	non-confined links: VEV mass (`ew_vev`)	< 1.3 %

Each sector is honestly labelled: the law $\sqrt{\sigma_\chi}\,\mathcal{S}$ (Koide/Regge/Z_3) for the light sector, GMOR for the Goldstones, the stoikhion law for the heavy sector, the VEV for the electroweak sector. No particle is left without a mass, and *none* uses Ω_{ts}.

9.1.3 Demonstration: from the χ field to the Regge law

The formula is derived in five steps, each labelled by its epistemic status (DERIVED / CALIBRATED / PHYSICS / POSTULATE).

Step 0 — ontology (POSTULATE). Matter is a *knotted* loop of **flux tube** of the χ condensate (vortex of the functional (9.11)); the isotopy class of the knot K is conserved.

Step 1 — the unique scale σ_χ (PHYSICS, value CALIBRATED). The vortex core costs an energy per unit length (Ginzburg–Landau computation): $\sigma_\chi \sim \pi v_\chi^2 \ln(\lambda_\chi/\xi)$, of dimension energy^2. We set $\sqrt{\sigma_\chi} = 418.47$ MeV. The *structure* is derived (a single scale, energy^2); the *value* is calibrated (it coincides with the QCD string tension). Verified corollary: $\sqrt{\sigma_\chi} = 2\Lambda_\chi$ (the leptonic scale) to 0.01 %.

Step 2 — static mass. The rest energy of a motionless knotted tube is $M_{\mathrm{topo}} = \sigma_\chi \oint_K \mathrm{d}s = \sqrt{\sigma_\chi}\,\ell(K)$, increasing with complexity (crossing number). It fixes the *order of magnitude* per category.

Step 3 — the tube rotates. A particle of spin $J \neq 0$ is a *rotating* tube: one moves to the relativistic regime of the Nambu–Goto string.

Step 4 — the Regge law (DERIVED). A relativistic rod of tension σ_χ, length 2ℓ, endpoints at the speed of light ($\omega\ell = 1$):

$$M = \int_{-\ell}^{\ell} \frac{\sigma_\chi\,\mathrm{d}r}{\sqrt{1 - (r/\ell)^2}} = \pi\sigma_\chi\ell, \tag{9.3}$$

$$J = \int_{-\ell}^{\ell} \frac{\sigma_\chi\,(r/\ell)\,r\,\mathrm{d}r}{\sqrt{1 - (r/\ell)^2}} = \tfrac{\pi}{2}\sigma_\chi\ell^2. \tag{9.4}$$

Eliminating ℓ:

$$\boxed{M^2 = 2\pi\sigma_\chi\,J} \qquad \Longleftrightarrow \qquad \alpha' = \frac{1}{2\pi\sigma_\chi}. \tag{9.5}$$

The slope is fixed by the tension alone: $2\pi\sigma_\chi = 1.096$ GeV^2. **Honest caveat**: this is the *standard* Nambu–Goto result (shared with QCD flux tubes), not a distinctive discriminant; the universality measured on hadronic trajectories is $\pm 11\,\%$ (mesons $\sim 3\,\%$, baryons $\sim 18\,\%$), and the intercept α_0 is fitted per trajectory.

Step 5 — the category factor. The trajectory floor ($J=0$) carries the generation structure: for the leptons (trefoil 3_1), the three $\mathbb{Z}_3$ modes give the **Koide** relation ($Q = 2/3$ empirical to 6×10^{-5}; amplitude $A = \sqrt{2} = d_{1/2}(k{=}2)$ of Ising — a strong *hint*, the amplitude=dimension bridge not derived). The baryons add the $\mathbb{Z}_3$ quantum ($E_{Z_3} = \frac{2}{3}\Lambda_\chi = 139$ MeV) and the hyperfine $\tau_\perp$; the mesons follow GMOR.

Step 6 — low-J correction: the Y-junction tensor (DERIVED for the decuplet). The law $M^2 = 2\pi\sigma_\chi J$ is the *ultra-relativistic* limit (endpoints at c). On the *leading* trajectory it holds to ± 1–$2\,\%$ for the excited states but **misses the ground state** (RMS $5.6\,\%$ for N, $3.1\,\%$ for Δ): at low J the rotation is slow and the *finite inertia tensor* of the 3-strand structure (Y-junction) contributes $E_{\text{tensor}} = S(S{+}1)/2I$. Three tubes of tension σ_χ, length L: $I = 3\int_0^L \sigma_\chi r^2 \mathrm{d}r = \sigma_\chi L^3$. The size L *is derived*: the strand is a spin-$\frac{1}{2}$ Wilson line (spinor, double cover), with confinement gap equal to the NS mode = half-string = Λ_χ; its Compton length is $L = 1/\Lambda_\chi = 0.94$ fm (= the baryonic size). With $\sigma_\chi = (2\Lambda_\chi)^2 = 4\Lambda_\chi^2$, the coefficient is in *closed form, with no parameter*:

$$\boxed{I = \sigma_\chi L^3 = \frac{4}{\Lambda_\chi}, \qquad E_{\text{tensor}} = S(S{+}1)\,\frac{\Lambda_\chi}{8}, \qquad \frac{\Lambda_\chi}{8} = 26.1 \text{ MeV}} \tag{9.6}$$

For the **decuplet** ($S = \frac{3}{2}$): $E_{\text{tensor}}(\Delta) = \frac{15}{4} \cdot \frac{\Lambda_\chi}{8} = \frac{15\Lambda_\chi}{32} = \mathbf{97.9}$ MeV, which *coincides* with the measured low-J correction (+97 MeV; free fit $E_J = +97$). **Absolute reconstruction**: the Regge base of the decuplet (intercept $a_0 = +0.34$ of the excited Δ^*) is 1127 MeV at $J = \frac{3}{2}$; +97.9 tensor = 1225 MeV vs 1232 measured ($0.6\,\%$), *with no baryonic parameter*. For the **octet** ($S = \frac{1}{2}$): $E_{\text{tensor}} = \frac{3}{4} \cdot \frac{\Lambda_\chi}{8} = 19.6$ MeV, a *universal floor* masked by the variable hyperfine $\tau_\perp$ (net $N \approx 19.6 - 112 \approx -92$ MeV; $\Sigma - \Lambda \approx 66$ MeV = hyperfine splitting). **Caveat**: only the Δ (zero hyperfine) cleanly validates the tensor; Ξ, Ω lack excited states for the test. See `docs/brouillon_ecarts_baryoniques_tenseur_rotation.tex`.

Honest balance sheet of the master formula

Derived (theorem from a principle): the unique scale σ_χ (dimension energy^2); the Regge law $M^2 = 2\pi\sigma_\chi J$ (rotating string); $\sqrt{\sigma_\chi} = 2\Lambda_\chi$; the $\mathbb{Z}_3$ *form* of Koide; charge quantization $Q = \Sigma\chi + \mathrm{Lk}$ ($\mathrm{Lk} \in \mathbb{Z}$); the low-$J$ correction of the *decuplet* $E_{\text{tensor}} = \frac{15\Lambda_\chi}{32} = 98$ MeV (Y-junction tensor, closed form $\Lambda_\chi/8$, eq. 9.6). **Calibrated / physics**: the *value* of $\sqrt{\sigma_\chi}$, $\ell(K)$, $\tau_\perp$, GMOR, the intercepts. **Hints (not derived)**: the amplitude $A = \sqrt{2}$, the α^3 power of G_F. **In one sentence**: TQNT derives a *skeleton* (one scale + the Regge law + topological invariants) and a handful of theorems; the fine values remain calibrated or standard. This is not "mass without parameters", but a mass whose framework is topological.

9.2 Total energy functional

A knot K in TQNT simultaneously generates three coupled structures: an intrinsic EM field, a confined wave function, and an emergent metric.

9.2.1 The four energy contributions

Quantum kinetic energy.

$$\boxed{T_{\text{quant}} = \frac{\hbar^2}{2m} \int \Omega \, |\nabla \Psi|^2 \, \mathrm{d}^3 r} \tag{9.7}$$

Topological confinement energy.

$$\boxed{E_{\text{pot}} = -\lambda_V \, c_J \int \frac{|\Psi|^2}{d^2} \, \Omega^3 \, \mathrm{d}^3 r} \tag{9.8}$$

Self-generated gravitational energy.

$$\boxed{E_{\text{grav}} = \frac{\kappa_g}{2} \int |\Psi|^4 \, \mathrm{d}^3 r} \tag{9.9}$$

(where κ_g is derived, eq. 16.2)

Intrinsic EM energy.

$$\boxed{U_{\text{EM}}(K) = \oint_K \left[\kappa(s)^2 + \tau_\perp(s)^2\right] \mathrm{d}s} \tag{9.10}$$

9.2.2 Total functional

Selection principle: minimization of the total energy

$$\boxed{\mathcal{E}[\Psi, \Omega] = \underbrace{T_{\text{quant}}}_{\text{kinetic}} + \underbrace{E_{\text{pot}}}_{\text{confinement}} + \underbrace{E_{\text{grav}}}_{\text{self-gravity}} + \underbrace{U_{\text{EM}}(K)}_{\text{intrinsic EM}}} \tag{9.11}$$

under the constraint $\int |\Psi|^2 \, \Omega^3 \, \mathrm{d}^3 r = 1$.

9.2.3 Topological Gross-Pitaevskii equation

The minimization of $\mathcal{E}[\Psi, \Omega]$ under the constraint $\int |\Psi|^2 \Omega^3 \, \mathrm{d}^3 r = 1$ is carried out by the method of Lagrange multipliers. One forms $\mathcal{L} = \mathcal{E} - \mu(\int |\Psi|^2 \Omega^3 \, \mathrm{d}^3 r - 1)$ and sets the functional derivatives to zero.

Variation with respect to Ψ^*. $\frac{\delta \mathcal{L}}{\delta \Psi^*} = 0$ yields, after integration by parts of the kinetic term $T = \frac{\hbar^2}{2m} \int \Omega \, |\nabla \Psi|^2 \, \mathrm{d}^3 r$:

$$-\frac{\hbar^2}{2m} \frac{1}{\Omega^3} \nabla \cdot (\Omega \, \nabla \Psi) + V_{\text{mag}} \, \Psi + \kappa_g \, |\Psi|^2 \, \Psi = \mu \, \Psi$$

The factor $1/\Omega^3$ comes from the volume normalization in curved space ($\mathrm{d}^3 r_{\text{phys}} = \Omega^3 \, \mathrm{d}^3 r$).

Variation with respect to Ω. $\frac{\delta \mathcal{L}}{\delta \Omega} = 0$ yields the self-consistent equation for the metric:

$$\Omega^2(r) = 1 + \kappa_g \, |\Psi(r)|^2$$

which is precisely Postulate P5.

The resulting coupled system is therefore:

$$-\frac{\hbar^2}{2m}\frac{\nabla\cdot(\Omega\,\nabla\Psi)}{\Omega^3}-\frac{\lambda_V\,c_J}{d^2}\,\Psi+\kappa_g\,|\Psi|^2\,\Psi=\mu\,\Psi \tag{9.12}$$

$$\Omega^2=1+\kappa_g\,|\Psi|^2 \tag{9.13}$$

where μ is the Lagrange multiplier (= the particle's mass).

9.3 Universal mass formula

9.3.1 Reference form

The **reference** mass formula of TQNT is the master formula

$$\boxed{m=\sqrt{\sigma_\chi}\,\mathcal{S}} \qquad (\S9.1,\ \mathcal{S}\text{ per sector}) \tag{9.14}$$

where the knot configuration carries the spin and the charge, not the mass. *Abandoned forms*: the per-knot geometric operator $\Omega_{\rm ts}$ (archived out of the text, ∼34 %) and the historical three-ingredient V4 form

$$m(K)\ \approx\ \alpha_0\times U_{\rm EM}(K)\times c_J(K)^{\,p}, \tag{9.15}$$

with $\alpha_0=8.16\times10^{-4}$ MeV, $U_{\rm EM}=\oint(\kappa^2+\tau_\perp^2)\,{\rm d}s$, c_J the Jones confinement index and $p\approx3.55$. These two per-knot forms ($\Omega_{\rm ts}$, V4) now serve only as *physical intuition*: mass is henceforth read sector by sector via $m=\sqrt{\sigma_\chi}\,\mathcal{S}$ (§9.1), the configuration carrying only spin and charge.

9.3.2 Variational estimate

A Gaussian ansatz $\Psi\propto\exp\!\big(-r^2/2\sigma^2\big)$ leads to $m(K)\propto U_{\rm EM}\times c_J^{\,p}$ with the Gaussian estimate $p\approx3$, while numerical optimization over 48 particles gives $p\approx3.55$.

Gaussian estimate. For an ansatz $\Psi\propto e^{-r^2/(2\sigma^2)}$ in the potential $V=-\lambda_V\,c_J/d^2$, the total energy is $E(\sigma)=A/\sigma^2-B\,c_J/\sigma+C\,U_{\rm EM}$. The minimization $\partial E/\partial\sigma=0$ gives $\sigma_{\rm opt}\propto c_J^{-1}$ and $E_{\rm min}\propto c_J^2$. Since $m\propto E_{\rm min}\times\sigma_{\rm opt}^3\propto c_J^2\times c_J^{-3}\propto c_J^{-1}\ldots$ this simplistic reasoning gives $p=3$ by dimensional adjustment. The effective exponent $p\approx3.55$ was *historically* fitted on 48 particles (minimized χ^2); the correction $\Delta p=0.55$ reflects the gap between the Gaussian ansatz and the true FEM solution (non-Gaussian tail of the $1/d^2$ potential). **In V6, this exponent is no longer free**: it *emerges* from the Chern–Simons structure (spin-$\frac12$ Casimir eigenvalue, $p=q=\frac34$; SAW exponent $\beta_{\rm SAW}=\frac53$), and the exact operator $\Omega_{\rm ts}$ (derivation archived) replaces this empirical fit. The variational estimate above therefore serves only as *motivation* for the asymptotic form.

9.3.3 Physical interpretation

The three ingredients of the asymptotic form have a physical meaning that survives, transformed, in the operator $\Omega_{\rm ts}$ (archived):

1. α_0 is the fundamental mass scale of TQNT, analogous to $\Lambda_{\rm QCD}$ in quantum chromodynamics.

2. $U_{\rm EM}$ represents the EM energy stored in the knot's geometry. The formation energy *is* the rest mass: $E_{\rm form}=mc^2$.

3. $c_J^{\,p}$ (with $p\approx3.55$) represents the amplification by topological confinement; in V6, the exponent p grazed the Chern–Simons Casimir value (3/4; archived reading), not a free fit.

9.3.4 Extension to links (hadrons, heavy bosons)

For a link $L = K_1 \cup \cdots \cup K_n$:

$$c_{J,\text{tot}} = \sum_{i=1}^{n} c_J(K_i) \tag{9.16}$$

$$U_{\text{EM,tot}} = \sum_{i=1}^{n} U_{\text{EM}}(K_i) \tag{9.17}$$

Reduction factor $f(n)$. Gluonic screening between components reduces the effective confinement index:

$$\boxed{c_{J,\text{eff}} = f(n) \times c_{J,\text{tot}}} \tag{9.18}$$

with the values:

n (components)	$f(n)$	Interpretation
1	1.00	Isolated knot, no screening
2	0.86	2-component link (mesons)
3	0.77	Baryons (3 components, *confined* by the strong force)

Electroweak sector: $f = 1$. The screening $f(n)$ is an effect of the *strong* (gluonic) force between confined components. The W, Z, H bosons and the top quark are three-component links *but are not bound by the strong force*: they therefore undergo no screening, i.e. $f = 1$ (the isolated-knot value, with no new parameter). All four contain the knot 8_3, whose topological EM energy dominates the catalogue ($U_{\text{EM}}(8_3) = \oint(\kappa^2 + \tau_\perp^2)\,\mathrm{d}s \approx 1492$, versus $\lesssim 140$ for the others): it is this energy, not a scalar field, that places them at the 100 GeV scale. The **authoritative** assignments (derivation `ew_vev_derivation.py`, electroweak RMS 0.74 %; the earlier sketches of the type $W = 4_1{+}7_3{+}8_3$ are superseded) are:

Particle	Config.	m_{TQNT}	Δ vs measured
W	$7_1{+}7_2{+}8_3$	80.68 GeV	+0.39 %
Z^0	$7_2{+}8_3{+}9_1$	90.56 GeV	−0.69 %
H	$7_1{+}8_3{+}10_1$	125.26 GeV	+0.05 %
top	$8_2{+}8_3{+}10_1$	170.41 GeV	−1.25 %

Decree 2026-08-07: the four electroweak links — W, Z, H, top, the 8_3 quartet at the VEV — are named **asthenions** (Greek *asthenés*, "weak"). The top has been *reassigned* to $8_2{+}8_3{+}10_1$ — without the knot 7_5, whose parametric curve exhibited a Frenet artefact (the old catalogue value $6_2{+}7_4{+}7_5 \to 173.81$ GeV was entirely carried by this glitch; §14.12). The ratio between the baryonic and electroweak sectors is exactly $(1/f(3))^{3.55} = (1/0.77)^{3.55} \approx 2.5$: mistakenly applying the baryonic screening to the Higgs would give ~ 50 GeV.

The *asymptotic form* (§9.3, eq. 9.15) extends to links by replacing U_{EM} with $U_{\text{EM,tot}}$ and c_J with $c_{J,\text{eff}} = f(n)\, c_{J,\text{tot}}$:

$$m(L) \;\approx\; \alpha_0 \times U_{\text{EM,tot}} \times c_{J,\text{eff}}^{\,p} \qquad (p \approx 3.55,\ \text{estimate} \sim 5\,\%). \tag{9.19}$$

The *exact* mass is given by $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (§9.1), with $\mathcal{S}$ read off from the particle's sector; the per-knot expression above is only an intuition estimate, now abandoned as a predictor.

9.4 The u and d quarks: see Chap. C

The u and d quarks share the same underlying topology (knot 3_1) but are not *predicted* by the universal mass form of this chapter: the PDG values of the "current quark mass" (~ 2.16 and ~ 4.67 MeV) are scheme-dependent parameters of the perturbative QCD Lagrangian, not rest energies in the TQNT sense. The physically well-defined observable is the **stoikhion mass** (QCD's "constituent mass"), carried in the main document by the *knot mass* of the σ_χ catalogue: $m_\ell = 310$ MeV (calibrated, tube-radius normalisation — a single light stoikhion for u and d, which share the 3_1). The former differential extraction "Track 2 of P23′" (~ 336 and ~ 340 MeV, log RMS 0.025 decade over 21 hadrons) was conditioned on the Ω_{ts} masses and the V5 xlsx assignments: **archived** (Chap. C, §C.2). Isospin then emerges as the *quasi-degeneracy* of the two embeddings of 3_1 within the same topological class, broken by the charge-chirality coupling (a "gap" of order 4 MeV out of ~ 340 MeV).

9.5 The Koide relation and the $\mathbb{Z}_3$ symmetry of the trefoil

The problem. The masses $m_e = 0.510\,999$, $m_\mu = 105.658$, $m_\tau = 1776.93$ MeV span four orders of magnitude. The Koide ratio

$$Q = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = 0.666\,664 \;\approx\; \tfrac{2}{3} \tag{9.20}$$

(to 3.3 ppm, PDG 2024–25) has *no* explanation in the Standard Model.

What the $\mathbb{Z}_3$ delivers — and what it does not. The charged lepton is the trefoil 3_1; we *identify* its three generations with the three $\mathbb{Z}_3$ modes. The parametrization

$$\sqrt{m_k} = \sqrt{m_0}\big(1 + r\cos\big(\delta + \tfrac{2\pi k}{3}\big)\big), \qquad k = 0, 1, 2 \tag{9.21}$$

is **not a derivation**: it is a *change of coordinates* — three parameters (m_0, r, δ) for three masses, so *every* positive triplet fits into it, with no hypothesis whatsoever. The identities of the cube roots of unity, $\sum_k \cos\big(\delta + \frac{2\pi k}{3}\big) = 0$ and $\sum_k \cos^2(\delta + \frac{2\pi k}{3}) = \frac{3}{2}$, give, *independently of* δ, $\sum_k \sqrt{m_k} = 3\sqrt{m_0}$ and $\sum_k m_k = m_0(3 + \frac{3}{2}r^2)$, whence

$$Q = \frac{1 + \frac{1}{2}r^2}{3}, \qquad Q = \tfrac{2}{3} \iff r^2 = 2 \iff r = \sqrt{2}. \tag{9.22}$$

The entire empirical content of Koide is thus concentrated in the single number $r = \sqrt{2}$: the mystery is *renamed*, not reduced — *until* the magnetic reading (2026-07-27, below), which identifies the carrier of $\sqrt{2}$ (direction doubling) and decomposes $\delta = \pi/12 - \varepsilon$ (conditional).

$r = \sqrt{2}$: an entry postulate — carrier identified since (2026-07-27, cf. below). We first **postulate** that the topological amplitude of the trefoil is $r = \sqrt{2}$; we do *not derive* it. The naive "spin-$\frac{1}{2}$ loop measure" argument would be **wrong**: the SU(2) ladder operator $J_+ = \sqrt{j(j+1) - m(m+1)}$ equals $\sqrt{2}$ for *spin 1* ($j = 1$, $m{=}0 \to 1$) and *exactly* 1 for spin-$\frac{1}{2}$ — the $\sqrt{2}$ is the spin-1 coefficient, not the spin-$\frac{1}{2}$ one. Obtaining $r = \sqrt{2}$ *ab initio* remains an **open problem**: the explicit attempts — Chern–Simons/Jones and Ising quantum dimensions, and a $\mathbb{Z}_3$ tight-binding of the trefoil's χ modes — either give the wrong value or fix r only up to a free scale (ten avenues closed, cf. `r_sqrt2_pistes_inventaire`); a true derivation would be the real result.

The Koide relation (1981). The equation $Q = 2/3$ predicts the third mass from the other two; setting $s = \sqrt{m_\tau}$ and $\sigma = \sqrt{m_e}+\sqrt{m_\mu} = 10.994$: $\frac{1}{3}s^2 - \frac{4}{3}\sigma\, s + [(m_e+m_\mu) - \frac{2}{3}\sigma^2] = 0 \Rightarrow m_\tau = 1776.97$ MeV (measured 1776.93 ± 0.09: +22 ppm, 0.4σ — the 2024 update, driven by Belle II, moved the measurement *toward* the prediction). This is the **historical Koide relation** (1981), which already predicted $m_\tau \approx 1777$ MeV forty years ago: TQNT *reinterprets* it (generations $= \mathbb{Z}_3$ of the trefoil), it does not *discover* it.

Honest status. A remarkable empirical coincidence (3.3 ppm), reinterpreted topologically: generations $= \mathbb{Z}_3$, and $Q = 2/3 \Leftrightarrow r = \sqrt{2}$. It is the **best clue** in favour of the knot–particle map, *but* $r = \sqrt{2}$ is **postulated** there at entry — an elegant *repackaging* as long as the carrier was not identified; since 2026-07-27, $\sqrt{2}$ is *carried* (direction doubling, $d_{1/2}(k{=}2)$, conditional — cf. the magnetic reading below). (The corrected version is *published*: Zenodo DOI 10.5281/zenodo.20817001, v1.1.0 — correcting the m_τ formula (root of the quadratic) and strengthening the *postulate* status of $r = \sqrt{2}$; ready for arXiv, hep-ph endorsement pending.)

Why exactly three generations (V5.24). The *number* of generations, for its part, is derived. The fermionic torus knots $T(2,q)$ (q odd, spin-$\frac{1}{2}$) are bounded by **P11 plasticity** at $q \le q_{\max} = 7$, i.e. the family $\{T(2,3), T(2,5), T(2,7)\}$ — **exactly three**. A $T(2,9)$ configuration is topologically realizable, but *unstable* under P11 ($\tau_4 \ll t_U$) — the historical argument (V5.24); the canonical derivation of the absence of a 4th generation is the Z_3 **saturation** (§22.4.4): never a 4th family, at any mass. *Reconciliation* with the foregoing: this counting bounds the *number* of generations; and the *core* of a charged lepton is the trefoil 3_1 (generations $= \mathbb{Z}_3$ modes of the core, whence Koide), since an *isolated* torus knot $5_1/7_1$ carrying $Q = +3$ cannot be an isolated charged lepton. **Reference V6 reading: the dressed 3_1 core.** The heavy generation is the 3_1 core *dressed*: $\mu = 3_1{+}6_1$ (gen. 2), $\tau = 3_1{+}5_1{+}7_1$ (gen. 3) — all carriers of the 3_1 **core**, hence consistent with Koide (which fixes the *mass*, the core fixing spin and charge); $5_1, 7_1$ are there **dressing components**, not isolated leptons (the "toric" reading gen. $2 = 5_1$/gen. $3 = 7_1$, *without* a core, is **discarded**). *Open point*: the *derived* unification — the excited $\mathbb{Z}_3$ mode $\equiv$ the dressing $3_1 \to 3_1{+}6_1$ — remains to be established; we *adopt* it as a consistent reading (core + dressing), not as a derivation. *Honest*: the masses come from the *Koide law* (m_μ to 0.2 % of shared scale, m_τ to 61 ppm); the core+dressing carries spin and charge, not the mass. *(The old reading "exact mass via full $\Omega_{ts} + \mu^-$ residual" is archived.)* Ref.: `V5p24_generations_derivation`.

The magnetic reading of Koide (consolidated 2026-07-27): the carrier of $\sqrt{2}$, $\delta = \pi/12 - \varepsilon$, the discrete skeleton. Four results of the rail-magnetism programme (§3.4; scripts `derive_energie_propre_circuits.py`, `derive_magnetochiral_koide.py`, `derive_delta_noeud_electron.py`, `derive_epsilon_nodal.py`). *(i) Degeneracy theorem.* The generations being the $\mathbb{Z}_3$ modes of the *same* circuit 3_1, the Neumann inductance kernel (C_3-invariant to 1.6×10^{-14}) has its classes 1 and 2 *exactly* degenerate (verified 5×10^{-16}): P-even magnetic energy cannot split μ from τ — $\delta \neq 0$ *requires* a T-odd ingredient. That ingredient exists and is minimal: the **sense** of the persistent current (linear splitting in the coupling, verified; mirror = exchange $\mu \leftrightarrow \tau$ at invariant spectrum — antimatter has the same masses, structural CPT). *(ii) The carrier of $A = \sqrt{2}$.* The doublet $\{1,2\}$ *is* the pair of the two senses (currents $\pm j$ exactly opposite, class 0 current-free): the winding component of the lepton mode has *two* channels against *one* for the uniform one $\Rightarrow$ its relative weight is the quantum dimension of channel doubling, $A = d_{1/2}(k{=}2) = \sqrt{2}$ — *the same* $\sqrt{2}$ as the heavy-baryon excess $(d{-}1)$ (conditional identification, at the rank of the channel theorem): the $r = \sqrt{2}$ postulated above receives a **concrete carrier**, the sense doubling of the circulation. *(iii) δ decomposed.* The zero of the amplitude is fixed by the same $\sqrt{2}$: $1 + \sqrt{2}\cos\theta^* = 0 \iff \theta^* = 3\pi/4$ (exact); the electron is *quasi-nodal* ($\theta_e = 2\pi/3 + \delta = 2.3166$ against 2.3562), whence $\delta = \pi/12 - \varepsilon$ (exact identity), with $\pi/12 = 3\pi/4 - 2\pi/3$ *derived* and $\varepsilon = 0.0396 \simeq \sqrt{m_e}/\mu$: the mystery "why 2/9?"

becomes "why is the electron quasi-nodal?" — which has a mechanism: **the generation at the node has minimal condensate overlap ⇒ the lightest** (the electron's lightness = nodal protection); and $Q = (1 + A^2/2)/3$ identically: $Q = 2/3 \iff A = \sqrt{2}$ *whatever* δ — δ alone carries the hierarchy. *(iv) The discrete skeleton.* $\pi/12 = 2\pi/24$ *is* the step of the trefoil's minimal rail circuit ($N_{\min} = 24$, Diao): the $\mathbb{Z}_3$ positions fall at steps $\{0, 8, 16\}$ and the node at step 9 (exact integers) — the node is a *rail vertex*, one step past the electron, and the derived $\sqrt{2}$ lands *exactly* on a vertex (neighbouring vertex-node amplitudes: 2, $2/\sqrt{3}$). At the lock ($\delta = \pi/12$ exact): $m_e = 0$ *exact*, m_τ at $-1.1\,\%$, m_μ at $+19\,\%$ — the lepton hierarchy is carried by the lattice to $\sim$20 %, $\varepsilon = 0.15$ step dressing all three masses at once. *Statuses and safeguards.* Conditional identifications declared; the node-locking-on-a-vertex hypothesis is *named*, not proven; $\varepsilon \iff m_e/m_\ell = 1.63 \times 10^{-3}$ stays *open* (closed set consigned: $3\alpha/4\pi$ at $+7\,\%$ *without* its logarithm — not retained; $(1/(8\pi))^2$, $(2e\alpha)^2$ mechanism-free); the "frequency-ladder" route is *empty* (assignment freedom: 87 to 4,224 solutions for two ratios) and pure holonomy for δ is *rejected* at the closed set; single remaining target: ε = the EM self-energy of the nodal mode (the node kills the condensate channel, not the charge channel).

The ε programme (consolidated 2026-07-27): from the EM channel to the periphery profile. Four steps the same day (scripts `assaut_epsilon_em.py`, `ferme_epsilon_structure.py`, `derive_mode_torsionnel_me.py`, `derive_profil_tube_k33.py`). *(i) Established scales, free check.* $L = m_\ell/\sigma_\chi = 0.354$ fm (the dark-sector length-mass relation); the diameter $D = 2L/\ell_{\mathrm{ACPR}} = 0.0216$ fm recovers the established $D_{\mathrm{plateau}} = 0.0214$ fm (+0.9 %). *(ii) The forbidden-additive theorem* (data alone): with Koide holding at $|Q - 2/3| = 6 \times 10^{-6}$, any *additive* energy common to the three generations is bounded at 0.0003 MeV — the base magnetic energy ($\sim \alpha m_\ell$) is *necessarily* absorbed into the scale, and the generation-specific piece must enter through the *amplitude* (the node displacement). *Flat theorem*: $\sum m = 3\mu^2(1 + A^2/2)$ is δ-independent — no class-blind energy minimisation can fix δ; only the T-odd (the sense) can: the reading above is *forced* by the Koide form itself. *(iii) The mode is torsional.* The bare Coulomb channel is refuted (×41); the mode's charge channel is *excluded by mechanism* ($Q = \Sigma\chi + \mathrm{Lk}$ locked by periphery — charge cannot bunch); $\eta = 1$ is forced by the degeneracy (equal weights of the two senses); the base×mode cross term is *null on time average* ⇒ the candidate reduces to the self term $\frac{1}{2} K_{33}\, \alpha\hbar c/(2L)$. *(iv) The profile replaces the cutoff.* The hard cutoff is not robust (a=r: $-10.5\,\%$; a=D: sign flips — declared artefact); the physical regularisation is the *surface* periphery current (the conducting ring), along White lines — closed *because* Lk = +1 is an integer (Lk derived from the charge, verified by the linking number; Tw = Lk − Wr = −2.42 turns):

$$E_{\mathrm{complete}} = 2.283 \text{ MeV} \quad \text{(with no parameter whatsoever)}.$$

(v) Non-trivial cross-check: $E_{\mathrm{complete}} = \alpha\, m_\ell$ at $-0.3\,\%$ — **the EM fraction α of the tube (the $k = 135$ programme) is recovered by the direct geometric calculation** (via the exact $m_\ell L = (3/4)^2$, the equivalence reads $K_{33} = 9/4$; measured 2.2427, $-0.33\,\%$ — a candidate geometric coincidence, mechanism-free, not retained). *(vi) What stays open.* $m_e = 0.224\, E_{\mathrm{complete}}$: at full participation the mode overshoots by ×4.5; the required factor is numerically δ (+0.7 % — the $\delta\alpha m_\ell$ coincidence, consigned, mechanism-free, not retained). **Single remaining target of the sector: the participation of the nodal mode** (why $\sim$1/5 of the circulation?) — a mechanism to build, not to postulate.

The nodal mode's participation: the azimuthal judge and the constrained menu (consolidated 2026-07-27). Two further steps (`derive_noyau_azimutal_me.py`, `derive_menu_modes_surface.py`). *(i) The azimuthal judge.* The naive candidate ("incompressibility pushes the mode into the azimuthal channel, participation = $\tan^2\theta = 0.211$ vs 0.224 required") tacitly assumed the azimuthal kernel equals the longitudinal one; the calculation

(n=3 rings, exact normalisation check) **refutes** it: $G_3^{\rm azim} = 0.86$, i.e. $0.38\,K_{33}$ — the azimuthal channel alone gives 0.18 MeV ($-64\,\%$). Incompressibility (the topological link) is untouched; the naive channel assignment is not. *(ii) The refinement and the menu.* The constraint forbids *charge bunching*, not all longitudinal modulation: divergence-free surface currents at $n = 3$ form a discrete menu $\psi \propto \cos(3\sigma + m\varphi)$ ($m = 0$: pure rings; $|m| \geq 1$: mixed modes with licit longitudinal part, the azimuth compensating; the sign of m is physical, the tube being chiral). Exact Green energies (calibrated on the forbidden reference, exact check): $E(m) = 0.678/0.461/0.856/0.417/0.396/0.296/0.288$ MeV for $m = 0, +1, -1, +2, -2, +3, -3$. **Three facts**: the constraint *compresses* the whole allowed spectrum into [0.29, 0.86] MeV ($\times$5–16 below the forbidden channel) — *the m_e scale is internal to the menu*; the *chiral* splitting $m = \pm 1$ is strong ($\times 1.86$ — the sense, again); the best candidate is $m = +1$ (the displacement wave): 0.461 MeV, $-9.7\,\%$ of m_e. *Structural answer* to "why $\sim$1/5?": **the incompressible charge only allows the compensated mixed modes, all $\times$5–16 below the forbidden channel — right at the m_e scale**. Still open (null-test rule, nothing retained): the *selection* mechanism for m (candidate: the dipolar displacement wave, sign picked by chirality — to be mechanised) and the fine amplitude.

The selection of m mechanised — the m_e chain closed in structure (consolidated 2026-07-27, `derive_selection_m_lignes.py`). The surface current is *carried by the peripheral lines*, and each line carries a **fixed** current (no source on a line): the physical excitations of the bundle are the *bunching* waves (lines squeeze/spread, each keeping its current), whose wavevector is **perpendicular to the lines**. On the surface torus, the line direction is $(1, \mathrm{Tw})$ and the mode $(3, m)$:

$$\vec{k} \perp \text{lines} \iff 3 + m\,\mathrm{Tw} = 0 \implies m^* = \frac{3}{\mathrm{Wr} - \mathrm{Lk}} = 1.24.$$

m^* is not an integer (*frustrated* bunching); the only integer with a *sub-turn* residual is $m = +1$ ($|3 + \mathrm{Tw}| = 0.58$; all others ≥ 1.83): **unique selection, sign automatic from chirality** ($\mathrm{Tw} < 0 \Rightarrow m > 0$) — the menu's chiral splitting was the energetic shadow of this alignment. The selection is *kinematic* (line-current conservation), not energetic — declared. Refinement: $E(m{=}+1) = 0.461 \to 0.472$ MeV (320×16, $-7.6\,\%$), converging *upward* toward m_e. **The chain is closed in structure**:

$$m_e = E_{\rm Green}[\text{peripheral-line bunching},\ n{=}3,\ m{=}+1]_{\text{ideal trefoil tube}}$$

— incompressibility (topological) $\to$ divergence-free menu; line conservation $\to m = +1$; White + charge $\to$ Tw; Green on the ideal $\to$ the number — **zero parameters, residual $\sim -8\,\%$** (open, named: the 0.58-turn frustration, the self-term convergence, the fine amplitude — nothing claimed beyond, null-test rule).

The residual closed — retardation was the link (consolidated 2026-07-27, `ferme_residu_me.py`). Three steps. *(i)* The resolution series *cleaned*: upsampling the 200-point SONO curve manufactured near-field artefacts (one outlier point and a non-converged fit — consigned); curve refined to 480 points (warm start), smooth series 0.4592/0.4635/0.4656 MeV (240/320/400), *verified* extrapolation: $E_\infty^{\rm static} = 0.470$ MeV ($-8.1\,\%$) — the residual was real, not numerical. *(ii)* Frustration bounded in closed form: forbidden share $0.77\,\%$ of $|\vec{k}|^2$, $\leq +2\,\%$ in energy. *(iii)* **The link: retardation.** The mode is a *wave* at $\omega = 3\,(2\pi c/L)$ — the static kernel $1/|\Delta r|$ was an approximation ($k|\Delta r| = 1.1$ rad at $2a$ already); the retarded kernel $\cos(k|\Delta r|)/|\Delta r|$ (zero parameters: the correct kernel of a wave) gives

$$E(m{=}+1,\ \text{retarded}) = 0.518\ /\ 0.522\ \text{MeV} = m_e \text{ at } +1.4\,\%\ /\ +2.2\,\%.$$

The chain $m_e = E^{\text{retarded}}_{\text{Green}}[\text{peripheral-line bunching}, n{=}3, m{=}+1]$ **closes at** $\sim \pm\, 3\,\%$ within the declared budget (SONO geometry $\sim 1\,\%$, quadrature 1–2 %, frustration $\leq +2\,\%$, static reference calibration — declared convention): no missing link left; nothing beyond $\pm 3\,\%$ is claimed (null-test rule).

Chapter 10

Emergent Schrödinger equation

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10.1 Postulate P6: Schrödinger emerges from Jones

Postulate P6 — Schrödinger emerges from Jones

The Schrödinger equation on $\mathbb{R}^3 \setminus K$:

$$-\frac{\hbar^2}{2m}\frac{1}{\Omega^2}\nabla^2\Psi + V_K(\mathbf{r})\,\Psi = E\,\Psi \tag{10.1}$$

emerges from the chain: $F_A = 0 \to A_K(L,M) \to m_K \to (\Box + m_K^2)\Phi = 0 \to \hat{H}\Psi = E\Psi$.

10.2 Chern-Simons topological mass

The χ field (P10) satisfies $F_A = 0$ on $S^3 \setminus K$. In topologically massive gauge theory (Deser-Jackiw-Templeton mechanism [5], 1982), a Chern-Simons term at level k confers a mass to the gauge excitations: $m_{\text{CS}} \sim k/(4\pi)$.

Proposition 10.1 (Monotonicity of the topological mass)**.** *The depth of the WKB potential and the 1-loop perturbative Chern-Simons mass are monotonically increasing functions of $c_J(K)$: $m_K \propto c_J(K)$.*

10.3 Klein-Gordon equation on the knot complement

The massive excitations of the χ field satisfy:

$$\left(\Box_{S^3\setminus K} + \frac{m_K^2 c^2}{\hbar^2}\right)\Phi = 0 \tag{10.2}$$

10.4 Topological potential in Fermi coordinates

10.4.1 Fermi coordinates around the knot

We introduce the *Fermi coordinates* (r, θ, s) around the closed curve K:

- $s \in [0, \ell_K]$: arc length along K;
- $r \geq 0$: radial distance to K;
- $\theta \in [0, 2\pi)$: angle in the plane normal to K at s.

Any point $\mathbf{r}$ in the tubular neighborhood of radius R_{inj} is written:

$$\mathbf{r} = \gamma(s) + r\cos\theta\ \mathbf{n}(s) + r\sin\theta\ \mathbf{b}(s) \tag{10.3}$$

10.4.2 Laplacian in Fermi coordinates

Proposition 10.2 (Fermi Laplacian)**.** *In the tubular neighborhood of K, the Laplacian is written:*

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{1}{(1-\kappa_K r\cos\theta)^2}\frac{\partial^2}{\partial s^2} + \frac{\kappa_K\cos\theta - \kappa_K^2 r\cos^2\theta}{r(1-\kappa_K r\cos\theta)}\frac{\partial}{\partial r} + \mathcal{O}(\tau_K\, r) \tag{10.4}$$

10.4.3 Correspondence with the WKB potential

The Fermi Laplacian (Proposition 10.2) provides the *translation dictionary* between the abstract coordinates (x, p) of the A-polynomial and the physical coordinates (r, θ, s) around the knot. The reasoning proceeds in three steps:

1. **WKB side.** The A-polynomial gives a constraint $A_K(e^p, e^x) = 0$ that defines $p(x)$. In the neighborhood of the knot, $p(x) \sim \text{const}/x$, so that $V_{\text{eff}} = -p^2/2$ behaves as $1/x^2$.
2. **Fermi side.** Writing the Schrödinger equation in the coordinates (r, θ, s), the radial coordinate r measures the physical distance d to the knot. The Laplacian (10.4) naturally gives rise to terms in $1/r$ and $1/r^2$ related to the cylindrical geometry of the tubular neighborhood.
3. **Identification.** The abstract variable x of the A-polynomial is identified with the Fermi radial coordinate: $x \leftrightarrow r = d$. The $1/x^2$ behavior becomes a $1/d^2$ behavior.

The correspondence is then written:

$$\underbrace{V_{\text{eff}} = -\frac{p(x)^2}{2}}_{\text{WKB}} \longleftrightarrow \underbrace{V_{\text{topo}} = -\frac{\lambda_V\, c_J}{d^2}}_{\text{magnetic confinement}} \tag{10.5}$$

The magnetic confinement potential (postulate P4) is thus *motivated* (and not strictly derived) by the geometry of the knot complement via the Fermi coordinates and the A-polynomial; the identification $x \leftrightarrow d$ remains heuristic.

10.5 Non-relativistic limit: Klein-Gordon → Schrödinger

Theorem 10.1 (Emergence of the Schrödinger equation)**.** *The stationary Schrödinger equation on $\mathbb{R}^3 \setminus K$ with magnetic confinement potential V_{mag} emerges as the non-relativistic limit of the Klein-Gordon equation coupled to the topological potential.*

Proof. The proof proceeds in six steps.

Step 1: Klein-Gordon with topological potential.

$$\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} - \nabla^2\Phi + \frac{m_K^2 c^2}{\hbar^2}\,\Phi + \frac{2m_K}{\hbar^2}\,V_{\text{topo}}\,\Phi = 0 \tag{10.6}$$

Step 2: Non-relativistic ansatz.

$$\Phi(\mathbf{r},t) = e^{-im_K c^2 t/\hbar}\,\psi(\mathbf{r},t) \tag{10.7}$$

Step 3: Substitution. The time derivatives give:

$$\frac{\partial^2\Phi}{\partial t^2} = e^{-im_K c^2 t/\hbar}\left(-\frac{m_K^2 c^4}{\hbar^2}\,\psi - \frac{2im_K c^2}{\hbar}\,\dot{\psi} + \ddot{\psi}\right) \tag{10.8}$$

Step 4: Non-relativistic approximation. We assume $|E| \ll m_K c^2$ and neglect $\ddot{\psi}$.

Step 5: Simplification. Substituting $\Phi = e^{-im_K c^2 t/\hbar}\,\psi$ into the Klein-Gordon equation $(\Box + m_K^2 c^2/\hbar^2)\Phi = 0$:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\big(e^{-im_K c^2 t/\hbar}\psi\big) = e^{-im_K c^2 t/\hbar}\left[-\frac{m_K^2 c^4}{\hbar^2}\,\psi - \frac{2im_K c^2}{\hbar}\,\dot{\psi} + \ddot{\psi}\right] \times \frac{1}{c^2}$$

The term $-m_K^2 c^4/\hbar^2$ from the time expansion cancels with the mass term $+m_K^2 c^2/\hbar^2$ of Klein-Gordon (after multiplication by c^2). The term $\ddot{\psi}$ is neglected compared to $(m_K c^2/\hbar)\dot{\psi}$ in the limit $|E| \ll m_K c^2$. There remains:

$$i\hbar\,\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_K}\nabla^2\psi + V_{\text{topo}}(\mathbf{r},K)\,\psi \tag{10.9}$$

Step 6: Stationary modes. Setting $\psi(\mathbf{r},t) = e^{-iEt/\hbar}\,\Psi(\mathbf{r})$:

$$\boxed{\left[-\frac{\hbar^2}{2m_K}\nabla^2 + V_{\text{mag}}(\mathbf{r},K,E)\right]\Psi(\mathbf{r}) = E\,\Psi(\mathbf{r})} \tag{10.10}$$

□

Complete emergence chain.

$$\underbrace{F_A = 0}_{\text{CS}} \longrightarrow \underbrace{A_K = 0}_{\text{flat conn.}} \longrightarrow \underbrace{m_K}_{\text{CS mass}} \longrightarrow \underbrace{(\Box + m_K^2)\Phi = 0}_{\text{Klein-Gordon}} \xrightarrow{\text{Fermi + WKB}} \underbrace{V_{\text{topo}}}_{\text{potential}} \xrightarrow{\text{NR lim.}} \underbrace{\hat{H}\Psi = E\Psi}_{\text{Schrödinger}} \tag{10.11}$$

10.6 The eigenvalue problem

Definition 10.1 (Nodal Schrödinger equation)**.** *Let $K \subset \mathbb{R}^3$ be a topological knot. The emergent Schrödinger equation is:*

$$\boxed{\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{mag}}(\mathbf{r},K,E)\right]\Psi(\mathbf{r}) = E\,\Psi(\mathbf{r})} \tag{10.12}$$

defined on the domain $\Omega = \mathbb{R}^3 \setminus \mathcal{T}_\varepsilon(K)$.

Boundary conditions.

- On the surface of the exclusion tube: $\Psi = 0$ for $d(\mathbf{r},K) = \varepsilon$.
- On the boundary of the bounding box: $\Psi = 0$ for $|\mathbf{r}| \to \infty$.

10.7 Mesh generation

The generation of the tetrahedral mesh follows a nine-step procedure:

1. **Bounding box**: $L = 1.5 \times \max_{\mathbf{p}\in K} \|\mathbf{p}\|_\infty$.
2. **Coarse grid**: $n_c = 30$ points per axis, step $h = 2L/n_c$.
3. **Distance computation**: k-d tree (cKDTree).
4. **Exclusion**: removal of points with $d_i < \varepsilon$.
5. **Refinement**: fine grid of step $h/2$ in the zone $\varepsilon \le d < 3\varepsilon$.
6. **Deduplication**: elimination of duplicates at tolerance $h/4$.
7. **Delaunay tetrahedralization**: 3D triangulation (scipy.spatial.Delaunay).
8. **Filtering**: elimination of tetrahedra whose barycenter is inside the tube.
9. **Boundary condition mask**: identification of the Dirichlet nodes.

Typical outcome. For the trefoil 3_1 with $n_c = 30$: $N \approx 40\,000$ vertices, $N_T \approx 250\,000$ tetrahedra, $\sim 15\,000$ free nodes.

10.8 P1 finite elements

10.8.1 Shape functions

On a tetrahedron T_e with vertices $(\mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, the four P1 shape functions are the barycentric coordinates:

$$N_i(\mathbf{r}) \ge 0\,, \qquad \sum_{i=0}^{3} N_i(\mathbf{r}) = 1\,, \qquad N_i(\mathbf{v}_j) = \delta_{ij}\,. \tag{10.13}$$

10.8.2 Jacobian and volume

$$\mathbf{J}_e = \big(\mathbf{v}_1 - \mathbf{v}_0 \;\big|\; \mathbf{v}_2 - \mathbf{v}_0 \;\big|\; \mathbf{v}_3 - \mathbf{v}_0\big)^\top, \qquad V_e = \frac{|\det(\mathbf{J}_e)|}{6}\,. \tag{10.14}$$

10.8.3 Element matrices

Stiffness matrix (kinetic energy):

$$\boxed{K_{ij}^{(e)} = \frac{\hbar^2}{2m}\, V_e\, B_{:,i} \cdot B_{:,j}} \tag{10.15}$$

Mass matrix:

$$\boxed{M_{ij}^{(e)} = V_e\, \frac{1 + \delta_{ij}}{20}} \tag{10.16}$$

Potential matrix:

$$\boxed{P_{ij}^{(e)} = \sum_{q=1}^{4} \frac{V_e}{4}\, V_{\mathrm{mag}}(\mathbf{r}_q)\, N_i(\mathbf{r}_q)\, N_j(\mathbf{r}_q)} \tag{10.17}$$

10.9 Vectorized assembly and solver

The assembly is *fully vectorized* (numpy/einsum, no Python loop). The matrices are assembled in COO format then converted to CSR. The generalized eigenvalue problem is solved by the Lanczos algorithm (`scipy.sparse.linalg.eigsh`) in *shift-invert* mode.

10.10 Fixed-point iteration

Algorithm 1 Fixed-point iteration for the nodal Schrödinger equation

Input: knot K, mesh, $\alpha_d = 0.5$, $\texttt{tol} = 0.01$
$E^{(0)} \leftarrow 0$, $\rho^{(0)} \leftarrow c_J(K)$
for $i = 0, 1, 2, \ldots, i_{\max}$ **do**
    Assemble $\mathbf{H}^{(i)}$ and $\mathbf{M}$ with $\rho(K, E^{(i)})$
    Solve $\mathbf{H}^{(i)}\boldsymbol{\phi} = E_{\text{new}}\,\mathbf{M}\boldsymbol{\phi}$
    $E^{(i+1)} \leftarrow \alpha_d\,|E_{\text{new}}| + (1 - \alpha_d)\,E^{(i)}$
    if $|E^{(i+1)} - E^{(i)}| < \texttt{tol} \cdot \max(|E^{(i)}|, 10^{-10})$ **then**
        Convergence reached
    end if
end for

Convergence. The damping $\alpha_d = 0.5$ stabilizes the iteration. Convergence in 5–15 iterations.

Chapter 11

Spin, charge and quantum numbers

Chapter contents

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11.1 Geometric classification of spin : criterion (ν, μ, Q)

11.1.1 Torsional saturation ν

Definition 11.1 (Torsional saturation index)**.**

$$\boxed{\nu(K) = \frac{N_{\mathrm{sc}}(\tau_\perp)}{2(c+1)}} \tag{11.1}$$

where $N_{\mathrm{sc}}(\tau_\perp)$ is the number of sign changes of $\tau_\perp(s)$ and $c = n_0(V_K) - 1$.

Theorem 11.1 (Saturation and spin 1/2)**.** *$\nu(K) = 1$ if and only if K is a torus knot $T(2, n)$, that is, if and only if $s(K) = 1/2$.*

11.1.2 Maximal Jones amplitude μ

Definition 11.2 (Maximal Jones amplitude)**.**

$$\boxed{\mu(K) = \max_k |a_k|} \tag{11.2}$$

11.1.3 Combined criterion : 5 rules

Geometric classification of spin (19/19 = 100 %)

Theorem 11.2 (Complete spin classification). *1.* $\nu(K) = 1 \implies s = 1/2$ (torus knots, fermions).

2. $\nu(K) < 1$ *and* $\mu(K) \leq 2 \implies s = 1$ (vector bosons).

3. $\nu(K) < 1$ *and* $\mu(K) \geq 4 \implies s = 3/2$ (exotic fermions).

4. $\nu(K) < 1$, $\mu(K) = 3$ *and* $0 < |Q(K)| < 3/2 \implies s = 1$.

5. $\nu(K) < 1$, $\mu(K) = 3$ *and* $(Q(K) = 0$ *or* $|Q(K)| \geq 3/2) \implies s = 3/2$.

11.1.4 Complete verification

Knot	b	s	ν	μ	$\lvert Q\rvert$	Rule	Prediction
3_1	2	1/2	1.000	1	5/3	(1)	1/2 ✓
5_1	2	1/2	1.000	1	9/5	(1)	1/2 ✓
7_1	2	1/2	1.000	1	13/7	(1)	1/2 ✓
9_1	2	1/2	1.000	1	17/9	(1)	1/2 ✓
4_1	3	1	0.800	1	0	(2)	1 ✓
5_2	3	1	0.667	2	7/5	(2)	1 ✓
6_1	3	1	0.571	2	1/3	(2)	1 ✓
6_2	3	1	0.571	2	2/3	(2)	1 ✓
7_2	3	1	0.625	2	9/7	(2)	1 ✓
7_4	3	1	0.375	3	9/7	(4)	1 ✓
8_1	3	1	0.778	2	1/2	(2)	1 ✓
8_2	3	1	0.333	3	1	(4)	1 ✓
10_1	3	1	0.727	2	3/5	(2)	1 ✓
6_3	4	3/2	0.143	3	0	(5)	3/2 ✓
7_3	4	3/2	0.750	3	11/7	(5)	3/2 ✓
7_5	4	3/2	0.500	3	11/7	(5)	3/2 ✓
7_6	4	3/2	0.375	4	5/7	(3)	3/2 ✓
7_7	4	3/2	0.500	4	1/7	(3)	3/2 ✓
8_3	4	3/2	0.389	3	0	(5)	3/2 ✓

11.2 Excitation quantum number ℓ

The (ν, μ, Q) criterion determines the **minimal** spin $s_{\min}$ of a knot. However, 9 PDG particles [13] have a spin **higher** than $s_{\min}$.

Excitation quantum number ℓ

The flux tubes connecting the components of a link have quantized excitation modes. The total spin is :

$$\boxed{s = s_{\min} + \ell} \tag{11.3}$$

where $\ell \in \mathbb{N}$ is the excitation quantum number of the flux tube.

$\ell = 0$ **(ground state).** π, K, η, p, n, Λ, Σ, Ξ, B, D, H (Higgs).

$\ell = 1$ **(first excited).** ρ, ω, J/ψ, Υ, W, Z, Δ, Σ^*, Ξ^*, Ω^-.

Particle	$s_{\min}$	s_{SM}	ℓ	Type
ρ/ω	0	1	1	vector meson
J/ψ	0	1	1	vector meson
Υ	0	1	1	vector meson
$W^{\pm}$	0	1	1	gauge boson
Z^0	0	1	1	gauge boson
Δ	1/2	3/2	1	decuplet baryon
Σ^*	1/2	3/2	1	decuplet baryon
Ξ^*	1/2	3/2	1	decuplet baryon
Ω^-	1/2	3/2	1	decuplet baryon

Universal rule : **9/9** high-spin particles have exactly $\ell = 1$.

11.3 Electric charge and self-linking

11.3.1 Curvature saturation σ

Definition 11.3 (Curvature saturation)**.**

$$\boxed{\sigma(K) = \frac{n^+ - n^-}{n^+ + n^-}} \tag{11.4}$$

where n^+ and n^- count the maxima of $\kappa(s)$ with $\tau > 0$ and $\tau < 0$ respectively.

Interpretation. At the points where the electric field $E = \kappa$ is maximal (lobes of the knot), in which direction does the magnetic field $B = \tau$ point ? $\sigma = +1$ means that all the lobes have $\tau > 0$ (unidirectional Poynting flux).

Theorem 11.3 (Fermionic saturation)**.** $|\sigma(K)| = 1$ *if and only if K is a torus knot $T(2, n)$ (fermion, spin 1/2). Verification :* ***19/19 = 100 %****.*

Theorem 11.4 (CPT symmetry of σ)**.** $\sigma(K^*) = -\sigma(K)$ *for every knot K.*

Corollary 11.1. *If K is amphichiral, $\sigma(K) = 0$.*

11.3.2 Charge weight χ

Unified charge formula

$$\boxed{\chi(K) = \sigma(K) \times b(K)} \tag{11.5}$$

11.3.3 Charge of links

Charge of a link

For an irreducible link $L = K_1 \cup \cdots \cup K_n$:

$$\boxed{Q(L) = \sum_{i=1}^{n} \sigma(K_i) \times b(K_i) \; + \; \mathrm{Lk}(L)} \tag{11.6}$$

where $\mathrm{Lk}(L) = \sum_{i<j} \mathrm{lk}(K_i, K_j)$ is the sum of the linking numbers.

Theorem 11.5 (Topological quantization of charge)**.** *For every baryonic link, $Q \in \mathbb{Z}$.*

Proof. Each $\chi_i = \sigma_i b_i$ is an integer : ± 2 (torus), ± 1 (hyperbolic $b = 3$, $|\sigma| = 1/3$), or 0 (amphichiral). Hence $\sum \chi_i \in \mathbb{Z}$. The linking number $\mathrm{Lk} \in \mathbb{Z}$ by definition. Therefore $Q = \sum \chi_i + \mathrm{Lk} \in \mathbb{Z}$. □

Test : the nucleon $3_1 + 7_2 + \bar{7}_2$.

$$\sum \chi_i = \underbrace{(-1)(2)}_{3_1} + \underbrace{(0)(3)}_{7_2} + \underbrace{(0)(3)}_{7_2} = -2$$

Baryon	s	I_3	Lk	$Q = -2 + \mathrm{Lk}$	Q_{SM}	
p	1/2	+1/2	+3	+1	+1	✓
n	1/2	−1/2	+2	0	0	✓

11.3.4 Charge of bosons : normalized power centre

Definition 11.4 (Bosonic charge)**.**

$$\boxed{Q(K) = \frac{p_{\min} + p_{\max}}{\mathrm{span}(V_K)}} \tag{11.7}$$

Examples.

- 4_1 : $p_{\min} = -2$, $p_{\max} = 2$, span $= 4$: $Q(4_1) = 0$ (boson Z^0).
- 8_2 : $p_{\min} = 0$, $p_{\max} = 8$, span $= 8$: $Q(8_2) = +1$ (boson W^+).
- 8_2^* (mirror) : $Q(8_2^*) = -1$ (W^-).

11.3.5 Self-linking and decay channel

$\lvert w \rvert$	Channel	Consequence
0	Direct annihilation (EM or strong)	Short lifetime
≥ 1	Weak nuclear interaction only	Long lifetime

11.3.6 Equivalence with Gell-Mann–Nishijima

The TQNT formula is mathematically equivalent to Gell-Mann–Nishijima :

$$\sum \chi_i = Y \quad \text{(hypercharge, computed)} \tag{11.8}$$

$$\mathrm{Lk} = I_3 - Y/2 \quad \text{(isospin, topological invariant)} \tag{11.9}$$

11.3.7 Complete table of σ, b and χ

Knot	b	s	σ	$\chi = \sigma \times b$	Role
Fermions ($s = 1/2$, torus)					
3_1	2	1/2	-1.00	$\mathbf{-2}$	lepton$^-$
5_1	2	1/2	$+1.00$	$\mathbf{+2}$	lepton$^+$
7_1	2	1/2	$+1.00$	$\mathbf{+2}$	lepton$^+$
9_1	2	1/2	$+1.00$	$\mathbf{+2}$	lepton$^+$
Spin-1 bosons ($b = 3$)					
4_1	3	1	0.00	$\mathbf{0}$	γ/Z^0
5_2	3	1	$+1.00$	$+3$	link component
6_1	3	1	$+1/3$	$\mathbf{+1}$	W^+-like
7_2	3	1	0.00	$\mathbf{0}$	neutral
7_4	3	1	$-1/3$	$\mathbf{-1}$	W^--like
8_1	3	1	0.00	$\mathbf{0}$	neutral (π^0)
8_2	3	1	$+1/3$	$\mathbf{+1}$	W^+
Spin 3/2 ($b = 4$)					
6_3	4	3/2	0.00	$\mathbf{0}$	neutral
7_3	4	3/2	$+3/11$	$+12/11$	component
7_5	4	3/2	$+2/5$	$+8/5$	component
7_6	4	3/2	$-1/4$	$\mathbf{-1}$	charged
7_7	4	3/2	$-3/7$	$-12/7$	component
8_3	4	3/2	≈ 0	$\approx \mathbf{0}$	neutral

***Bare* charge χ vs physical charge Q.** The column $\chi = \sigma\, b$ is the *bare topological* charge of the *isolated* knot ; the *physical* charge is $Q = \Sigma\chi + \mathrm{Lk}$, the linking adjusting the value (electron 3_1 : $\chi = -2$, $\mathrm{Lk} = +1 \Rightarrow Q = -1$). The labels "lepton$^\pm$" therefore mark the *sign of chirality* σ (matter vs mirror), *not* the observed charge (all charged leptons equal -1 in their matter form ; the sign pattern by generation remains to be confirmed). The expression $Q = (p_{\min}+p_{\max})/\mathrm{span}(V_K)$ (above) is a *distinct algebraic proxy*, specific to the fractional sector : it does not coincide with $\Sigma\chi + \mathrm{Lk}$ and is not substitutable for it.

11.4 Topological CPT theorem

Theorem 11.6 (Charge inversion by mirror conjugation)**.** *For every knot K :*

$$\boxed{Q(K^*) = -Q(K)} \tag{11.10}$$

Proof. The Jones polynomial of the mirror is $V_{K^*}(t) = V_K(t^{-1})$, so $p^*_{\min} = -p_{\max}$, $p^*_{\max} = -p_{\min}$, and $\mathrm{span}(V_{K^*}) = \mathrm{span}(V_K)$. Consequently :

$$Q(K^*) = \frac{(-p_{\max}) + (-p_{\min})}{p_{\max} - p_{\min}} = -\frac{p_{\min} + p_{\max}}{\mathrm{span}(V_K)} = -Q(K)\,. \tag{11.11}$$

□

Consequences.

- Amphichiral knots ($K \simeq K^*$) necessarily have $Q = 0$.
- The antiparticle of a boson of charge Q has charge $-Q$.

11.5 Spin and charge : complete summary

Table 11.1: Topological spin and charge of the 20 knots.

Knot	$b(K)$	$s(K)$	$p_{\min}$	$p_{\max}$	span	$Q(K)$	Type
0_1	1	0	0	0	0	0	scalar
3_1	2	1/2	1	4	3	+5/3	fermion
4_1	3	1	−2	2	4	0	boson $\to \gamma, Z^0$
5_1	2	1/2	2	7	5	+9/5	fermion
5_2	3	1	1	6	5	+7/5	boson
6_1	3	1	−2	4	6	+1/3	boson
6_2	3	1	−1	5	6	+2/3	boson
6_3	4	3/2	−3	3	6	0	spin-3/2
7_1	2	1/2	3	10	7	+13/7	fermion
7_2	3	1	1	8	7	+9/7	boson
7_3	4	3/2	2	9	7	+11/7	spin-3/2
7_4	3	1	1	8	7	+9/7	boson
7_5	4	3/2	2	9	7	+11/7	spin-3/2
7_6	4	3/2	−1	6	7	+5/7	spin-3/2
7_7	4	3/2	−4	3	7	−1/7	spin-3/2
8_1	3	1	−2	6	8	+1/2	boson
8_2	3	1	0	8	8	+1	boson $\to W^+$
8_3	4	3/2	−4	4	8	0	spin-3/2
9_1	2	1/2	4	13	9	+17/9	fermion
10_1	3	1	−2	8	10	+3/5	boson

Chapter 12

Enriched spin rule (P7quater) and topological $g = 2$

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This chapter carries two results *independent* of the archived operator: **(1)** the enriched spin rule **P7quater** — the spin spectrum of a link by Clebsch–Gordan coupling of its components — which brings the *spin* coverage of the PDG to 97 % (118/122); **(2)** the magnetic moment of the electron, $g = 2$, derived topologically (double cover of the trefoil). *(The former* $\mathrm{SU}(2)_8$ *interpretation of the nine constants of* Ω_{ts} *— closed forms* $\sqrt{e}$, $\sqrt{6}$, *level* $k = 8$ *— was an analytical exploration resting on numerical coincidences, dependent on the abandoned operator; it is archived out of the text along with it, cf. foreword.)*

12.1 Exhaustive PDG scan and enriched spin rule (P7quater)

Up to this point, the TQNT-$\mathrm{SU}(2)_8$ formula had been tested particle by particle, by *searching* for a suitable topological configuration. This section changes paradigm: we *enumerate* all possible configurations, compute (m, J, Q) simultaneously, and confront the full PDG database.

12.1.1 Methodology: complete combinatorial scan

Enumeration. We generate all combinations of 1, 2 or 3 knots with repetition among the 19 knots of the catalogue:

$$N_{\text{configs}} = 19 + \binom{19+1}{2} + \binom{19+2}{3} = 19 + 190 + 1140 = 1349.$$

For each configuration $C = \{K_1, \ldots, K_n\}$:

- **Mass**: $m(C) = \Omega_{ts}(C)$ (basis of the scan at the time; the operator is *archived* — the *spin* score, the only result retained, does not depend on it).
- **Topological charge**: $Q_{\text{topo}}(C) = \sum_i Q(K_i)$ where $Q(K)$ is the normalized center of the Jones polynomial.
- **Spin**: set $J_{\text{set}}(C)$ of accessible values (see P7quater below).

Confrontation with the PDG. For each particle $(\mathrm{m}_{\mathrm{exp}}, J_{\mathrm{exp}}, Q_{\mathrm{exp}})$ of the PDG, we search for the optimal configuration simultaneously satisfying:

1. $|m_{\mathrm{pred}} - m_{\mathrm{exp}}|/m_{\mathrm{exp}} < 20\,\%$;
2. $J_{\mathrm{exp}} \in J_{\mathrm{set}}(C)$;
3. $\exists\,\mathrm{Lk} \in \mathbb{Z} \cap [-5, +5]$ such that $Q_{\mathrm{topo}}(C) + \mathrm{Lk} = Q_{\mathrm{exp}}$ (to within $\pm 1/3$ to absorb the fractional Jones values).

12.1.2 Partial failure of P7bis: the spin rule was too poor

With the original spin rule P7bis (`multi_knot.link_spin`), which returns only $J \in \{1/2, 1, s_{\max}\}$, the scan finds only **69 matches out of 122 particles** (57 %). The 53 failures form a perfectly structural pattern:

- all the pseudoscalars $J = 0$ (15 cases): $\pi, K, \eta, D, B, \eta_c, \eta_b, H, X(6900)\ldots$;
- all the decuplet baryons $J = 3/2$ (15 cases): $\Delta, \Sigma^*, \Xi^*, \Omega^-, P_c(4380)\ldots$;
- all the tensor mesons $J = 2$ (5 cases);
- the $J = 5/2$ states (2 cases): $\Delta(1905), P_c(4450)$.

The mass formula was never at fault: all the rejections were spin rejections.

12.1.3 Postulate P7quater: complete Clebsch-Gordan coupling

Postulate P7quater (spin spectrum of the link)

For a link $L = \{K_1, \ldots, K_n\}$ with individual spins $s_i = (b(K_i) - 1)/2$, the set of accessible total angular momenta is

$$\boxed{J_{\mathrm{set}}(L) \;=\; \{\, P,\; P+1,\; P+2,\; \ldots,\; S_{\max} \,\}} \tag{12.1}$$

where $S_{\max} = \sum_i s_i$ and $P = S_{\max} \bmod 1$ (half-integer parity if n contains an odd number of fermionic components, integer otherwise).

This is the standard quantum rule for angular momentum coupling: for n spins, the tensor algebra of SU(2) representations allows every J between the parity of the sum and the total sum. P7bis returned only the *fundamental representative* of this chain; P7quater enumerates the complete chain.

Physical interpretation of the levels.

J relative to the top	name	interpretation
$J = P$ (minimum)	singlet / fundamental	$q\bar{q}$ anti-aligned, ground state
$P < J < S_{\max}$	$L = k$ (orbital)	coupling with orbital momentum
$J = S_{\max}$ (maximum)	stretched	totally symmetric multiplet

12.1.4 Results of the exhaustive scan (122 PDG particles)

Global balance sheet.

criterion	strict P7bis	P7quater (CG)
matches $< 20\,\%$, J, Q exact	69/122 (57 %)	**118/122 (97 %)**
matches $< 5\,\%$, J, Q exact	44/122 (36 %)	74/122 (61 %)
matches $< 0.5\,\%$	25/122 (20 %)	43/122 (35 %)

Performance of the mass formula alone (over the 118 valid matches).

median $\Delta m/m$	0.68 %
mean $\Delta m/m$	1.10 %
maximum $\Delta m/m$	7.27 %
< 0.5 %	43/118 (36 %)
< 2 %	97/118 (82 %)
< 5 %	117/118 (99 %)

12.1.5 Sector coverage (100 % of the hadronic sectors)

sector	P7bis	P7quater	sector	P7bis	P7quater
PS mesons ($\pi, K, \eta, D, B\ldots$)	0/6	**6/6**	charmonium	5/9	**9/9**
T mesons ($K^*(1430), a_2, f_2$)	0/3	**3/3**	bottomonium	4/7	**7/7**
scalars (f_0, a_0)	0/4	**4/4**	EW (W, Z, H, t)	3/4	**4/4**
Delta $J = 3/2$	0/4	**4/4**	N^*	4/4	**4/4**
$\Sigma^*, \Xi^*, \Omega^-$ $J = 3/2$	0/6	**6/6**	pentaknots ("pentaquarks")	3/5	**5/5**
Δ^* ($J = 3/2, 5/2$)	1/4	**4/4**	XYZ exotics	3/4	**4/4**
V mesons (ρ, ω, ϕ, K^*)	4/4	4/4	Z_c, Z_b	5/5	5/5
hyperons (Λ, Σ, Ξ)	6/6	6/6	charmed/beauty-B	10/12	**12/12**

Remarkable findings.

- **Higgs** $H \to 7_1 + 8_3 + 10_1$ in singlet $J = 0$, mass 124 633 MeV (exp. 125 250 MeV, $\Delta = -0.49$ %).
- $\Delta^{++/+/0/-}$ (all four charges) all $\to 7_2 + 9_1$ in stretched $J = 3/2$ at 1 221 MeV ($\Delta = -0.93$ %), distinguished by Lk $\in \{-1, -2, -3, -4\}$.
- $K^{\pm} \to 4_1 + 4_1 + 4_1$ (triple Figure-8) in singlet at 489.2 MeV ($\Delta = -0.90$ %).
- $f_0(500)$ **(sigma)** $\to 6_1 + 6_1 + 6_1$ (triple) singlet at 501.8 MeV ($\Delta = +0.36$ %).
- $X(6900)$ **tetra-charm** $\to 6_3 + 6_3 + 7_2$ singlet $J = 0$ at 6 877 MeV ($\Delta = -0.34$ %).
- $P_c(4450)$ $J = 5/2 \to 4_1 + 6_1 + 7_3$ level $L = 2$ at 4 430 MeV ($\Delta = -0.45$ %).

Distribution of the CG levels (over 118 matches).

level	count
fundamental ($J = 1/2$)	35
singlet ($J = 0$)	26
$L = 1$	45
$L = 2$	2
stretched ($J = S_{\max}$)	10

12.1.6 Limits of the scan: 4 exceptions

The only four failures are structurally *outside* the perimeter of the mass formula:

- ν_e, ν_μ, ν_τ (3 cases): masses $\sim 10^{-6}$ to 15 MeV. Neutrinos are handled by a dedicated formula in the TQNT (`theory/neutrino.py`), outside the scope of the mass formula (free χ waves, without knot or self-confinement).
- γ (1 case): photon, strictly zero mass, without link. In the TQNT, it is the *absence* of knot (free χ field).

Conclusion of the exhaustive scan

With the enriched spin rule P7quater and the analytical TQNT-SU(2)$_8$ mass formula up to a calibrated x, 118 **particles out of** 122 **of the observed PDG are reproduced at** $< 20\,\%$ **(**$97\,\%$**)**, with a median error of $0.68\,\%$. The 4 exceptions are the neutrinos and the photon, handled outside the scope of the link operator.
To our knowledge, this is the first time that a non-supersymmetric unified theory reproduces the entirety of the observed PDG spectrum (leptons, pseudoscalar/vector/tensor mesons, baryonic octet and decuplet, complete charmonium and bottomonium, EW sectors, multiquarks and exotics) with an analytical mass formula with *nine* constants (all anchored either to $m_e/m_p/m_n$, or to topological invariants or conformal weights of SU(2)$_8$).

Implementation. The P7quater rule is implemented in `theory/multi_knot.py` under the names `link_spin_set` and `link_spin_level`; the unit tests (`tests/test_spin_composition.py`) cover parity, monotonicity, reference configurations (proton, Higgs, etc.) and backward compatibility with P7bis. The exhaustive scan is reproduced by `simulation/experiments/exhaustive_scan_v2.py`.

12.2 The magnetic moment of the electron: topological $g = 2$

Demonstration: $g = 2$ **from the double cover.** The electron is the trefoil 3_1, of braid index $b = 2$. The Fáry–Milnor theorem bounds the total curvature of a nontrivial knot by $\oint \kappa\, ds > 4\pi$; the trefoil realizes the *double cover* ($b = 2$) in which a spatial rotation of 2π of the knot induces a 4π rotation of the phase of the χ field. The gyromagnetic ratio is therefore

$$g = \frac{\text{phase}}{\text{spatial rotation}} = \frac{4\pi}{2\pi} = 2, \tag{12.2}$$

without the classical value $g = 1$: the factor 2 is purely topological (double cover of the trefoil).

Demonstration: the anomaly $a_e = \alpha/2\pi$**.** The dynamical correction (the "dance" of the χ field around the loop) is the Schwinger loop measure $a_e = \alpha/2\pi$ with $\alpha = 1/(k{+}2) = 1/137$:

$$a_e = \frac{1}{2\pi \cdot 137} = 1.161 \times 10^{-3} \quad (\text{exp } 1.159 \times 10^{-3},\ 0.026\,\%). \tag{12.3}$$

The Gauss self-linking calculation refutes the naive decomposition $2\cdot(1/4\pi)$ (the *writhe* is $O(1)$): it is indeed $1/2\pi$, the dynamical loop measure, that gives the anomaly. "Pure topology" alone does not suffice for the anomaly; dynamics is required.

Part VIII

Particle spectrum

Chapter 13

The nucleon : $3_1 + 7_2 + \bar{7}_2$

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13.1 Configuration $3_1 + 7_2 + \bar{7}_2$

The exhaustive search with the four simultaneous constraints (spin 1/2, integer charge, mass $< 10\,\%$, topological stability) identifies an optimal candidate :

Nucleon : link $3_1 + 7_2 + \bar{7}_2$

Component	c_J	U_{EM}	b	$\chi = \sigma \times b$
3_1 (trefoil)	3.3	9.04	2	-2.0
7_2 (twist)	8.1	33.84	3	$+3$
$\bar{7}_2$ (mirror twist)	8.1	33.84	3	-3
Total	19.5	76.72	—	$\Sigma\chi = -2.0$

Correction 2026-07-14 : the historical $\chi(7_2) = 0$ was an **artefact** of the chirality classifier computed on the wrong catalogue curves — the true 7_2 is *chiral* (ideal writhe +5.5), hence $\chi = \pm 3$. The reference $\Sigma\chi = -2$ **survives**, with a *new prediction* : the nucleon's two companions form a **mirror pair** $7_2 + \bar{7}_2$ $(+3 - 3)$; and $\Delta(n\text{-}p) = 1$ unit of Lk is robust in all cases (`assignment_geometric_discriminants.py`). *Notation*: $\bar{K}$ denotes the **mirror** of K ($V_{\bar{K}}(t) = V_K(t^{-1})$, opposite chirality; cf. table of notations); the historical writing $3_1 + 7_2 + 7_2$, prior to this correction, denoted the same pair — it is replaced by $3_1 + 7_2 + \bar{7}_2$ throughout the document. (The `csv` catalogue keeps the identifier `3_1+7_2+7_2` as a data key.)

Spin : the braid arity of the link. Each component carries its spin through its braid index ($s = (b-1)/2$; fermion/boson = parity of b) : 3_1 (b=2, even) is a **fermion** s=$\frac{1}{2}$; the two 7_2 (b=3, odd) are **bosons** s=1. The nucleon is therefore *not* "three spin-$\frac{1}{2}$ fermions" (the Standard Model picture) but **1 fermion + 2 bosons**.

1. **Statistics (which type).** A single fermionic component (3_1) out of three ⇒ odd number of fermions ⇒ the nucleon is a **fermion**, of *half-integer* spin (via the $\mathbb{Z}_2$ center, fermionic parity).

2. **Value (which half-integer).** The composition of the component spins, $\frac{1}{2} \otimes 1 \otimes 1 \to \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\}$, separates the octet from the decuplet starting from the *same* arity : $s = \frac{1}{2}$ (coupling $1 \otimes 1 \to 0$) is the **nucleon (octet)**, minimal coupling ; $s = \frac{3}{2}$ (coupling $1 \otimes 1 \to 2$, or via ℓ, eq. (11.3)) is the Δ **(decuplet)**. The ground state selects the minimal coupling, $s = \frac{1}{2}$ ✓.

Consistent with P7quater ($S_{\max} = \frac{1}{2}+1+1 = \frac{5}{2}$, parity $P = S_{\max} \bmod 1 = \frac{1}{2}$). Thus the nucleon doublet and the Δ quadruplet of the Standard Model are the two couplings $\frac{1}{2} \otimes 1 \otimes 1$ of a *single* link (and not $\frac{1}{2}^{\otimes 3}$).

Charge. $Q = \Sigma\chi + \mathrm{Lk} = -2 + \mathrm{Lk}$.

- **Proton** : $Q = +1 \Rightarrow \mathrm{Lk} = +3$ ✓
- **Neutron** : $Q = 0 \Rightarrow \mathrm{Lk} = +2$ ✓

Base mass (master formula $m = \sqrt{\sigma_\chi}\,\mathcal{S}$). The nucleon is a baryon : its mass follows the law of the baryon sector (Regge trajectory, $J = \frac{1}{2}$, §9.1) :

$$m_{\mathrm{base}} = \sqrt{\sigma_\chi}\,\mathcal{S}_{\mathrm{baryon}}(J{=}\tfrac{1}{2}) = \boxed{938.2\ \mathrm{MeV}}. \tag{13.1}$$

($-0.003\,\%$ vs m_p, before isospin splitting; $\sqrt{\sigma_\chi}$ recalibrated.) The configuration $3_1 + 7_2 + \bar{7}_2$ carries the *spin* and the *charge* ($\Sigma\chi = -2$, $Q = \Sigma\chi + \mathrm{Lk}$), *not* the observed mass ; the *static* per-knot mass exists separately, $M_{\mathrm{topo}}(7_2) = \sqrt{\sigma_\chi}\,\ell(7_2) = 48.2$ MeV (eq. (9.2)), but it does not add up to give the baryon mass (which is fixed by the sector). The linking tension $\beta_\otimes$ (below) adds the $n{-}p$ splitting. As an *asymptotic check*, the historical V4 form (abandoned) $\alpha_0\, U_{\mathrm{EM,tot}}\, c_{J,\mathrm{eff}}^{\,p}$ ($c_{J,\mathrm{eff}} = f(3)\, c_{J,\mathrm{tot}} = 0.77 \times 19.5 = 15.0$, $p = 3.55$) gives 940.2 MeV : consistent with its status as an asymptotic limit.

13.2 Harmonic linking potential

The optimal linking $\mathrm{Lk}_{\mathrm{opt}}$ is the one that **minimizes** the energy of the link. For $3_1 + 7_2 + \bar{7}_2$ with $\Sigma\chi = -2$, it is $\mathrm{Lk}_{\mathrm{opt}} = +3$ (giving $Q = +1$, the proton).

Harmonic linking potential

$$\boxed{E_{\mathrm{tension}} = \beta_\otimes\,(\mathrm{Lk} - \mathrm{Lk}_{\mathrm{opt}})^2\,\langle\sqrt{c_J^{(i)}\,c_J^{(j)}}\rangle} \tag{13.2}$$

with :

- $\beta_\otimes = 0.210$ MeV (tension constant, calibrated on $\Delta m(n{-}p) = 1.293$ MeV),
- $\langle\sqrt{c_J^{(i)}\,c_J^{(j)}}\rangle = 6.15$ (geometric mean over the 3 pairs of the nucleon),
- $\mathrm{Lk}_{\mathrm{opt}} = +3$ (proton = minimal energy).

Interpretation. The proton ($\mathrm{Lk} = +3 = \mathrm{Lk}_{\mathrm{opt}}$) is the lowest-energy state. The neutron ($\mathrm{Lk} = +2$, $\Delta\mathrm{Lk} = -1$) has a **one-turn linking deficit** which stores 1.293 MeV of potential energy.

13.3 Results for the nucleon

	Lk	Q	ΔLk	E_{tens} (MeV)	m_{TQNT} ($\sqrt{\sigma_\chi}\mathcal{S}$)	m_{SM} (MeV)
Proton	+3	+1	0	0.00	938.2	938.27 (**0.0** %)
Neutron	+2	0	−1	1.292	939.5	939.57 (**0.0** %)
$\Delta m(n{-}p)$	—	—	—	—	1.292	1.293 (0.1 %; structure derived, scale $\beta_\otimes$
r_p ($= 4\,\hbar c/m_p$)	—	—	—	—	0.841 fm	0.841 fm (0.0 %)

Full mass : sector base + linking tension. The mass is written as the sector base ($\sqrt{\sigma_\chi}\,\mathcal{S}$, §9.1) plus the harmonic linking tension :

$$m(L,\mathrm{Lk}) = \underbrace{\sqrt{\sigma_\chi}\,\mathcal{S}}_{\text{sector base}} + \underbrace{\beta_\otimes\,(\mathrm{Lk}-\mathrm{Lk}_{\text{opt}})^2\,\big\langle\sqrt{c_J^{(i)}\,c_J^{(j)}}\big\rangle}_{\text{linking tension } (n\text{-}p \text{ differential})} \tag{13.3}$$

with $\beta_\otimes = 0.210$ MeV and $\langle\sqrt{c_J c_J}\rangle = 6.15$ for the nucleon. **Honest status** : the *absolute* is at 0.003 % ($\sqrt{\sigma_\chi}$ recalibrated 2026-07-18; Regge base $\sqrt{\sigma_\chi}\,\mathcal{S}$) ; the $n{-}p$ *differential* = 1.293 MeV is *derived in structure* at 0.1 % (harmonic form + Lk_{opt} : the proton, at the floor, receives no tension, m_p = base) ; the *scale* $\beta_\otimes$ is *calibrated* on this same differential (uniform status throughout the text) : the *independent* support is the fluxoid (+6.6 % with no parameter) and the SONO relaxed nucleon (+0.63 %).

The nucleon : $\sqrt{\sigma_\chi}\,\mathcal{S}$ (V6) and abandoned forms

	V4 $\alpha_0 U_{\text{EM}} c_J^{3.55}$ (abd.)	Ω_{ts} (abd.)	$\sqrt{\sigma_\chi}\mathcal{S}$ (V6)
Nucleon base	940.2 MeV	938.54 MeV	**938.2** MeV
Error m_p	+0.20 %	+0.03 %	**−0.00** %
Error m_n	+0.20 %	+0.03 %	**−0.00** %
$\Delta m(n{-}p)$	1.293 MeV	1.292 MeV	1.292 MeV (structure derived, $\beta_\otimes$ calibrated)
r_p (charge)	0.839 fm	0.841 fm	0.841 fm
$\beta_\otimes$ tension	0.210	0.210	0.210 MeV
Lk_{opt}	+3	+3	+3

Note. The old 0.03 % (per-knot operator Ω_{ts}) is *abandoned* ; $\sqrt{\sigma_\chi}\,\mathcal{S}$ predicts the absolute at 0.003 % (Regge, $\sqrt{\sigma_\chi}$ recalibrated 2026-07-18), the $n{-}p$ differential remaining derived at 0.1 %.

Stability of the calibration. The tension $\beta_\otimes$ and Lk_{opt} are unchanged (the $n{-}p$ differential does not depend on the base) ; the switch to $\sqrt{\sigma_\chi}\,\mathcal{S}$ does not recalibrate the global parameters : $\sqrt{\sigma_\chi}$ is the single scale (from which m_e derives), $\beta_\otimes$ is topological and unchanged, $\alpha = 1/137.036$ is universal, and $G_F \cdot m_p^2$ varies by −0.01 % (cross-check, base recalibrated 938.2, 2026-07-18). The recalibration is therefore *strictly local* to the nucleon prediction ; the other predictions (lifetimes, nuclear binding energy, etc.) remain valid to better than 0.1 %.

13.4 Proton stability and neutron decay

13.4.1 Proton stability: a barrier, not an exact law

Proton stability — topological barrier, *finite* lifetime

There is no exact topological baryon conservation in TQNT. The χ field is SU(2) ; its only robust charge is the center $\mathbb{Z}_2$ (fermionic parity), *identical* for e^- and the nucleon. Baryon number is therefore *not* a topological invariant, and the twist knot 7_2 has unknotting number $u = 1$ (unknottable by a single crossing change) : **no absolute prohibition**. The proton is stable because it is the **linking ground state** ($\mathrm{Lk}_{\mathrm{opt}} = 3$, the easy relaxation channel closed). Its only remaining decay (undoing the two 7_2, $\Delta c \approx 14$) is suppressed by $\Gamma_p \sim \alpha_{\mathrm{EM}}^{18}/\tau_{\mathrm{weak}}$ (with $18 = c(3_1) + 2\,c(7_2) + 1$, *the same exponent as emergent gravity* $G \sim \alpha_{\mathrm{EM}}^{18}$), hence a **finite** lifetime :

$$\boxed{\tau_p \approx 8 \times 10^{33}\ \mathrm{yr} \quad (\mathrm{not}\ \infty)}, \qquad \tau_p/\tau_n = \alpha_{\mathrm{EM}}^{-18} \approx 3 \times 10^{38}. \tag{13.4}$$

This value lies *in the vicinity of the Super-Kamiokande limit* ($\tau_p > 1.6 \times 10^{34}$ yr for $p \to e^+\pi^0$) — a tension to watch, the TQNT topological channel not necessarily being $e^+\pi^0$ — and is **testable at Hyper-Kamiokande**. **TQNT therefore does NOT predict a rigorous prohibition** : the old "$\tau_p = \infty$" wrongly treated baryon number as an exact Chern–Simons charge. Remaining hypotheses (even-n_{7_2} rule, weak prefactor) are detailed in `brouillon_proton_neutron_chaine`.

13.5 Test on the Σ multiplet

The Σ multiplet ($5_1 + 8_1$, $\Sigma\chi = +2$) provides a test of the harmonic potential. $\mathrm{Lk}_{\mathrm{opt}} = -1$ (Σ^+, $Q = +1$). $\langle\sqrt{c_J^{(i)}\, c_J^{(j)}}\rangle = 7.15$.

	Lk	ΔLk	Δm_{TQNT} (MeV)	Δm_{SM} (MeV)
Σ^+	-1	0	0	0
Σ^0	-2	-1	1.50	3.27
Σ^-	-3	-2	6.02	8.08

The ratio $\Delta m(\Sigma^-)/\Delta m(\Sigma^0) = 4.0$ (prediction $(\Delta\mathrm{Lk})^2 = 4/1$) vs 2.5 (SM). The order of magnitude and the hierarchy are correct.

13.6 Topological gluons : flux tubes between components

In TQNT, **gluons** are not exchanged particles (as in QCD) but **flux tubes** of the χ field connecting the components of a link.

- The linking number $|\mathrm{Lk}_{ij}|$ measures the **number of turns** of one tube around another : it is the topological analogue of the number of exchanged gluons.
- The tension $\beta_\otimes \approx 0.210$ MeV is the energy per linking turn.
- The normalized tension $\beta_\otimes \times \sqrt{c_J^{(i)} c_J^{(j)}}$ gives ~ 1.3 MeV/turn for the nucleon, which is exactly $m_n - m_p$.

13.6.1 Neutron decay

$n \to p + e^- + \bar{\nu}_e$ is a change of Lk ($+2 \to +3$) with no topological modification : it is a **reconnection** of one strand (weak nuclear interaction, chap. 22), *not* the exchange of a mediator. Its **lifetime** is therefore not computed via an energy barrier + tunneling through a *heavy mediator* (the Standard Model picture), but via **Fermi/Sargent** theory with G_F = vev (a topological scale, not a mediator mass) and the energy $Q = \Delta m$. The V6 mechanism is the **release of the trapped loop** by the ΔLk reconnection (cf. the β-decay figure); the precise value follows Fermi/Sargent theory (G_F = vev, $Q = \Delta m$, §15.2). The old knot-count "topological fragility law" is **abandoned** (foreword).

Evaluation. With the TQNT formula (Fermi-Sirlin, G_F from the vev + SM constants) (§22 : $G_F = 1/(\sqrt{2}\,E_{\rm relax}^2) = 1.168 \times 10^{-5}$ GeV^{-2} with $E_{\rm relax} = 246$ GeV = vev — now *derived* (open-crossing on the anchor, +28 ppm, §22.4.4) : the historical "sector wall" is *lifted* ; the three natural routes *closed* that had certified it (2026-07-14, `erelax_writhe_doublet_test.py` : nucleonic fluxoid, factor 140 ; twist elastic of the 8_3 doublet, factor 10–24 ; Compton self-consistency, degenerate in scale — with, in passing, the exact identity $\Phi_\chi^2/(2\mu_0\hbar c) = \alpha\pi/2$). *By contrast, the **structure** of Fermi is derived* (quantitative P15, 2026-07-14, `p15_reconnexion_quantitative.py`) : *scale-free* surgery vertex ⇒ dimensionless amplitude ; second order through the barrier ⇒ contact $\propto 1/E_{\rm relax}^2$; hence $\Gamma = g_{\rm topo}^4 m^5/\,(384\pi^3 E_{\rm relax}^4)$ with $\boxed{g_{\rm topo} = 1}$ (measured 0.99890, unity to 0.11 % ; the deviation = QED radiative corrections) : **there is no weak coupling constant**, only the barrier. τ_τ at +12 % with $N_{\rm eff} = 2 + 3$ ($3 = Z_3$ modes of the hadronic channel : *derived* counting — relative Z_3 phase of the nucleated pair, pinned to one bit by τ_τ, §22.4.4). Relativistic kinematics itself is now derived from the spacetime weave (conditional, §3.5); v is now *derived* (open crossing on the anchor, §22.4.4 — the vev *extracted* from the W/Z links, 242 GeV, remains a ~ 3 % check), then the Fermi-Sirlin formula with the *SM inputs* g_A, $|V_{ud}|$, f_N and m_e^5), one obtains

$$\boxed{\tau_n \approx 880.3\ \text{s} \quad \text{vs} \quad \tau_n^{\rm bottle} = 878.4 \pm 0.5\ \text{s} \quad (+0.001\ \text{decade},\ 0.2\,\%).}$$

Neutron-puzzle NB: the reference is the *bottle* average (PDG); *beam* measurements give $\tau_n^{\rm beam} = 888.0 \pm 2.0$ s — an unresolved $\sim 4\sigma$ discrepancy. The prediction 880.3 s falls *between the two* (+0.2 % from the bottle, −0.9 % from the beam). *NB*: this 0.2 % is the *lifetime* deviation (Fermi-Sirlin chain), unrelated to the former 0.2 % mass deviation absorbed by the $\sqrt{\sigma_\chi}$ recalibration (2026-07-18): its inputs ($Q = \Delta m(n{-}p)$ derived, base-independent; experimental m_e, g_A, $|V_{ud}|$, f_N) do not see the scale — $\tau_n = 880.3$ s is unchanged. *NB* : the old P13/P15 fragility law is *abandoned* (history and reasons in the foreword); the hierarchy is driven by the Q-value (Sargent, $\mathrm{corr}(N, \log Q) = -0.99$) and the slowness of β decay comes from the smallness of G_F (large vev) and the small Q, not from a barrier to cross.

13.7 Topological interpretation of β decay

In TQNT, β decay is a purely topological process :

1. **Initial state** : neutron ($3_1 + 7_2 + \bar{7}_2$, Lk = +2, $E_{\rm tens} = 1.293$ MeV).
2. **Transition** : one of the linking bonds flips (Lk : $+2 \to +3$). The tension energy stored in the linking deficit is released.
3. **Final state** : proton (Lk = +3, $E_{\rm tens} = 0$) + electron (emitted 3_1) + antineutrino (free wave).

The topology of the link is not modified : the three components (3_1, 7_2, 7_2) remain the same. Only the linking number changes. This is why the neutron is **metastable** ($\sim$ 15 min) rather than unstable.

13.8 Complete table of nucleon observables

Observable	**V6 derivation ($\sqrt{\sigma_\chi}\mathcal{S}$)**	**Theory**	**Experiment**	Δ
Mass m_p	$\sqrt{\sigma_\chi}\mathcal{S}$ (Regge base)	938.2 MeV	938.27 MeV	0.0 %
Mass m_n	$m_p + E_{\text{tens}}$ (E_{tens} via calibrated $\beta_\otimes$)	939.5 MeV	939.57 MeV	0.0 %
$\Delta m(n{-}p)$	$\beta_\otimes \times 1 \times 6.15$ (*calibrates* $\beta_\otimes$)	1.293 MeV	1.293 MeV	*calib.*
Charge Q_p	$\Sigma\chi + \text{Lk}$	+1	+1	exact
Charge Q_n	$\Sigma\chi + \text{Lk}$	0	0	exact
Spin	1 fermionic component/3	1/2	1/2	exact
Radius r_p	$4\hbar c/m_p$	0.841 fm	0.841 fm	0.0 %
τ_n	Fermi-Sirlin (SM constants)	880.3 s	878.4 s	0.2 %
m_p/m_e	$\sqrt{\sigma_\chi}\mathcal{S}/m_e$	1836.1	1836.15	0.0 %
Stability p	barrier α_{EM}^{18} (P15)	$\approx 8{\times}10^{33}$ yr	$> 1.6{\times}10^{34}$ yr	*tension*

Note. This table uses the reference V6 formula $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (Regge base, §9.1). The absolute (m_p, m_n, r_p) is at $\leq 0.01\,\%$ ($\sqrt{\sigma_\chi}$ recalibrated 2026-07-18) ; the $n{-}p$ differential remains derived at 0.1 %.

Honesty (what is predicted vs calibrated). In this table, $\Delta m(n{-}p)$ is the *calibration* of $\beta_\otimes$ (a global parameter), *not* a prediction — hence the mention "calib." ; and $m_n = m_p + \Delta m$ inherits it (its residual deviation is in fact that of m_p, the Δm part being exact by construction). The *genuine* predictions are m_p and r_p ($\sqrt{\sigma_\chi}\,\mathcal{S}$; $\beta_\otimes$ *plays no role there* since the proton sits at Lk_{opt}, zero tension), the charge ($Q = \Sigma\chi + \text{Lk}$) and the spin (topological). Once calibrated on Δm, $\beta_\otimes$ becomes *predictive* for its *other* uses (proton stability, widths of the strong resonances).

Chapter 14

Complete spectrum ($m = \sqrt{\sigma_\chi}\,\mathcal{S}$)

14.1 The source formula: derivation and user guide

This chapter accumulated the mid-2026 results (torsion, recoupling ladder, metaxions, bands). This section gives the complete chain, self-contained and in order, for a reader arriving fresh: where the formula comes from, how to apply it, and the status of each piece.

Step 1 — one scale: the χ string. Every massive particle is a confined defect of the spacetime weave (mass = confined frequency, §3.3). The *static* energy of a knot is the length of its tube under the single tension σ_χ ($\sqrt{\sigma_\chi} = 418.47$ MeV, recalibrated):

$$m_K = \sqrt{\sigma_\chi}\,\ell(K) \qquad \text{(the } \textit{knot masses}\text{)}: \tag{14.1}$$

$m_\ell = 310$ MeV (trefoil 3_1, calibrated — the framework's transmutation constant), $m_s = 494.8$ (calibrated on the data; geometric hint 5_2, +5.6 % dressing open — ACPR audit 2026-07-22; rail-side support: combinatorial ratio $36/24 = 3/2$ exact, §3.4; formerly: $m_\ell\,\ell(5_2)/\ell(3_1)$), $m_c = 1\,557$, $m_b = 4\,952$ (calibrated). This is the "$\mathcal{S}$" of the title: a discrete factor of topological class on the single scale.

Step 2 — rotation: the Regge slope. A rotating state adds, per orbital turn L, *one* rotation quantum in M^2:

$$\Delta M^2 = 2\pi\sigma_\chi \cdot L = 1.100\ \text{GeV}^2 \cdot L, \tag{14.2}$$

the *derived* Regge slope — so that the spin of excited states is *read* from the masses alone (31/32 correct, §14.6(d)).

Step 3 — torsion: the $\pi\sigma_\chi$ quantum and the recoupling ladder. Flips of the peripheral *ribbon* cost *half*-turns: $\pi\sigma_\chi = 0.550$ GeV^2 each (verified universal on the vector–pseudoscalar hyperfine, $\pi \to B_s$, 0.7 %, §14.2). The number n of half-turns of a state is fixed by its quantum numbers — the *recoupling ladder*, each term of which is a theorem (conditional on the canonical identifications):

$$n = \underbrace{1}_{\text{Y junction (triality)}} + 3^g\Big(\underbrace{1}_{\text{flip } (V\text{-}P \text{ quantum})}\cdot[\text{flip}] + \underbrace{(\sqrt{2}{-}1)}_{\text{exchange } (d-1)}\cdot[\text{identical}]\Big)_{\text{sat.}} + \underbrace{\tfrac{1}{2}}_{\text{alignment}}\,[J{=}\tfrac{3}{2}], \tag{14.3}$$

with $g = 0$ (charm), 1 (bottom) — *the Z_3 generation step* (factor exactly 3, measured 2.98 ± 0.03) — and the *saturation*: the excess of one flip cannot exceed 1 half-turn (the budget of a single exchange): $\min(3^g(\sqrt{2}{-}1),\,1)$. The base 1 exists only if there is a junction (mesons: 0); the excess $(\sqrt{2}{-}1)$ only if the pair is of the same knot type; the Pauli lock forbids the antisymmetric ground state of identical pairs (ss, cc): no "Λ_{ss}".

Step 4 — radial and the P bands. Radial excitations cost $1.38 \cdot 2\pi\sigma_\chi$ per radial node (calibrated ratio). The P bands of heavy-light mesons: $n_{\text{band}} = 2 + 3^g(2 + \frac{1}{2})$ — the tube's orbital (generation-blind) + the companion's rotation *relative to the heavy knot* (string locking + heavy-frame inertia, §14.3); the strange shift $s = \Delta\text{Wr}/\pi = 0.359$ (*derived* 2026-07-22: the 5_2's ideal writhe excess read in radians, 0.55σ) (measured, qualified) adds to it.

Step 5 — compositions and the boundary. Composite states assemble: heavy mesons = sums of knot masses (the $\ell \to s$ substitution costs *half* on the shared tube: $\text{core}(H_s) = m_Q + (m_\ell{+}m_s)/2$); baryons = Y junction (*full* substitution + ladder); metaxions = sums of the formula's *outputs* ($X(3872) = D + D^*$, etc.). **Declared boundary**: Coulomb-dominated bindings (quarkonia, B_c) are in *compression* ($\tau < 0$): outside the formula on principle (Cornell sector).

The assembled formula and the user guide.

$$\boxed{M^2 \;=\; \Big(\sum_i m_i\Big)^2 \;+\; 2\pi\sigma_\chi\Big(L + \tfrac{n}{2} + 1.38\,n_r\Big)} \tag{14.4}$$

Four steps for any mass: *(i)* identify the sector (light → sector anchors; heavy → knot-mass sums; Coulomb → outside); *(ii)* write the core $\sum m_i$; *(iii)* read (L, n, n_r) from the quantum numbers (Step 3 ladder); *(iv)* apply. **Three worked examples**:

- $a_2(1320)$ (light, rotation): anchor $\rho = 775.3$, L=1: $M = \sqrt{0.775^2 + 1.100} = 1\,304$ MeV (measured 1 318: $-1.1\,\%$).
- Σ_b (heavy baryon, ladder): core $m_b + 2m_\ell = 5\,572$; identical $\ell\ell$ pair, symmetric, J=$\frac{1}{2}$, g=1: $n = 1 + \min(3(\sqrt{2}{-}1), 1) + 3 \cdot 1 = 5$: $M = \sqrt{5.572^2 + 5 \times 0.550} = 5\,814$ MeV (measured 5 813: $+0.01\,\%$).
- $X(3872)$ (metaxion, tetraknot): $D = m_\ell{+}m_c = 1\,867$; $D^* = \sqrt{1.867^2 + 0.550} = 2\,009$; $M = D + D^* = 3\,876$ MeV (measured 3 872: $+0.1\,\%$).

Statuses (for the reader: what is what).

Derived	$2\pi\sigma_\chi$ (slope), $\pi\sigma_\chi$ (quantum, V-P 0.7 %), the full ladder (base/triality, flip/V-P, $\times 3/Z_3$, $\frac{1}{2}$/junction, excess/loop value), the saturation, the half-substitution, the bands $2{+}3^g(5/2)$, the shift $s = \Delta\text{Wr}/\pi$ and the intra-doublet splitting $(3/8)\sigma_\chi/m_Q$ ($\kappa_u = h_{1/2}(k{=}2)$; conditional, 2026-07-22)
Calibrated	$\sqrt{\sigma_\chi}$, m_ℓ, m_s, m_c, m_b, radial ratio 1.38 (global count: §27.2)
Measured/read	— (the j_K=1/2 sign is *derived* 2026-07-22: ladder monotonicity, null alignment at the floor)
Outside	quarkonia and B_c (Coulomb compression, Cornell sector); scalars f_0/a_0 (own sector)

Overall result: 89 catalogue states at median 0.13 %; 57 heavy states at ∼0.3 % (49 with no anchor at all); 226 states at 1.43 % (bands at main document 2026-07-22: D_0^* goes from -8.6 to $+1.8\,\%$ under X=2, $D_1(2430)$ added, B_s at 19/2+s) — and seven dated falsifiable predictions (Ω_b^*, Ξ_{cc}^*, $\Omega_{ccc} = 4\,782$ strict, B_0^*/B_1' — and the Ω_{cc} *verified*: LHCb 06/2026, $\sim 3\,727 \in [3\,715, 3\,788]$ — saturation at $g{\geq}1$, 4th Z_3 rung = refutation).

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14.2 The torsion of peripheral vertices (S4″): the $\pi\sigma_\chi$ quantum and the recoupling ladder

(Consolidated 2026-07-21; scripts `calcul_torsion_peripherique_socle.py`*,* `calcul_reparation_sigmac_torsion.py`*,* `attaque_anomalie_sigmab.py`*,* `derive_p_charme_noeuds.py`*,* `calcul_224_particules_socle.py`*; source draft:* `brouillon_concepts_socle` *§6–7.)*

The torsion quantum. In the S4′ reading, a knot depletes the spacetime weave at its location and the peripheral teeth carry the compensation. A bound or excited state *twists* these peripheral rings, and this torsion is quantized by the derived slope:

$$1 \text{ turn} \;\Rightarrow\; \Delta M^2 = 2\pi\sigma_\chi = 1.100 \text{ GeV}^2 \quad \text{(the Regge slope re-read)}; \qquad \tfrac{1}{2} \text{ turn} \;\Rightarrow\; \pi\sigma_\chi = 0.5501 \text{ GeV}^2.$$

The half-turn is the flip of the *ribbon* — the same 4π fact as spin (§3.5), applied to the peripheral vertices. The category factor is $\tau = \Delta M^2/(2\pi\sigma_\chi)$, in turns.

Key test: the vector–pseudoscalar hyperfine. Over the six pairs $\pi{\to}\rho$, $K{\to}K^*$, $D{\to}D^*$, $D_s{\to}D_s^*$, $B{\to}B^*$, $B_s{\to}B_s^*$: $\Delta M^2(V{-}P) = 0.496 \pm 0.033$ turn — **the half-turn at 0.7 %, universal from the pion to the B_s**, zero parameters. The phenomenological "$\Delta M^2(V{-}P) \approx$ constant" *is* the spacetime weave half-turn (string-model antecedent $\sim 1/(2\alpha')$ declared; here the value comes from the recalibrated σ_χ, with no fit).

The heavy-baryon recoupling ladder — closed forms. The bare stoikhion cores ($m_c{+}2m_\ell$, etc.) fell short by -4 to -11 %: they lack the *spin-recoupling* torsion. The pair structure is imposed by the knots: $\ell\ell$ = twice the *same* knot 3_1, distinguished by Lk (u/d) — exchanging knots of the same type *interferes*; ℓs = *different* knots (3_1, 5_2) — bare flip; ss = two *identical* 5_2 — braided Pauli (derived spin-statistics) *forbids* the ground antisymmetric state: no "Λ_{ss}" (confirmed by the PDG — a structural prediction), and the symmetric ground state carries only the exchange excess. Closed system, *zero parameters, zero anchors*:

$$n \;=\; 1 \;+\; 3^g\Big(\underbrace{1}_{\text{flip} = V-P \text{ quantum}}\cdot[\text{flip}] \;+\; \underbrace{(\sqrt{2}-1)}_{\text{exchange interference}}\cdot[\text{identical pair}]\Big) \;+\; \tfrac{1}{2}\,[J{=}\tfrac{3}{2}], \quad (14.5)$$

with $g = 0$ (charm), 1 (bottom), and $M^2 = M^2_{\text{core}} + n\,\pi\sigma_\chi$. Checks: physical splittings $\Xi_c'{-}\Xi_c = 1.014$ (1 expected), $\Xi_b'{-}\Xi_b = 3.051$ (3), $\Sigma_c{-}\Lambda_c = 1.439$ ($\sqrt{2}$); **the generation factor is exactly** 3 ($p_b/p_c = 2.98 \pm 0.03$ across the three families; $m_b/m_c = 3.18$ disfavoured at +6 % systematic, the core ratio excluded) — one generation step = one Z_3 step, the same Z_3 as N_{gen} (§22.4.4); the $\frac{1}{2}$ for $J{=}\frac{3}{2}$ is the $\frac{1}{2}$ junction bond of the Y (the N-Δ hyperfine). Result: **15 heavy baryons at 0.23 % median** (max 0.73 %), the *two* Ω (Pauli lock, excess only: $n = \sqrt{2}$ and $3\sqrt{2}{-}2$) at -0.01 % each, Ξ_{cc} at -0.2 %. Validated catalogue touch-up (2026-07-21): Λ_c and Λ_b move to stoikhion+torsion ($n{=}1$): +0.6 and +0.0 % (`catalogue_sigma_chi.csv/tex`).

The base derived: the triality half-turn. The base term n=1 is itself a conditional theorem (2026-07-21, `derive_base_jonction_y.py`), whose ingredients are all already canonical: (i) *triality* (§22.4.4): the three strands carry the three *distinct* Z_3 characters of the gate — the neutrality $3 \times \frac{2\pi}{3} = 2\pi \equiv 0$ is what allows the Y junction to exist; (ii) around the junction, the χ condensate phase therefore winds **one full turn** (winding = 1, not 0); (iii) the ribbon frame is the 4π object (§3.5): it rotates by *half*, π — one half-turn; (iv) the quantum gives $n_{\text{base}} = 1$ *exactly*. For free: the meson = character + conjugate (compensated relative Z_3 phase) ⇒ base 0 — measured 0.11 ± 0.16 against 0.88 ± 0.09 for baryons, the separation *is* the junction; *generation invariance* ($\text{base}_b/\text{base}_c = 0.94$, not 3): the base is *portal* (the gate's characters, identical for any content), whereas the pair excess is dynamical; the antibaryon (-2π) is even in energy; heavy dibaryons carry base 2 — a structural prediction *computed* 2026-07-22: H_{cc}^{++} doublet 0^+ $4\,497 \pm 10$ (deeply bound, displaced vertex) / 1^+ $4\,572 \pm 10$ (on the $\Lambda_c\Lambda_c$ threshold; bridge derived ℓ=D); H_{bb}^0 bound; light H *absent* (40 years of nulls postdicted); §14.3. Numerical corollary, **verified 2026**: Ω_{cc} predicted 3 715–3 788 MeV (bracket = the declared Ξ_{cc} lock/flip ambiguity) — **observed by LHCb** (Beauty 2026, 03/06/2026, $\Omega_c^0\pi^+$): ∼3 727 MeV, *inside* the bracket, at +0.3 % from the *lock* edge ⇒ the ambiguity is *settled by the data* (the *cc* pair at the lock, like *ss*) and $\Omega_{ccc} = \mathbf{4\,782}$ MeV becomes a *strict* prediction (lattice benchmark ∼ 4.8 GeV compatible). *Declared chronology*: announcement 03/06 < consolidation 21/07 — blind derivation *in practice* (the main document treated the state as unobserved), not a pre-registration. (`confrontation_lhcb_2026.py`.); Ω_{bbb} *outside* the torsion frame (Coulomb compression — negative binding expected, the sign the categorization predicts).

The $\sqrt{2}-1$ hint: an identification (2026-07-21). Three independent determinations of the exchange excess (Σ_c, $\ell\ell - \ell s$ flip: 0.4256; Ω_c, offset − base: 0.4159; Ω_b, /3 of the Z_3 step: 0.4191) give **excess** $= 0.4202 \pm 0.004$ **(stat)** $\pm\sim 0.01$ **(cores)**: a structural constant, flavour-blind. Mechanism: the identical-knot pair has *two fusion channels* (sym/antisym — for *ss*/*cc* the ground antisym is locked, but the *channel* exists); the quantum-dimension theorem then *forces* $d = \sqrt{2}$ — this is $d_{1/2}(k{=}2)$, **the same number as the Koide amplitude** $A{=}\sqrt{2}$: the main document's two hints are *one* structure, the channel doubling of identical topological objects; excess $= d-1$ (+1.4 % from the measurement). Eliminated competitors: the Ising weight $h_\psi = \frac{1}{2}$ *excluded* by the data ($\sim 4\sigma$); L2 co-rotation ruled out by geometry (axes at 120 at the junction: cost 1, it would take 90); 5/12 (+0.8 %) declared but *without mechanism* (null rule). The link "residual twist = $(d-1)$ half-turn" is *proven* below. (`derive_indice_racine2.py`.)

The link proven: the loop value (2026-07-21). Three pieces close the link: *(1) geometric fact* (verified to 10^{-16}): a half-turn of the ring around the two-strand cable *exchanges* the two members — so the excess attaches to the half-turn unit, and *only* for identical pairs (distinct members: the exchanged configuration is a *different* state, no closure condition, no residue); *(2) loop value* (Temperley–Lieb, exact): the Markov closure of the exchange element e on two strands is worth d (one loop; $\text{tr}(\mathbf{1}) = d^2 = 2$ = the number of channels, consistent), the classical closure (distinguishable) is worth 1 — **excess** $= d - 1$; the identification "closed loop ↔ stored half-turn" is *that of the S4″ quantum itself* (no new hypothesis — one identification, two uses); *(3) exact identities*: $d - 1 = \tan(\pi/8) = \tan(2\pi h_\sigma)$ [h_σ=1/16] — the excess is also the tangent of the topological-spin angle (k=2 over-determination declared, no test in k). **Discriminant**: the only competing reading, the raw channel phase (3/8 = 0.375), is *rejected* by the measurement (0.4202 ± 0.004: +12 %, $\sim 4\sigma$ with the core systematics). (`demontre_maillon_twist.py`.)

The $\Sigma_b-\Lambda_b$ tension resolved: saturation (2026-07-21). The bottom $\ell\ell$ flip is a physical splitting (core-free): 4.0212 ± 0.011 half-turns against $3\sqrt{2} = 4.2426$ (−5.2 %, $\sim 21\sigma$ experimental — missing structure, not noise). The resolution comes from the link's own geometric fact: *one ring half-turn = one exchange bounds* the residual content of *one* flip to **one** half-turn (the

budget of a single exchange). At g=0: $(\sqrt{2}-1) = 0.414 < 1$, no saturation (charm intact); at g=1: $3(\sqrt{2}-1) = 1.243 > 1 \Rightarrow$ *saturated at* 1 $\Rightarrow$ bottom $\ell\ell$ flip $= 3 + 1 = 4$ *exactly* (measured +0.5 %). **The dynamic/static distinction is tested**: the lock (Ω, static ground of the ring, no single exchange event) is *not* bounded — Ω_b measures 1.257 vs 1.243 (+1.1 %, intact); the bare ℓs flip is insensitive. Masses: Σ_b moves to $n = 5$ (integer): $+0.21 \to +0.01$ %; Σ_b^* to $n = \frac{11}{2}$: $+0.28 \to +0.08$ %. Final closed rule (zero parameters — the bound *is* the geometric fact):

$$n = 1 + 3^g\,[\text{flip}] + \min\big(3^g(d-1),\,1\big)\,[\text{identical, flip}] + 3^g(d-1)\,[\text{identical, lock}] + \tfrac{1}{2}\,[J{=}\tfrac{3}{2}]. \quad (14.6)$$

Falsifiable consequence: any identical-pair flip at $g \geq 1$ saturates at the integer (never $3\sqrt{2}$); at g=0, never any saturation. (`resolve_tension_sigmab.py`.)

Metaxions (the LHC "molecules") and the 226-state extension. *Decree 2026-08-06:* these states — two *whole* hadrons pinned just below their threshold — form the **metaxy** (Greek *metaxú*, "the in-between"); such a state is a **metaxion** (the "molecules" of the literature), and its composition is given as a *knot count*: *tetraknot* (meson+meson), *pentaknot* (baryon+meson); the dibaryon H_{cc} is a *fused hexaknot* (connected network, outside the metaxy). The metaxions = spacetime weave hadrons *with* their (half-)turns (standard molecular contents): $X(3872){=}D\bar{D}^*$ +0.1 %, T_{cc} +0.0, $Z_c(3900/4020)$ $-0.3/-0.2$, $Z_b(10610/10650)$ $-0.3/-0.2$, pentaknots P_c/P_{cs} < 1 % (the historical -6 % was *entirely* the bare Σ_c core), $X_0(2900){=}\bar{D}^*K^*$ +1.3 % — median 0.3 %. The full extension (226 states, all known particles + recent LHC, one spacetime weave reading per state: mass, spin, fraction $m/M_{\rm Pl}$) holds at **global median** 1.43 % (21/07 clean-up: half-substitution, saturation; 22/07: bands at main document — derived values, $X(1/2){=}2$), with 31/32 trajectory spins *read* from the masses alone (inverse Regge). The τ categorization: 0 ground states; $\frac{1}{2}$ meson flip (universal); the baryon recoupling ladder; L orbital; $1.38\,n$ radial; integers for metaxions; $\tau < 0$ quarkonia = Coulomb *compression* — the sign of τ separates torsion from compression and explains why Cornell is its own sector.

Falsifiable predictions. Ω_b^* ($J{=}\frac{3}{2}$, unobserved): $M = 6067$ MeV, splitting +22 MeV above Ω_b (LHCb-accessible); $\Xi_{cc}^* = 3659$ MeV. Consistency: no generation beyond (the Z_3 saturation, §22.4.4) — the factor 3 is unique like the generation step; any fourth rung would refute it.

Remark

Declared statuses: *derived* — the $\pi\sigma_\chi$ quantum (verified V-P at 0.7 %), the bare flip = 1 (the V-P quantum itself), the $\times 3$ (Z_3), the *ss* Pauli lock, and the *base* 1 (triality half-turn, conditional theorem — dedicated paragraph); and the exchange excess $(d-1)$ ($d = \sqrt{2}$ forced by the two-channel fusion; link *proven* 2026-07-21 by the loop value, cf. dedicated paragraphs) — **the ladder no longer has a free link**: base, flip, $\times 3$, $\frac{1}{2}$ and excess, five theorems conditional on the canonical identifications. The $\Sigma_b{-}\Lambda_b$ tension (-5 %) is *resolved* by the exchange-budget saturation (cf. dedicated paragraph: flip = 4 exact, Σ_b at +0.01 %). Declared residues: Ω_c^* -0.7 % (the D_s/B_s cores are repaired by the half-substitution, §14.3). Scale null: the rung spacing (110 MeV at charm, 47 at bottom) would make a per-state fit empty — the closed forms run through the 15 states with no fitted number.

14.3 The spacetime weave equation: vacuum, particle, master functional

(Consolidated 2026-07-21; scripts `verif_equation_socle.py`, `calcul_masses_equation_socle.py`*; source draft:* `brouillon_concepts_socle` *§10–11. The added value is the* single writing*: all content is canonical — P5, QGF, S4′–S4″ unified into equations.)*

Without a particle — the free spacetime weave. The χ wave background (level α^0), graduated in action (each tooth = h):

$$\Box\,\chi = 0, \qquad \omega = c\,|\mathbf{k}| \text{ (exact: zero dispersion = structural Lorentz)}, \tag{14.7}$$

uniform amplitude A_0 — the quartet's G line (the bare spacetime weave's amplitude). Nothing forms, nothing depletes: no wells, no operating G.

With a particle — S4′ as an equation. Depletion $A(\mathbf{r}) = A_0\,(1 - \delta(\mathbf{r}))$; the **differential compensation law** is the P5 Poisson in local form, and **mass is the charge** of the depletion field:

$$\nabla^2 h \;=\; -\,\kappa_g\,|\Psi|^2 \quad (h = \Omega^2 - 1), \qquad m \;=\; \frac{c^2}{2G}\,\lim_{r\to\infty} r\,h(r). \tag{14.8}$$

Verified: GPS 45.7 μs/day exact ($\delta f/f = h/2$); double smallness at the proton $h(r_p) = 2.96 \times 10^{-39}$; Newton and the QGF are the gradient readings.

The mode and the master functional. The defect is a confined χ wave ($\omega_0 = mc^2/\hbar$; the self-consistent solver *is* this equation — microscopic layer). Its static energy is the length: $M_{\text{topo}} = \sqrt{\sigma_\chi}\,\ell(K)$ — the geometric law *re-derived* as the defect's energy functional; the excitations are the S4″ terms, whence:

$$\boxed{M^2 \;=\; \Big(\sum_i m_i\Big)^2 \;+\; 2\pi\sigma_\chi\Big(L \;+\; \tfrac{n}{2} \;+\; 1.38\,n_r\Big)} \tag{14.9}$$

with $m_i = \sqrt{\sigma_\chi}\,\ell(K_i)$ the knot masses, L the orbital turns, n the recoupling-ladder half-turns (§14.2), n_r the radial.

The masses from the equation. Run over the spectrum with, as *sole* dimensional inputs, the 5 canonical scales ($\sqrt{\sigma_\chi}$ + 4 knot masses — m_s now calibrated, ACPR audit 22/07), and per state its topological numbers (L, n, n_r — integers and closed forms, never fitted):

Class	N	$\lvert\Delta m\rvert$ median
heavy mesons (ground+vectors, half-substitution)	10	0.2 %
heavy baryons (S4″ ladder)	15	0.23 %
LHC metaxions (composed outputs, $+DK/D^*K$)	15	0.3 %
heavy baryons (orbital band)	8	4.5 %
P doublets j_K=3/2 (read bands[b])	8	0.4 %
with no per-state anchor	**49**	**$\sim$ 0.25 %**
+ P doublets (3 read bands)	57	$\sim$ 0.3 %

[b]bands read as anchors (bottom *universal* 19/2 — B and B_s together, ±0.3 %; charm 9/2/$\sim$ 4.9): *identified* 2026-07-21 — $n = 2 + 3^g\,(5/2)$, cf. dedicated paragraph. Reclassified: $D_{s0}^*(2317)/D_{s1}(2460) \to DK/D^*K$ metaxions (+1.8 % coherent, common binding −43 MeV declared); $B_c/B_c(2S) \to$ *compression* (half-quarkonium, negative binding, sign tested); $D_0^*(2300)$ ($\Gamma \sim 230$ MeV) residual. **Half-substitution** (2026-07-21): in a meson (one shared K+$\bar{K}$ tube) the $\ell \to s$ substitution costs $(m_s - m_\ell)/2$ (measured 0.50 ± 0.03); in a baryon (own legs), the full amount (measured 0.98/0.93) — core$(H_s) = m_Q + (m_\ell + m_s)/2$: D_s +4.2 → −0.45 %, B_s +1.5 → −0.23 %, $D_s^*/B_s^* < 0.9$ %, cascade Z_{cs} +2.0 → −0.36 % (`resolve_coeurs_dsbs.py`, `resolve_bandes_p.py`).

The band values identified (2026-07-21). The j_K=3/2 band splits as $n_{\text{band}} = 2 + 3^g\,(5/2)$ ($+\,s$, *derived* 2026-07-22 — cf. the dedicated paragraph): the bare orbital (2 = the Regge slope, generation-blind — the light mesons' only share, verified a_2/K_2^*) + the companion excitation *coupled to the heavy knot* ($X = 5/2$, $\times 3$ per generation — **third appearance of the Z_3 step**, measured 2.96; the puzzling centroid ratio $\sim$2.1 = (2+7.5)/(2+2.5): the $\times$3 was *hidden* by the orbital). The read bands 9/2 and 19/2 come out of *one* number; the 8 doublet states at $\leq 0.8\,\%$ (B/B_s at $\pm 0.2\,\%$). j_K=1/2: $X = 2$ (null alignment — *derived* 2026-07-22 by the ladder's monotonicity, cf. the dedicated paragraph; charm centroid at $-0.4\,\%$). **Prediction**: the bottom j_K=1/2 doublet (B_0^*/B_1', unobserved, broad): centroid 5 665 MeV (a point). The 5/2 is *proven* below, the s shift *derived* ($\Delta\text{Wr}/\pi$) and the intra-doublet splitting has a *closed* structure (Compton response, measured coefficient κ_{red} — 2026-07-22, cf. the dedicated paragraphs). (`derive_valeurs_bande.py`.)

The 5/2 proven (2026-07-21). Three pieces: *(1) geometric fact* (verified: $1.000000 \times 2\pi$) — the *material* frame of a rotating string makes one full turn per revolution; the light end, locked to the tube, turns *with* it: 2 half-turns; *(2) inertia asymmetry* — the *heavy* knot's frame is inertial (the pivot): the *relative* tube/heavy rotation accumulates at *its* ring $\Rightarrow 3^g$ (the relative-to-heavy); the *tube*'s own turn = the Regge slope (portal, generation-blind); **corollary**: light-light = both ends dragged = zero relative = pure n=2 — *the light sector is a corollary, not an exception*; *(3) alignment* j_K=3/2: $+\frac{1}{2}$ (the ladder rung). Whence $X = 2 + \frac{1}{2} = 5/2$. Refinements: the j_K=1/2 case admitted three alignment readings ($0/-\frac{1}{2}/-1$) — *settled* 2026-07-22 by the ladder's monotonicity (X=2 exact, cf. the dedicated paragraph; the apparent D_0^* preference for $-\frac{1}{2}$ was a member-vs-centroid artefact): B_0^*/B_1' prediction at the *point* 5 665 MeV; L=2: *settled* 2026-07-23 (the frame turn is per period, cf. the dedicated paragraph — D 1D top 2 759, +0.09 %; the old $[2.66, 2.86]$ bracket is superseded). Named conditions: string locking (light end), inertia of the heavy frame. (`demontre_cinq_demis.py`.)

The strange shift derived: $s = \Delta\text{Wr}/\pi$ (2026-07-22). The strange band shift ($s = 0.3577(23)$, three determinations including one with no TQNT core at all) is the *ideal writhe excess* of 5_2 over the trefoil, read in radians: $s = [\text{Wr}(5_2) - \text{Wr}(3_1)]/\pi = 1.1277/\pi = 0.3590$ (0.55σ; writhes converged to the 4th digit on the ACPR-verified katlas base), i.e. in mass $\Delta M^2 = \sigma_\chi\,\Delta\text{Wr}$ — the writhe converted at the ladder's rate ($2\pi\sigma_\chi$ per turn $\equiv \sigma_\chi$ per radian). The chain, in four theorems/measurements: *(1) Fuller holonomy* (theorem *verified*, $\leq 4 \times 10^{-3}$ rad by discrete transport): the parallel (twist-free) frame around the closed curve returns rotated by $2\pi\text{Wr}$ (mod 2π) — the writhe *is* a transported angle; the turn-unit readings are *excluded* by the data (full defect 825σ, double cover 335σ, defect mod 2π — the "naive" reading — 44σ: the shift reads the *full* writhe, in radians); *(2) uniformization* (exact theorem): elastic equilibrium under the Fuller constraint gives a *constant* density $\omega = \text{Wr}$ (rad/rad) — the junction receives only this dimensionless number; *(3) locality*: the total holonomy is *global*, a point-like reading can only depend on ω ($s = f(\omega)$, zero at the trefoil — the gate defines the frame; linearity measured at 0.4 %); *(4) the measured aperture*: the density→angle conversion rate is $a = \pi s/\Delta\text{Wr} = \mathbf{0.9965 \pm 0.0078}$ rad — **the identity at** 0.5σ; every angular scale the portal *owns* is excluded ($\pi/3$ ribbon$\times Z_3$, the closest competitor: 6.5σ; $2\pi/3$: 141σ; π, 2π: $> 270\sigma$; *metric* footprint: excluded by the sign), and the identity is the unique *scale-free* rate (action-graduated bedrock). The structural discriminant: the shift exists at the *mesonic* portal of zero winding (base 0, triality) — readings via the triality winding or a Y-junction sector are excluded; the reading is *portal-independent* (consistent: $s(D_s)$=$s(B_s)$, no $\times 3^g$; the rotation share multiplies the *reading*, the aperture is invariant). **Status**: derivation *conditional* on the identification "scale-free rate = identity" (at the rank of σ=b in the v chain and loop↔half-turn in S4″): *the shift chain has no free link left.* **Falsifiable prediction**: the orbital bands of strange *baryons* (Σ^*/Ξ^*, L=1) carry *the same* $s = 0.358(2)$ — a value $\Delta\text{Wr}/3 = 0.376$ (Z_3-sector reading) would refute; *tested 2026-07-22* on

the λ-mode doublets: sign and order **confirmed** (+0.32/+0.25, cf. the LHCb paragraph below). (`demontre_maillon_fuller.py`, `demontre_aperture_portail.py`.)

The intra-doublet splitting: structure derived, coefficient pinned (2026-07-22). The splitting between the two members (J=1/J=2) of the j_K=3/2 doublets (D_1/D_2^*: 39.0 MeV; D_{s1}/D_{s2}^*: 34.0; B_1/B_2^*: 13.4; B_{s1}/B_{s2}^*: 11.2) has its structure *derived* in four pieces: *(a) degeneracy* at $m_Q \to \infty$ — a corollary of the pivot (the inertial heavy frame makes the band ladder *blind* to S_Q: this is *why* the doublet is a doublet, cf. the shared B/B_s band n=19/2); *(b) the* $1/m_Q$ *law = Compton response*: the heavy ribbon responds to the rotating tube's torque on its Compton time — *the same identification* as the canonical magnetic moment (tube torsion at the $\lambda\!\!\!^{-}_C$ radius, §14.15) $\Rightarrow \Delta m = \kappa/m_Q$, *verified*: $\kappa = \Delta m \cdot m_Q$ constant to ±9 % while Δm varies by ×2.9; *(c) the ordering*: the aligned state (J=2) above, 4/4; *(d) the strange dilution = half-substitution of the* moment: on the meson's shared tube, $\ell \to s$ carries only half of the moment: $\kappa_s/\kappa_\ell = (1{+}m_\ell/m_s)/2 = 0.813$ vs 0.852 ± 0.019 measured (+4.8 %); applying it *tightens* the dispersion from 9 to 4 %. **Coefficient derived (2026-07-22)**: per *unit* of $\langle S_Q \cdot j_K \rangle$ (the Clebsch gap between members is 2), $\kappa_u = 3/16 = h_{1/2}(k{=}2) = j(j{+}1)/(k{+}2)$ — the *conformal weight* of the spin-$\frac{1}{2}$ Wilson line at level 2, **the same object whose quantum dimension is** $d = \sqrt{2}$ (Koide, the $d{-}1$ excess): the *third invariant of the ribbon*; the topological spin $\theta = e^{2\pi i h}$ is the ribbon's response to a 2π rotation — the very physics of the splitting (the two J = the ribbon co-/counter-rotating with j_K). Mechanism selection: $s = \Delta\mathrm{Wr}/\pi$ excluded *structurally* ($\Delta\mathrm{Wr} = 0$ for non-strange doublets: the trefoil is the portal's zero — it would predict a null splitting for D_1/D_2^* and B_1/B_2^*); $1/e$ unanchored (null rule); the *closed* set of k=2 invariants scanned: 3/16 at 0.5σ (measured $\kappa_u = 0.1857 \pm 0.0038$), all others excluded (Ising 1/16: 32σ; $(d{-}1)/2$: 5.6σ; h_ψ, $1/d^2$: 82σ). Form:

$$E(\text{per unit of } \langle S \cdot j \rangle) = h_{1/2}\,\sigma_\chi\,\lambda\!\!\!^{-}_C(m_Q) = \tfrac{3}{16}\,\sigma_\chi/m_Q$$

— three separately canonical factors ($h_{1/2}$: the k=2 chain; σ_χ; $\lambda\!\!\!^{-}_C$: Compton response = the magnetic-moment identification); the assembly (a product with no free scale) is the declared identification, at the rank of $\sigma{=}b$ and the identity aperture. Residues: B +1.0 %, D_s −0.9 %, B_s +3.5 %; the charm-sector doublet (D_1/D_2^*, c knot) −7.5 % (dominant gap, $1/m_c$ correction of the Compton response — open). *Predictions*: $\Delta m = (3/8)\,\sigma_\chi/m_Q$ for any j_K=3/2 doublet (×0.813 for a strange end); strange *baryonic* P bands: *full* dilution (m_ℓ/m_s, separate legs — a joint discriminant with the shift-s prediction); B_0^*/B_1' split by **6.63** MeV (1 unit; the charm j_K=1/2 sector, broad and disputed, stays declared distorted — coupled channels). (`derive_splitting_doublet.py`, `derive_kappa_red_mecanisme.py`.)

The j_K=1/2 alignment sign: dissolved by monotonicity (2026-07-22). The three readings of the previous paragraph (X = 2+0, 2−$\frac{1}{2}$, 2−1), degenerate at j_K=3/2, are separated by a *monotonicity theorem*: every contribution of the canonical ladder is ≥ 0 (closed rule: base, flip, ×3, $\frac{1}{2}$, excess — all positive; $V{-}P$: the anti-aligned P is the *floor*, the V counts +1; baryonic rung: J=1/2 counts 0, never $-\frac{1}{2}$): the stored torsion is an *excess over the floor*, with no negative rung. The anti-aligned j_K=1/2 *is* the floor of the $L \otimes s_K$ couple $\Rightarrow$ alignment cost 0: $\boxed{X(1/2) = 2}$ exactly — the sign question *dissolves*. Data: the band predicts ($2J{+}1$)-weighted *centroids* (the sector's practice; 3/2 check: −0.2 %) — at the measured charm centroid (2 394.8: D_0^* 2343(10), D_1' 2412(9)), X=2 gives **−0.4** %; the $-\frac{1}{2}$ (−2.9 %) and −1 (−5.4 %) readings are *excluded*. The old "D_0^* prefers $-\frac{1}{2}$" compared the low member alone: a member-vs-centroid artefact, corrected. Members with the derived splitting (21.1 MeV): D_0^* +1.1 %, D_1' −0.9 % — of the expected sign of the declared DK distortion (threshold 2 363 MeV, double pole). **The prediction sharpens**: centroid B_0^*/B_1' = **5 665** MeV (a point — 5 591 was the $-\frac{1}{2}$ reading, 5 517 the −1 reading: observing them would refute monotonicity), members ∼5 660/5 666 (splitting 6.6 MeV). (`derive_signe_alignement.py`.)

Section balance: centroids ($2{+}3^g(5/2)$, the 5/2 proven), shift s ($\Delta\mathrm{Wr}/\pi$), splitting (Compton structure + $\kappa_u = h_{1/2}$), 1/2 alignment (monotonicity) — **the band sector is fully derived, zero read numbers** (theorems, declared conditional identifications, independent measurements); joint predictions: B_0^*/B_1' (centroid + splitting), strange baryonic bands (shift *and* full dilution).

First LHCb confrontation (2026-07-22). The section's predictions confronted with existing LHCb/PDG data (`confrontation_lhcb_2026.py`): *(i) the strange baryonic shift* (prediction of the dedicated paragraph) tested on the λ-mode P doublets (spin-0 diquark, one $\ell \to s$ substitution): $\Delta n(\Xi_c) - \Delta n(\Lambda_c) = +0.316$ and $\Delta n(\Xi_b) - \Delta n(\Lambda_b) = +0.251$ vs $s = 0.359$ — **sign and order confirmed in two independent sectors** (far from zero); the decimal stays open ($-12/-30\,\%$: systematics of the grounds/cores, or a partial dilution to examine); the Ω_c (2017/2023 quintet) and Ω_b (2020 quartet) multiplets, with a spin-1 ss diquark, are *outside direct comparison* (multiplet rule half-derived — dedicated paragraph below); *(ii) the ss lock of the grounds re-verified on data*: $n(\Omega_c) = 1.4159$ vs $1{+}(d{-}1) = 1.4142$ ($+0.12\,\%$), $n(\Omega_b) = 2.271$ vs $1{+}3(d{-}1)$ ($+1.24\,\%$); *(iii) the baryonic P bands measured*: $\Delta n(\Lambda_c) = 2.94$, Ξ_c 3.25, Λ_b 6.25, Ξ_b 6.50 — *derived the same day* (two-strand cable, $4(d{-}1)$: dedicated paragraph below). For the Ω_{cc} observed inside the bracket (the Ξ_{cc} ambiguity settled, $\Omega_{ccc} = 4\,782$ strict): §14.2.

The Ω_c/Ω_b multiplet rule (2026-07-22, derived — the alignment ladder as a conditional identification). *Content derived*: the Pauli lock forces the ss pair into spin 1; in P wave, $j = L \otimes S_{ss} = \{0,1,2\}$, then $\otimes S_Q$: **five states** ($\{\frac{1}{2}\}_{j0}$, $\{\frac{1}{2},\frac{3}{2}\}_{j1}$, $\{\frac{3}{2},\frac{5}{2}\}_{j2}$), the $j{=}0$ floor by monotonicity — the Ω_c quintet (LHCb 2017/2023) *conforms*; in the Ω_b quartet (2020), **the fifth is missing**. *The alignment ladder is heavy-blind* (measured): step $j1 \to j2 = 0.528$ (c) vs 0.470 (b) in n ($-11\,\%$: a light-sector term, like the band step); candidate $\sim \frac{1}{2}$ per level ($\pm 6\,\%$ on $j1 \to j2$); the $j0 \to j1$ step (0.66, $+32\,\%$) not pinned (non-uniform floor or the 3000 assignment (2S?) — null rule). *Heavy splittings* (previous §: $h_{1/2}\sigma_\chi/m_Q \times$ full dilution m_ℓ/m_s): charm *supported* (ratio $j2/j1 = 1.90$ vs Clebsch 5/3, $+14\,\%$; amplitudes $-23/-12\,\%$); bottom: $j2$ at $\mathbf{-2}\,\%$, $j1$ inverted — *declared candidate*: $O(1/m_b)$ **mixing** of the two $J{=}\frac{3}{2}$ states (6 330.3/6 339.7, 9.4 MeV apart), which inflates the apparent $j1$ splitting and preserves $j2$. **Predictions**: $\Omega_b(1P, j{=}0, J{=}\frac{1}{2}) =$ **6 297–6 304** MeV (the fifth, below the quartet; larger width expected — plausibly why it is missing); strong mixing of the two $\Omega_b(3/2)$ (testable: $\Xi_b K/\Xi_b' K$ ratios; the quartet's spins will settle it). *The centroid, derived the same day* (the puzzle $+0.79$ charm vs ~ 0 bottom): two structure theorems *from the main document* resolve it — *(a) the shift is intensive*: the aperture (§shift) reads a *density*, uniformized *per ring*; two strange legs present the same density $\Delta\mathrm{Wr}$ as one $\Rightarrow$ no second s (Ω carries the same $+s$ as Ξ; corollary: three strange legs would still give $+s$, never $+3s$); *(b) the lock un-triples in P wave*: the $\times 3^g$ applies to what accumulates at the *heavy* ring (inertia asymmetry); in the ground state the ss pair sits *at* the junction ($3^g(d{-}1)$, measured); in P wave it *orbits*: the excess migrates to *its own* ring $\to (d{-}1)$ **flat**. Whence the rule:

$$n(\text{level } j) = 1 + (d{-}1) + \Delta n_{\text{band}}(\Xi_Q) + x_j \;\Rightarrow\; \text{excess}(\Omega{-}\Xi) = A - (3^g{-}1)(d{-}1),$$

verified level by level (*uniform* offsets: -0.093 ± 0.000 at charm — all three levels —, -0.044 ± 0.029 at bottom); **the observed c/b gap** $0.809 = 2(d{-}1) = 0.829$ **at** $-2.4\,\%$ — the inconsistency *was* the un-tripling. The fifth Ω_b is *reconstructed* at 6 297 MeV (a route independent of the transfer). *Falsifiable*: any identical pair in P wave carries the *flat* lock $(d{-}1)$ (future Ξ_{cc}/Ω_{cc} P waves will test it). *The alignment ladder, finally, has its mechanism* (2026-07-22): the locked pair = *two* ribbons; aligning against L is counted **per member**, at the rung $\frac{1}{2}$ — the already-canonical alignment quantum (mesonic band $+\frac{1}{2}$ for one ribbon, §17; baryonic $J{=}3/2$ $+\frac{1}{2}$; $V{-}P$ $+1$, the full pair flip): $x_j = j \times \frac{1}{2}$, ladder $(0, \frac{1}{2}, 1)$, *zero new numbers* — monotonicity and heavy-blindness automatic. *Support*: the $j1 \to j2$ step brackets $\frac{1}{2}$ at both generations (0.528 c /

0.470 b, ±6 %); *declared tension*: charm x_1 (0.661, +32 %), concentrated on the assignment of *one* state ($\Omega_c(3000) = j0$; parity forbids the 2S excuse, $1/2^+$ vs $1/2^-$: it is the assignment, not the dynamics); the numerical competitor $(2/3, 7/6)$ has no mechanism (null rule, not retained). **Favoured conditional identification**: if it holds, the multiplet rule is *complete with no measured number at all. Discriminants*: the fifth Ω_b's mass — **6 304** (flips) vs 6 297 (charm-measured ladder), 7 MeV apart, observation will decide; the Ω_c quintet *spins* (a re-assignment of the second J=1/2 recomputes x_1); the Ξ_{cc}/Ω_{cc} P waves (ladder $(0, \frac{1}{2}, 1)$ + flat lock predicted). (`derive_multiplet_omega.py`, `derive_centroide_multiplet.py`, `derive_echelle_alignement.py`.)

The baryonic orbital bands derived: the two-strand cable and $4(d-1)$ (2026-07-22). The last named structural residue falls. The mesonic form $2 + 3^g X$ *fails* for baryons (X not universal: 0.94 c vs 1.42 b) — the companion is not a knot, it is a **two-strand cable** (the diquark). *The identification*: the relative part is $(\Delta n_b - \Delta n_c)/2 = 1.654$ vs $4(d-1) = 1.657$: $\mathbf{-0.16}$ %. Mechanism (the link, §ladder): each half-turn of the ring around a two-strand cable *exchanges* the members (proven theorem), excess $(d-1)$ per exchange; per revolution, *four* half-turns act on the cable (2 from the tube + 2 from the dragged frame) $\to 4(d-1)$, stored $\times 3^g$ — the *fourth appearance* of $d-1$; declared condition: the exchange has no portal-blind component (canonical rank). Whence the rule:

$$\Delta n = b + 3^g \cdot 4(d-1)\ [+\, s \text{ if strange, intensive}]\,, \qquad b = 2 - \langle x \rangle = 23/18 \text{ (derived 2026-07-23, below)}$$

Nine values, zero read numbers (b derived below): Λ_b −0.1 %; Ξ_c/Ξ_b −1.3/−1.7 % (the shift deficit, §LHCb); Ω levels via the *full chain* $b \to \Lambda \to \Xi \to \Omega$ (+ flips + flat lock): Ω_c $j1/j2$ +0.5/+0.9 %, Ω_b $j1/j2$ $\mathbf{+0.4/+0.03}$ %; only gap: Ω_c $j0$ (−2.9 %), *the* assignment suspect. *Bonus*: the Ω's **select the flip ladder** ($A = 0.722$, *identified* $= \langle x \rangle = 13/18$ at +0.03 %, below) — mutual support. *Predictions*: the Σ_c/Σ_b 1P multiplets are computable (spin-1 $\ell\ell$ diquark, no s); Ξ_{cc} P waves (cc cable, flat lock, same $4(d-1)$). The N^*/Δ^* stay in the proper Regge sector (declared). *The status of b: DERIVED* (mechanized sum rule, 2026-07-23; the 2026-07-22 pinning stands: 1.281 ± 0.015, exact-3 ratio at 0.03 %). *The key*: $A_{\text{flips}} = 13/18$ is the **trace average** of the mechanized ladder $x_j = j/2$ over the coupling space L=1⊗cable axis (weights $2j+1 = 1, 3, 5$): $\langle x \rangle = (0 + \frac{3}{2} + 5)/9 = 13/18$ *exact* — the measured A stops being a number. A four-piece theorem: **(1) blindness forces the trace** (the tube turn is j-blind, the pivot corollary: a blind deposit cannot select a $j \to$ uniform average); **(2) conservation of the blind slope** (the turn is worth 2, the mesonic theorem): $b = 2 - \langle x \rangle = \mathbf{23/18} = 1.2778$ (0.2σ from the measurement); *the sum rule is measured independently*: b (Λ/Ξ bands) $+A$ (Ω centroids) $= 2.003$ (+0.16 %) — two different systems sum to the mesonic slope; **(3) the carrier rule** (state/transport): locked pair (Σ/Ω) = stored → the ladder $(0, \frac{1}{2}, 1)$ appears and the centroid restores exactly 2; unlocked pair (Λ, spin 0) = no carrier → migrates (flat): the deficit is *visible* in the Λ band (that is the very measurement); **(4) the mesonic control**: no cable, no diversion (the 2 intact, D/B bands). *The scan's cluster is broken*: $2-2s$ killed structurally ($\Delta\text{Wr} = 0$ on trefoil legs — the same kill as κ_u); 9/7, $4/\pi$ mechanism-free (null); 23/18 alone mechanized → selected. Status: a conditional theorem (blindness → trace; slope conservation; rank $\sigma = b$). Discriminants: the Σ_c 1P multiplet unchanged to ∼0.3 MeV ($j0/j1/j2 = 2\,772/2\,821/\ 2\,869$ MeV — $\Sigma_c(2800)$ a $j0/j1$ blend candidate); *the sum rule's own discriminant*: $\Delta n(\Sigma_c\,1\text{P centroid}) - \Delta n(\Lambda_c\,1\text{P}) = 13/18$ (resolving the multiplet measures the split).

The flip's un-tripling, settled: state vs transport ($\Sigma_b(6097)$, 2026-07-22). The main document's **state / transport** dichotomy (the saturation one) extends to P-wave migration with no new hypothesis: *the exchange excess* is *transport* (generated by the ring's rotation around the cable, the link theorem): it resides on the rotating ring → in P wave it *migrates* to the orbiting pair's ring → flat $(d-1)$ [verified on the Ω centroids]; *the flip* is a *state* (the spin-1 configuration,

stored on the pair→heavy path: the angular-momentum balance with the heavy): it does not migrate → $\times 3^g$ **preserved**. *The test*: the four bookkeepings vs $\Sigma_b(6097)$ (LHCb 2019, $\Lambda_b\pi$, $\Gamma \sim 30$, unknown J) — flat flip: max levels 6 030/6 057, **excluded** (40–70 MeV below 6 097); $\times 3$ + saturated excess: 6 097 = $j0$ (+0.09 %); $\times 3$ + flat excess: **6 097 = j1** (**+0.02** %) — the rule *and* the fit select the same option. **The Σ_b 1P quintet predicted**: $j0$ ($J{=}\frac{1}{2}$) 6 076; $j1$ 6 092/6 102 (centroid 6 098 = $\Sigma_b(6097)$, unresolved doublet under Γ); $j2$ 6 111/6 128. *Falsifiable*: $J(6097) \in \{\frac{1}{2}, \frac{3}{2}\}$ (not $\frac{5}{2}$); the $j0/j2$ levels in $\Lambda_b\pi$; mixing of the two 3/2's at ∼19 MeV (the Ω_b pattern). Consistency: charm is blind to the question (3^0; $\Sigma_c(2800)$ unchanged); the Ξ_b' 1P (ℓs pair, flip without excess): $j0 \sim$ 6 243 — $\Xi_b(6227)$ at −0.26 %, a declared candidate with reserve (ρ-mode/metaxion debated). (`derive_detriplement_bascule.py`.) **The heavy spectroscopic sector is at ZERO read numbers**: theorems and conditional identifications — b derived (23/18, sum rule), the $L{=}2$ extension derived (next paragraph); what remains: Ω_c^* −0.7 %. (`derive_bandes_baryoniques.py`, `derive_partie_aveugle_b.py`, `derive_b_regle_somme.py`.)

The $L{=}2$ extension derived: the frame turn is per period (2026-07-23). At $L{=}1$, "per L" and "per revolution *period*" coincide — the attribution of the per-revolution bricks was degenerate. The 1D data (six systems: D, D_s, Λ_c, Λ_b, Ξ_c, Ξ_b) settle it: everywhere the second rung costs less (all-linear excluded +7 %, all-fixed −23 %); the deficit is worth ∼one exchange unit and *triples* with generation (2.93) — **the subtracted unit = the material frame turn, one per period whatever** L (the exact geometric fact of the 5/2 theorem, L-blind by nature); the tube winding is per L. Closed forms (rung $\frac{1}{2}$ *inside* the $\times 3^g$, the 19/2 convention): meson $n(L, j_K) = 2L{+}3^g(L{+}1{+}\frac{1}{2}[\text{top}])$ [$+ s{\times}1$ strange]; baryon $\Delta n(L) = \frac{23}{18}L{+}3^g(3L{+}1)(d{-}1)$ [$+ s{\times}L$ strange] — at $L{=}1$: 5/2 and $4(d{-}1)$, *the established 1P unchanged*. The shift s follows the *half-substitution* (a second echo): common tube → $\times 1$ ($D_s(2860)$ selects, −0.1 vs +1.1 %); proper legs → $\times L$ (Ξ_b selects). **Eight 1D systems at ≤ 0.4 % in mass, zero parameters**: D +0.09, D_s −0.10, $\Lambda_c(2860/2880)$ −0.10, $\Lambda_b(6146/6152)$ −0.07, $\Xi_c(3055/3080)$ +0.37, $\Xi_b(6327/6333)$ +0.07 % (in n, the Ξ's carry their known 1P tension, −2 %) — and **two reversals**: $B_J(5970)$ (unknown J) = the bottom 1D top at **+0.02** % (5 972 vs 5 971), $B_{sJ}(6064)$ = the strange 1D top (+0.11 %) — falsifiable: $J^P(5970) = 3^-$. Null: the isolated deficits ($\pi/2$ meson at 0.1 %, $\sim\pi/8$ baryon) have no mechanism, no link, no extrapolation — the per-period reading is *one* rule, two sectors, automatic $\times 3^g$, predictive at $L{=}3$. *Predictions*: bottom 1D doublet 5 903/5 972 (∼69 MeV); D 1D floor 2 709 (broad, declared 2S distortion); $L{=}3$: D 1F 3 043, Λ_c 1F 3 101, Λ_b 1F 6 366 MeV. Conditional identification: the fixed unit = the frame turn (forced by its L-independence). (`derive_extension_L2.py`.)

The base-2 dibaryon: the double-junction category populated (2026-07-22, complete card). *The structure, by theorem*: the **Steiner** theorem (every internal vertex of a minimal uniform-tension tube network is 3-valent at 120°; a degree-≥ 4 vertex always splits while shortening the network) forbids the 6-valent vertex: the compact dibaryon = **double-Y + bridge** — base 2 survives (two triality windings; the base counts windings, not vertices, §14.2); the angular geometry is baryon-standard (the $\frac{1}{2}$ liaison transfers — *no* 6-valent unknown); the *bridge* = a Z_3-character conduit (a mesonic tube between two baryonic junctions), and its length is *derived* by the **contact theorem** (2026-07-23): an *exact* impenetrability floor (the J_1/J_2 legs are non-adjacent tubes → axes ≥ D apart, and the junction points lie on those axes ⇒ $d(J_1, J_2) \geq D$; verified $d_{\min} = 1.0000\,\ell$ in both geometries); *no anchor above* (the $(\Sigma m)^2$ law carries no leg length: tension pulls to the floor); the conduit counts *in full* (the only new tube, spanning D) → $\ell = D$, $E_{\text{bridge}} = \sigma_\chi D = 19.0$ MeV — the $[0, +19]$ window closes on its *upper* edge (competitors: $\ell{\to}0$ impossible, $D/2$ mechanism-free; residual systematic $\delta D/D$, ±0.4 MeV). *The partition* (forced by spin): the 6 legs split — ground state **(cll)–(cll)**: the two heavies on separate junctions, no cable, no lock, Pauli settles into spin 0 → $J^P = 0^+$, $n = 2$; excited **(ccl)–(lll)**: cc cable, lock

$3(d-1)$, $J^P = 1^+$. What *binds*: the $(\Sigma m)^2$ coherence of the connected network — *the* law of the catalogue (declared identification). *The cards*: $\mathbf{H_{cc}^{++}}$: 0^+, Q=+2, I=0, M = **4 497 ± 10** MeV — **deeply bound** (−76 below $\Lambda_c\Lambda_c$, −63 below $\Xi_{cc}N$): weak decay only ($\tau \sim 1.0 \times 10^{-13}$ s, $c\tau \sim 30$ μm, **displaced vertex**, channels $\Lambda_c^+ pK^-\pi^+ / \Lambda_c^+ \Lambda\pi^+ /\ \Xi_c^+ p\bar{K}^0$, double-charm production); **the excited** 1^+ **at** 4 572±10 — **pinned ON the** $\Lambda_c\Lambda_c$ **threshold** (−0.5 MeV at central: the established threshold motif, T_{cc} at −0.4 of D^0D^{*+}, $X(3872)$ at −0.0 of $D\bar{D}^*$), +12.5 above $\Xi_{cc}N \to$ a narrow $\Xi_{cc}N$ resonance just below $\Lambda_c\Lambda_c$ — **a** $0^+/1^+$ **doublet split by** ∼75 **MeV**; $\mathbf{H_{bb}^0}$: 0^+, Q=0, M = 11 212±10 — *bound* (−27 below $\Lambda_b\Lambda_b$), $c\tau \sim 220$ μm; **light H**: 2 481, +250 above $\Lambda\Lambda \to$ *absent*, robust (the derived bridge moves it *away* from threshold: the postdiction *strengthens*) — **the machinery postdicts the 40 years of H-dibaryon nulls** (a free consistency test, passed). *Falsifiable*: a displaced-vertex structure at 4 497 ± 10 in $\Lambda_c^+ pK^-\pi^+$ (LHCb Run 3, double-charm); the ∼75 MeV doublet; never a peak above 4 573 (exact at central); the absence of the light H. The dibaryon sector has *no free systematic left* — the only input is the stoikhion chain (±10). (`calcul_dibaryon_base2.py`, `derive_liaison_6valente.py`, `calcul_hcc_carte_complete.py`, `derive_liaison_pont.py`.)

The light sector follows the *same* equation on its sector anchors (Regge/radial/orbital of chapter 14); quarkonia are excluded *on principle* (Coulomb compression, $\tau < 0$).

Remark

Declared limits: (i) the absolute scale m_ℓ is a *transmutation* constant — no differential equation of the framework fixes the zero rung: the law fixes *ratios* and *structure*; (ii) the pair (eigenmode + compensation) is not a *new* equation: it is P5+QGF+S4 *unified in writing*; (iii) the remaining residue — Ω_c^* (−0.7 %) — forms the work list (the L=2 extension is *derived*: frame turn per period, 8 systems, cf. the dedicated paragraph) (the flip's un-tripling is *settled*: state/transport, $\Sigma_b(6097) = j1$, cf. the dedicated paragraph) (the baryonic orbital bands are *derived*: two-strand cable, $4(d-1)$, cf. §14.3); **the band sector is closed, zero read numbers**: Σ_b saturation, D_s/B_s half-substitution, bands $n = 2 + 3^g$ (5/2) with the 5/2 *proven* (locking + inertia), the s shift *derived* ($\Delta\mathrm{Wr}/\pi$, identity aperture), the intra-doublet splitting *derived* (pivot + Compton response + moment half-substitution structure, coefficient $\kappa_u = h_{1/2}(k{=}2) = 3/16$) and the j_K=1/2 sign *dissolved* (ladder monotonicity: null alignment at the floor) — joint predictions: B_0^*/B_1' (5 665, splitting 6.6 MeV), strange baryonic bands (shift *and* dilution).

14.4 Principle of the table

For each particle, the topological configuration is determined by the exhaustive combinatorial search among the 1539 possible configurations, with the constraints:

- **Mass**: $|m_{\mathrm{TQNT}} - m_{\mathrm{SM}}|/m_{\mathrm{SM}} < 5\,\%$.
- **Spin**: verified by (ν, μ, Q) and $s = s_{\min} + \ell$.
- **Charge**: $Q = \Sigma\chi_i +$ Lk integer.

The number of knot combinations grows so fast with the number of crossings that the bracketing by the characteristics of a particle alone (mass, spin, charge) is in general **not unique**: it may **encompass several distinct combinations** satisfying the same constraints. In the following table, we have retained for each particle the **most probable** combination (the simplest compatible one); it remains experimentally possible that *another* combination realizes the same particle.

14.5 Complete table

Part.	**Config.**	J	Q	m_{exp}	$m = \sqrt{\sigma_\chi}\mathcal{S}$	$\Delta\%$	method
			Base (χ waves)				
γ	—	1	0	0	0	0.0 %	zero
ν	—	$\frac{1}{2}$	0	0	0	0.0 %	zero
			Leptons (Koide)				
e^-	3_1	$\frac{1}{2}$	-1	0.5	0.511	-0.0 %	koide
μ^-	3_1	$\frac{1}{2}$	-1	105.7	105.7	-0.0 %	koide
τ^-	3_1	$\frac{1}{2}$	-1	1 776.9	1 776.9	+0.0 %	koide
			Mesons				
π^0	$3_1{+}5_1$	0	0	135.0	135.0	—	GMOR
$\pi^\pm$	$3_1{+}5_1$	0	+1	139.6	139.6	—	GMOR
$K^\pm$	$3_1{+}7_1$	0	+1	493.7	493.7	—	GMOR
K^0	$3_1{+}7_1$	0	0	497.6	497.6	—	GMOR
η	$6_1{+}6_1{+}6_1$	0	0	547.9	567.0	+3.4 %	GMO
η'	$3_1{+}3_1{+}6_1$	0	0	957.8	958.0	—	anomalie_U1A
ρ	$4_1{+}5_2{+}6_1$	1	+1	775.3	755.0	-2.6 %	Regge(a0=0.48)
ω	$3_1{+}4_1{+}5_1$	1	0	782.6	755.0	-3.5 %	Regge(a0=0.48)
ϕ	$7_5{+}7_5$	1	0	1 019.5	1 019.5	+0.0 %	Regge+etrange
$D^\pm$	$3_1{+}7_6$	0	+1	1 869.7	1 867.0	-0.1 %	constituant
D^0	$3_1{+}5_2{+}9_1$	0	0	1 864.8	1 867.0	+0.1 %	constituant
$D_s^\pm$	$7_2{+}9_1{+}9_1$	0	+1	1 968.3	1 959.4	-0.5 %	const.+half-subst.
J/ψ	$4_1{+}4_1{+}8_1$	1	0	3 096.9	3 114.0	+0.6 %	Cornell/const
Υ	$4_1{+}5_2{+}7_4$	1	0	9 460.3	9 800.0	+3.6 %	Cornell/const
$B^\pm$	$6_2{+}7_3{+}7_3$	0	+1	5 279.3	5 262.0	-0.3 %	constituant
B^0	$6_2{+}7_3{+}7_3$	0	0	5 279.7	5 262.0	-0.3 %	constituant
B_s^0	$3_1{+}6_2{+}6_3$	0	0	5 366.9	5 354.4	-0.2 %	const.+half-subst.
			Baryons				
p	$3_1{+}7_2{+}\bar{7}_2$	$\frac{1}{2}$	+1	938.3	938.2	-0.0 %	octet
n	$3_1{+}7_2{+}\bar{7}_2$	$\frac{1}{2}$	0	939.6	938.2	-0.1 %	octet
Λ	$3_1{+}4_1{+}7_2$	$\frac{1}{2}$	0	1 115.7	1 114.7	-0.1 %	Regge+Delta_s
Σ^+	$3_1{+}6_1{+}7_2$	$\frac{1}{2}$	+1	1 189.4	1 192.8	+0.3 %	Regge+Delta_s+tau_perp
Σ^-	$3_1{+}6_1{+}7_2$	$\frac{1}{2}$	-1	1 197.5	1 192.8	-0.4 %	Regge+Delta_s+tau_perp
Ξ^0	$3_1{+}6_2{+}7_2$	$\frac{1}{2}$	0	1 314.9	1 339.8	+1.9 %	Regge+Delta_s+tau_perp
Ξ^-	$3_1{+}6_2{+}7_2$	$\frac{1}{2}$	-1	1 321.7	1 339.8	+1.4 %	Regge+Delta_s+tau_perp
Δ	$3_1{+}5_2{+}7_4$	$\frac{3}{2}$	+1	1 232.0	1 232.1	+0.0 %	decuplet
Ω^-	$5_1{+}7_2{+}7_2$	$\frac{3}{2}$	-1	1 672.5	1 673.1	+0.0 %	Regge+Delta_s+tau_perp
Λ_c	$3_1{+}4_1{+}8_1$	$\frac{1}{2}$	+1	2 286.5	2 299.9	+0.6 %	const.+torsion(n=1)
Λ_b	$4_1{+}6_3{+}7_2$	$\frac{1}{2}$	0	5 619.6	5 621.2	+0.0 %	const.+torsion(n=1)
			Electroweak				
W	$7_1{+}7_2{+}8_3$	1	+1	80 377.0	80 680.0	+0.4 %	EW_VEV
Z	$7_2{+}8_3{+}9_1$	1	0	91 188.0	90 560.0	-0.7 %	EW_VEV
H	$7_1{+}8_3{+}10_1$	0	0	125 250.0	125 260.0	+0.0 %	EW_VEV
t	$8_2{+}8_3{+}10_1$	$\frac{1}{2}$	+1	172 690.0	170 410.0	-1.3 %	EW_VEV

Note. Masses in MeV; $m = \sqrt{\sigma_\chi}\,\mathcal{S}$, the law being that of the sector (Koide/Regge/GMOR/stoikhion/VEV, last column). The configurations carry spin and charge ($Q = \Sigma\chi + \mathrm{Lk}$); bare quarks ($u, d, s, \dots$) do not appear as particles (Rule 0: knot eigenstates, not fundamental). Reference assignments of the σ_χ catalogue (EW: `ew_vev`). *Update 2026-07-21*: the $D_s/B/B_s$ rows are realigned onto the σ_χ catalogue (source of truth); Λ_c and Λ_b carry the recoupling torsion n=1 (validated touch-up, §14.2).

14.6 The heavy sector completed by torsion (S4″)

The torsion of the peripheral vertices (§14.2) extends the table to the whole heavy sector *with no new parameter*: $M^2 = M^2_{\text{core}} + n\,\pi\sigma_\chi$, n in half-turns.

(a) Spin partners (one half-turn, the V-P quantum).

State	m_{exp}	$m_{\text{spacetimeweave}}$	$\Delta\%$
$D^*(2007)$	2 006.9	2 008.9	+0.1
$D^*(2010)^\pm$	2 010.3	2 008.9	−0.1
D_s^*	2 112.2	2 095.1	−0.8
B^*	5 324.7	5 314.0	−0.2
B_s^*	5 415.4	5 405.5	−0.2
$\Sigma(1385)$	1 383.7	1 404.6	+1.5
$\Xi(1530)$	1 531.8	1 531.4	−0.0

D_s/B_s cores via the half-substitution (§14.3, 2026-07-21).

(b) The heavy-baryon recoupling ladder (closed forms $\{1, \sqrt{2}, \sqrt{2}-1\}$, zero anchors).

State	n	m_{exp}	$m_{\text{spacetimeweave}}$	$\Delta\%$
Λ_c	1	2 286.5	2 299.9	+0.6
Ξ_c	1	2 467.9	2 475.5	+0.3
Λ_b	1	5 619.6	5 621.2	+0.0
Ξ_b	1	5 791.9	5 804.4	+0.2
Σ_c	$1+\sqrt{2}$	2 453.5	2 463.2	+0.4
Σ_c^*	$\frac{3}{2}+\sqrt{2}$	2 518.1	2 518.4	+0.0
Ξ_c'	2	2 578.4	2 584.3	+0.2
Ξ_c^*	$\frac{5}{2}$	2 645.6	2 636.9	−0.3
Ω_c	$\sqrt{2}$	2 695.2	2 695.0	−0.0
Ω_c^*	$\frac{1}{2}+\sqrt{2}$	2 765.9	2 745.6	−0.7
Ξ_{cc}	$1+\sqrt{2}$	3 621.6	3 612.7	−0.2
Σ_b	5 (sat.)	5 813.1	5 813.6	+0.0
Σ_b^*	$\frac{11}{2}$ (sat.)	5 832.5	5 837.2	+0.1
Ξ_b'	4	5 935.0	5 944.9	+0.2
Ξ_b^*	$\frac{9}{2}$	5 953.8	5 967.9	+0.2
Ω_b	$3\sqrt{2}-2$	6 045.2	6 044.5	−0.0

Median 0.23 % over the 15 baryons (+Ξ_{cc}); the two Ω at −0.01 %.

(c) Metaxions — the LHC "molecules" (spacetime weave hadrons *with* their half-turns, standard contents).

State	Content	m_{exp}	$m_{\text{spacetimeweave}}$	$\Delta\%$
$X(3872)$	$D\bar{D}^*$	3 871.7	3 875.9	+0.1
$T_{cc}(3875)$	DD^*	3 874.8	3 875.9	+0.0
$Z_c(3900)$	$D\bar{D}^*$	3 887.5	3 875.9	−0.3
$Z_c(4020)$	$D^*\bar{D}^*$	4 024.3	4 017.9	−0.2
$Z_b(10610)$	$B\bar{B}^*$	10 607.2	10 576.0	−0.3
$Z_b(10650)$	$B^*\bar{B}^*$	10 652.2	10 628.0	−0.2
$P_c(4312)$	$\Sigma_c\bar{D}$	4 311.9	4 330.2	+0.4
$P_c(4380)$	$\Sigma_c^*\bar{D}$	4 380.0	4 385.4	+0.1
$P_c(4440)$	$\Sigma_c\bar{D}^*$	4 440.3	4 472.2	+0.7
$P_c(4457)$	$\Sigma_c\bar{D}^*$	4 457.3	4 472.2	+0.3
$P_{cs}(4338)$	$\Xi_c\bar{D}$	4 338.2	4 342.5	+0.1
$P_{cs}(4459)$	$\Xi_c\bar{D}^*$	4 458.8	4 484.5	+0.6
$X_0(2900)$	$\bar{D}^*K^*$	2 866.0	2 902.1	+1.3
$Z_{cs}(3985)$	$D_s\bar{D}^*$	3 982.7	3 968.3	−0.4
$D_{s0}^*(2317)$	DK	2 317.8	2 360.7	+1.9
$D_{s1}(2460)$	D^*K	2 459.5	2 502.6	+1.8

Median 0.3 %: the historical −6 % of the P_c was *entirely* the bare Σ_c core.

(d) The spins READ from the masses alone (inverse Regge). The spacetime weave's reading theorem (§3.3): the rate ω is hidden, but the *value* of j is written in energy in the mass and is read via

$$j \;=\; j_0 + \frac{M^2 - M_0^2}{2\pi\sigma_\chi} \qquad (\textit{the} \text{ derived slope, one anchor per trajectory}), \tag{14.10}$$

with rounding on the sector's grid (integers for mesons, half-integers for baryons). Ground states carry their spin through the *braid arity* (P7quater, previous chapter). Over the 13 trajectories of the 226-state table:

Traj.	State	j read	grid	J	Traj.	State	j read	grid	J
ρ/a	$a_2(1320)$	2.03	2	2	b/π_2	$\pi_2(1670)$	2.16	2	2
	$\rho_3(1690)$	3.05	3	3		$b_3(2030)$	3.37	3	3
	$a_4(1970)$	3.97	4	4		$\pi_4(2250)$	4.23	4	4
	$\rho_5(2350)$	5.39	5	5	a_1	$a_3(1875)$	2.82	3	3
	$a_6(2450)$	5.91	6	6	N	$N(1680)$	2.28	$\frac{5}{2}$	$\frac{5}{2}$
ω/f	$f_2(1270)$	1.92	2	2		$N(2220)$	4.18	$\frac{9}{2}$	$\frac{9}{2}$
	$\omega_3(1670)$	2.97	3	3		$N(2700)$	6.32	$\frac{13}{2}$	$\frac{13}{2}$
	$f_4(2050)$	4.14	4	4	Δ	$\Delta(1950)$	3.58	$\frac{7}{2}$	$\frac{7}{2}$
	$f_6(2510)$	5.97	6	6		$\Delta(2420)$	5.44	$\frac{11}{2}$	$\frac{11}{2}$
K^*	$K_2^*(1430)$	2.12	2	2		$\Delta(2950)$	8.03	$\frac{17}{2}$	$\frac{15}{2}$ (×)
	$K_3^*(1780)$	3.15	3	3	Λ	$\Lambda(1820)$	2.38	$\frac{5}{2}$	$\frac{5}{2}$
	$K_4^*(2045)$	4.07	4	4		$\Lambda(2350)$	4.39	$\frac{9}{2}$	$\frac{9}{2}$
	$K_5^*(2380)$	5.43	5	5	Σ	$\Sigma(1915)$	2.55	$\frac{5}{2}$	$\frac{5}{2}$
ϕ/f'	$f_2'(1525)$	2.15	2	2		$\Sigma(2455)$	4.69	$\frac{9}{2}$	$\frac{9}{2}$
	$\phi_3(1850)$	3.18	3	3	Σ^*	$\Sigma(2030)$	3.51	$\frac{7}{2}$	$\frac{7}{2}$
					Ξ	$\Xi(1950)$	2.38	$\frac{5}{2}$	$\frac{5}{2}$
					Ω	$\Omega(2250)$	3.57	$\frac{7}{2}$	$\frac{7}{2}$

31/32 spins read correctly from the masses alone (mean deviation from exact j: 0.2). The single deviation, $\Delta(2950)$ (j read 8.03 vs $\frac{15}{2}$), is not held against the reading: the state is *poorly established* (2-star PDG, $\Gamma \sim 330$ MeV, mass from old analyses $\sim$2 950 ± 100), and the same universal slope that reads the other 31 lands on $\frac{15}{2}$ *exactly* for $M = 2\,849$ MeV — within 1σ of the published mass (the −3.4 % on the mass is the same fact, counted once). The apparent "high-J curvature" of the last segment of the Δ trajectory (local slopes 1.14/1.03/1.42 GeV^2/unit) rests on this single point, while at the same altitudes $a_6(2450)$ (deviation −0.09) and $N(2700)$

(-0.17) stay on the derived slope. **Attached prediction**: the mass of $\Delta(2950)$ must *come down* to $\sim$2 850 MeV in modern partial-wave analyses; a mass confirmed at $2\,950 \pm 30$ with $J^P = \frac{15}{2}^+$ would put the derived slope in tension ($+7\,\%$ required on the Δ trajectory alone). The pion is *not* anchored on Regge (it is Goldstone): the sector division of the mass law is *visible* in the spin reading. Note: $\Sigma(2030)$ only reads correctly on the *decuplet* trajectory $\Sigma(1385)$ (j read 3.51): the spin reading *diagnoses* trajectory membership.

Predictions. $\Omega_b^* = 6\,067$ MeV ($+22$ above Ω_b); $\Xi_{cc}^* = 3\,659$ MeV (§14.2); Ω_{cc}: **verified** (LHCb 06/2026: $\sim$3 727 MeV $\in [3\,715, 3\,788]$ predicted, lock edge — base derived, the triality half-turn, §14.2); $\Omega_{ccc} = 4\,782$ MeV (*strict*: the Ξ_{cc} ambiguity settled by the Ω_{cc}); Ω_{bbb} *outside* the torsion frame (Coulomb compression, negative binding expected); the bottom j_K=1/2 doublet B_0^*/B_1' (broad): centroid 5 665 MeV, members $\sim$5 660/5 666 (a point: the alignment sign dissolved by monotonicity 2026-07-22; 5 591/5 517 would refute, §14.3); $\Omega_b(1P, j{=}0, J{=}\frac{1}{2}) = 6\,297$–$6\,304$ MeV (the fifth of the 2020 quartet) and strong mixing of the two $\Omega_b(3/2)$ (multiplet rule, §14.3); H_{cc}^{++} (base-2 dibaryon): 0^+ doublet at $4\,497 \pm 10$ MeV (deeply bound, 30 μm displaced vertex, $\Lambda_c^+ p K^- \pi^+$) / 1^+ at $4\,572 \pm 10$ (pinned on the $\Lambda_c\Lambda_c$ threshold, $T_{cc}/X(3872)$ motif); H_{bb}^0 0^+ at $11\,212 \pm 10$ (bound, $c\tau$ 220 μm) (§14.3; light H *absent*: nulls postdicted; bridge *derived* $\ell{=}D$, $E = 19.0$ MeV — contact theorem). Bottom 1D doublet 5 903/5 972 MeV ($\sim$69) — $B_J(5970)$ identified as the top ($J^P = 3^-$ falsifiable), $B_{sJ}(6064)$ = the strange 1D top ($+0.11\,\%$); L=3: D 1F 3 043, Λ_c 1F 3 101, Λ_b 1F 6 366 MeV (§14.3). *Dated TQNT-vs-QCD discriminants* (the masses decide): $\Xi_{cc}^* = 3\,659$ vs $\sim$3 690 (lattice/quark models) — the Ξ_{cc}^+ ground is observed (03/2026), LHCb will get the star; $B_0^*/B_1' = 5\,660/5\,666$ vs $\sim$5 700–5 750 (models). The states *proper* to TQNT (strict DM, realizables) are predicted **absent** from colliders (creation suppressed as $\alpha^{\Delta c} \sim 10^{-11}$–$10^{-24}$ from hadronic bricks $c \leq 7$ — the V5.19 pattern, echo of the ATLAS null): their window is *astro* (Fermi-LAT lines 1.10/1.64 GeV). (`scan_lhcb_tqnt_vs_sm.py`.) The full extension (226 states, global median 1.43 %, spins read 31/32): `calcul_224_particules_socle.py`. The whole heavy sector is written as *one* functional — the spacetime weave equation (§14.3): **49 states with no per-state anchor, median** $\sim 0.25\,\%$ (57 with the 3 read bands at $\sim 0.3\,\%$).

14.7 Lifetimes of the spectrum (Fermi/Sargent + topological Q)

The lifetimes complete the table above. They follow the Fermi/Sargent theory of the SM, the TQNT contribution being the energy $Q = \Delta m$ and the Sargent law (chap. 15, §15.2): G_F scale of the topological relaxation (= vev; weak and EW), linking tension $\beta_\otimes$ (strong), $\alpha_{\rm EM}$ (EM); the couplings (g_A, V_{ud}, f_π, α_s) remain *measured* SM inputs. Stable: e^-, ν, u, p ($D = 0$, the lightest ones); confined: d, s, c, b.

Particle	**Config.**	**Channel**	$\tau_{\rm TQNT}$ (s)	$\tau_{\rm exp}$ (s)	$\Delta \log_{10}$
μ^-	$3_1{+}6_1$	weak	2.2×10^{-6}	2.2×10^{-6}	-0.00
τ^-	$3_1{+}5_1{+}7_1$	weak	3.2×10^{-13}	2.9×10^{-13}	$+0.05$
π^0	8_1	EM	8.5×10^{-17}	8.4×10^{-17}	$+0.00$
$\pi^\pm$	8_1	weak	2.7×10^{-8}	2.6×10^{-8}	$+0.01$
$K^\pm$	$4_1{+}4_1{+}5_2$	weak	1.3×10^{-8}	1.2×10^{-8}	$+0.01$
K_S^0	$4_1{+}4_1{+}5_2$	weak	2.9×10^{-9}	9.0×10^{-11}	$+1.52$
η	$3_1{+}4_1{+}5_1$	EM	1.5×10^{-18}	5.0×10^{-19}	$+0.48$
η'	$5_2{+}5_2{+}6_1$	strong	1.9×10^{-23}	3.5×10^{-21}	-2.26
ρ/ω	$5_2{+}7_7$	strong	4.4×10^{-22}	7.8×10^{-23}	$+0.75$
ϕ	$3_1{+}4_1{+}6_2$	strong	2.4×10^{-23}	1.6×10^{-22}	-0.81
J/ψ	$4_1{+}5_2{+}9_1$	strong	1.8×10^{-23}	7.1×10^{-21}	-2.59
Υ	$3_1{+}7_4{+}9_1$	strong	1.7×10^{-23}	1.2×10^{-20}	-2.85

... continued

Particle	Config.	Channel	τ_{TQNT} (s)	τ_{exp} (s)	$\Delta \log_{10}$
D^0	$6_2{+}7_7$	weak	1.8×10^{-12}	4.1×10^{-13}	+0.65
D^+	$6_2{+}7_7$	weak	1.8×10^{-12}	1.0×10^{-12}	+0.25
B	$4_1{+}10_1$	weak	2.7×10^{-12}	1.5×10^{-12}	+0.24
n	$3_1{+}7_2{+}\bar{7}_2$	weak	8.8×10^{2}	8.8×10^{2}	+0.00
Λ	$4_1{+}7_4$	weak	1.2×10^{-9}	2.6×10^{-10}	+0.66
Σ^+	$6_2{+}7_2$	weak	5.6×10^{-10}	8.0×10^{-11}	+0.84
Σ^0	$6_2{+}7_2$	EM	5.6×10^{-20}	7.4×10^{-20}	−0.12
Σ^-	$6_2{+}7_2$	weak	5.4×10^{-10}	1.5×10^{-10}	+0.56
Ξ^0	$4_1{+}4_1{+}7_2$	weak	6.4×10^{-10}	2.9×10^{-10}	+0.34
Ξ^-	$4_1{+}4_1{+}7_2$	weak	6.1×10^{-10}	1.6×10^{-10}	+0.57
Δ	$3_1{+}5_2{+}6_2$	strong	2.2×10^{-23}	5.6×10^{-24}	+0.60
Σ^*	$5_2{+}6_2{+}7_2$	strong	1.7×10^{-23}	1.8×10^{-23}	−0.04
Ξ^*	$4_1{+}5_2{+}7_2$	strong	1.9×10^{-23}	7.2×10^{-23}	−0.57
Ω^-	$5_2{+}5_2{+}7_2$	weak	2.5×10^{-10}	8.2×10^{-11}	+0.49
Λ_c	$6_3{+}8_1$	weak	1.8×10^{-12}	2.0×10^{-13}	+0.96
Σ_c	$4_1{+}5_1{+}7_1$	strong	2.0×10^{-23}	3.5×10^{-22}	−1.24
Ω_c	$3_1{+}6_2{+}6_2$	weak	1.8×10^{-12}	2.7×10^{-13}	+0.83
Λ_b	$5_2{+}5_2{+}8_2$	weak	2.7×10^{-12}	1.5×10^{-12}	+0.26
W	$7_1{+}7_2{+}8_3$	EW	3.2×10^{-25}	3.2×10^{-25}	+0.01
Z^0	$7_2{+}8_3{+}9_1$	EW	2.7×10^{-25}	2.6×10^{-25}	+0.01
H	$7_1{+}8_3{+}10_1$	EW	2.0×10^{-22}	1.6×10^{-22}	+0.10
t	$8_2{+}8_3{+}10_1$	EW	4.4×10^{-25}	5.0×10^{-25}	−0.05

RMS per channel: weak 0.61, strong 0.56 (broad states), EM 0.28, EW **0.06**; **global** 0.94 **decade** (34 unstable, *zero* calibration). The marked deviations ($K^0_S, \eta', J/\psi, \Upsilon$) are *known* suppressions ($\Delta I = 1/2$ rule, OZI, $U(1)_A$ anomaly) outside the topological scope; the heavy bosons (W, Z, t, H at 0.06 dec) confirm relaxation as the electroweak scale. Script: `spectrum_all_table.py`.

14.8 Statistical analysis of the spectrum

Criterion	Number of particles
$\Delta < 0.5\,\%$	20 (67 %)
$\Delta < 1.0\,\%$	23 (77 %)
$\Delta < 2.0\,\%$	27 (90 %)
$\Delta < 5.0\,\%$	30 (100 %)
Mean Δ	0.7 %
Median Δ	0.3 %
Maximum Δ	3.6 % (Υ)

Final score of the spectrum (V6: $m = \sqrt{\sigma_\chi}\,\mathcal{S}$)

Reference V6 formula $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (sector-by-sector law, §9.1): on the 30 states with non-calibrated deviation in the table above (update 2026-07-21: Λ_c/Λ_b with the recoupling torsion, $D_s/B/B_s$ rows realigned onto the catalogue), **100 % at** $< 5\,\%$, **median 0.3 %**, maximum 3.6 % (Υ, Cornell/compression; D_s/B_s repaired by the half-substitution). From the electron (0.5 MeV) to the top quark (173 GeV): **5.5 orders of magnitude**. The scaling-up is *done*: **226 states** (all known particles + recent LHC), global median 1.43 %, single Regge slope and the $\pi\sigma_\chi$ **torsion quantum** (§14.6): heavy baryons 0.23 % (closed forms), LHC metaxions 0.3 %, trajectory spins read 31/32.

Derived low-J correction (decuplet). The decuplet ground state ($\Delta, \Sigma^*, \Xi^*, \Omega$) receives the tensor correction of the Y junction $E_{\text{tenseur}} = \frac{15\Lambda_\chi}{32} = 98$ MeV (eq. 9.6), now *derived in closed form* and not calibrated: it coincides with the measured low-J gap (+97 MeV) and reconstructs the absolute Δ to 0.6 % (1225 vs 1232 MeV) from the intercept of the excited Δ^*, *with no baryonic parameter*. The global score of the spectrum is *unchanged* (median 0.35 %): the tensor explains the low-J structure of the baryons without shifting any catalogue value.

The quarks u and d share the 3_1 topology; the values 2.18 and 4.81 MeV (†) are V4 *calibrations* on three geometric parameters (R, a, b) per particle and not predictions, the SM "current quark mass" not being a TQNT observable (cf. §9.4). The reference stoikhion mass is the *knot mass* of the σ_χ catalogue: $m_\ell = 310$ MeV (calibrated, a single light stoikhion for u and d); the former extraction "Track 2 of P23′" ($\sim$ 336/340 MeV) is archived (chap. C, §C.2). The nucleon (octet, 3 strands $3_1 + 7_2 + \overline{7}_2$) is reproduced to **0.0 %** (938.2 MeV, octet Regge base) under the reference formula $m = \sqrt{\sigma_\chi}\,\mathcal{S}$, with the differential $\Delta m(n{-}p) = 1.293$ MeV *derived in structure* (0.1 %, linking tension $\beta_\otimes$ — calibrated scale).

14.9 Complete quantum numbers of a TQNT particle

Each particle is characterized by:

$$|K_1, \ldots, K_n;\ \text{Lk};\ \ell\rangle \tag{14.11}$$

Quantum number	**Origin**	**Observable**
$K_1, \ldots, K_n$ (topology)	Knot classification	Mass, minimal spin, chirality
Lk (linking)	Link invariant	Charge, isospin, fine mass
ℓ (excitation)	Flux tube mode	Additional spin
$\lvert w\rvert$ (self-linking)	Self-linking of the knot	Decay channel
σ (saturation)	Curvature + torsion	Chirality (matter/antimatter)

14.10 Comparative assessment: TQNT vs Standard Model

TQNT vs Standard Model

	Standard Model	**TQNT**
Mass	free parameter per particle	$m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (V6, 10 global param.)
Spin	field (Dirac, Proca, ...)	topology (ν, μ, Q) + excitation ℓ
Charge	postulated quantum numbers	$Q = \Sigma\chi + \mathrm{Lk}$ (derived)
Generations	3 postulated families	3 torus knots $T(2,3)$, $T(2,5)$, $T(2,7)$
Nucleon	3 quarks + QCD	link $3_1 + 7_2 + \bar{7}_2$
$\Delta m(n{-}p)$	lattice QCD (~ 1 MeV)	harmonic potential (1.293 MeV exact)
Confinement	not demonstrated analytically	topological (irreducible link)
m_p/m_e	unexplained	$= f(c_J, U_{\mathrm{EM}})$ derived
Lifetime	Fermi + QCD + diagrams	relaxation 246 GeV (34 via Fermi/Sargent + Q, 0.94 dec.)
Mass score	19 fitted param.	**89 states at median** 0.13 % ($\sqrt{\sigma_\chi}\mathcal{S}$, 10 global param.; recomputed 21/07, engine at main document: recalibration, ladder, half-substitution, $\frac{1}{2}$ V-P turn; Ω_{cc} out of the score — observed LHCb 06/2026, fine mass to come)
Gravitation	separate (GR)	emergent (G derived to 0.05 %, κ_g derived)

14.11 Coherent re-derivation under a single formula

NB (2026-07-21): the modern *unified re-derivation is the master functional of the spacetime weave equation (§14.3); this section documents the historical* $\Omega_{unified}$ *exercise.*

The table above mixes calibrations

The masses of the complete table (§ "Complete table") were calibrated *per entry*, with distinct mass implementations depending on the sections (V5 Ω_{ts} catalogue, axiomatic form, variants). Confronted with a *single formula* — $\Omega_{\mathrm{unified}}$, derived single-knot sector, a single collective parameter x — a notable fraction of the assignments in this table does not reproduce (e.g. $\phi = 3_1{+}4_1{+}6_2$ gives +79 %, $\Lambda = 4_1{+}7_4$ gives +143 %). This section re-derives the spectrum *under this single formula*, by the categorical procedure (core + radial/relativistic tower + width, §D.1). The file `docs/TQNT_masses_categorisation.xlsx` gives the tabulated detail (knots, links, category, mass, PDG, status).

Consistent assignments under Ω_{unified} (19/28)

Particle	Category	Assignment	$\Delta\%$
e^-	lepton (core 3_1)	3_1	0.0
ρ^0	diagonal meson	$6_2 + \bar{6}_2$	-1.4
ω	diagonal meson	$8_1 + \bar{8}_1$	-1.5
ϕ	diagonal meson	$7_5 + \bar{7}_5$	-3.6
K^*	vector meson	$5_2 + 7_5$	$+0.3$
J/ψ	diagonal meson ($c\bar{c}$)	$7_4 + \bar{7}_4$	$+1.0$
Υ	diagonal meson ($b\bar{b}$)	$7_6 + \bar{7}_6 + 4_1$	-1.3
p, n	octet baryon (core 7_2)	$3_1 + 7_2 + \bar{7}_2$	-2.3
Λ	octet baryon	$3_1 + 5_2 + 7_2$	-0.2
Σ	octet baryon	$5_2 + 7_2 + 7_2$	$+1.7$
Ξ	octet baryon	$3_1 + 3_1 + 7_2$	-6.2
Δ	decuplet baryon (core 5_2)	$5_2 + 6_3$	-0.6
Σ^*	decuplet baryon	$5_1 + 5_1 + 5_2$	$+0.8$
Ξ^*	decuplet baryon	$4_1 + 5_2 + 7_2$	-2.1
W, Z, H, t	EW boson (core 8_3)	(cf. §EW)	$\lesssim 1.3$

Eighteen particles obtain an assignment that is *both* consistent with the single formula ($\lesssim 4\,\%$) *and* conforming to their categorical core.

The walls of the re-derivation — honest

Three families *resist* the single formula, for *structural* reasons already identified:

- **light pseudoscalars** (π, η, η', K): they are the *Goldstones* of chiral breaking (§D.2), not ordinary knots; they do *not* fit Ω_{unified} by construction (the π is anomalously light, the mass of the η' comes from the topological susceptibility of the χ);
- Ω^-: Ξ is now resolved with a coherent core ($3_1{+}3_1{+}7_2$, $-6.2\,\%$, core 7_2); Ω^- remains a *real wall*. Its *physical* discriminant is clear — it is the *stable apex* of the $S = -3$ sector (*weak* decay, $\Gamma \sim 8 \times 10^{-6}$ MeV, six orders below the resonances Δ, Σ^*, Ξ^* at $\Gamma \sim 10$–120 MeV) — but this stability does *not resolve* the assignment. The hypothesis of an off-catalogue decuplet knot filling the gap ($1501 \to 1928$) is numerically *refuted*: a true knot with 9–10 crossings has a high curvature energy ($U_\kappa U_\tau \sim 2000$–$3600$ depending on the embedding) and gives $4_1{+}5_2{+}X \sim 8000$–12000 MeV, i.e. $+400$ to $+600\,\%$ above Ω^-; the required profile ($U_\kappa U_\tau \sim 200$, that of a *simple* knot) is incompatible with 9–10 crossings. *No* configuration with a decuplet core (charged or not) reaches 1672 without overshooting;
- **heavy flavoured mesons** (D, D_s, B, B_s): *under-determined* — many links $K + \bar{K}'$ match the mass (residual off-diagonal sector), with no topological criterion to decide.

These walls are the *same* as everywhere else: TQNT gives the *structure* (category, core, mechanism), the *values* of the chiral and flavoured sectors remaining out of reach of a single topological formula.

Fundamental caveat: U_κ, U_τ depend on the embedding

The Frenet energies $U_\kappa = \oint \kappa^2\, ds$ and $U_\tau = \oint \tau_\perp^2\, ds$, which enter Ω_{unified} via the geometric coupling G, are computed on *parametric curves* (Fourier). For the 19 charged knots, these curves were *calibrated* to reproduce targets (archived Ω_{ts} derivation); but for a knot *without a target* (off catalogue), $U_\kappa U_\tau$ varies by a factor ~ 2 depending on the chosen parametrization (test: three plausible curves for 9_3 give $U_\kappa U_\tau \in [1973, 3574]$). Part of the input to the mass formula is therefore *non-principled* — an embedding uncertainty that

affects every composite, not just Ω^-. This is a structural limit to keep in mind: the mass of a link is fully reproducible only for knots whose embedding is fixed.

14.12 V6 catalogue corrections

The Frenet glitch of the knot 7_5. The catalogue mass of 7_5 (5 959 MeV) was an *outlier*: the seven 7-crossing knots range from 50 to 493 MeV, except 7_5 at 12× the next one. The Jones factor $c_J(7_5) = 8.70$ is nevertheless normal, and the V4 form $\alpha_0 U_{\rm EM} c_J^{3.55}$ gives 86 MeV: the 5 959 required $U_\kappa U_{\tau_\perp} = 103\,000$ (vs 564, i.e. 183×), the signature of a *quasi-singular curve* (Frenet artefact). Properly recomputed, $7_5 = 141$ MeV; 210 configurations corrected. A systematic scan confirms that 7_5 is the *only* glitch.

Coherent reassignment of the top. The old top $6_3{+}7_1{+}7_5$ (+0.03 %) rested on the aberrant 7_5. Reassigned to the coherent knots $8_2{+}8_3{+}10_1$ (−1.25 %, fermion). Honest electroweak RMS: 0.74 % (W +0.39, Z −0.69, H +0.05, top −1.25 %).

Λ_c/Λ_b touch-up (recoupling torsion, 2026-07-21). The bare stoikhion cores left Λ_c at −4.8 % (the table's former maximum) and Λ_b at −0.8 %: they lacked the Y-junction *base* (n=1 half-turn, §14.2): $\Lambda_c \to 2\,299.9$ (+0.6 %), $\Lambda_b \to 5\,621.2$ (+0.0 %), method stoikhion+torsion(n=1), propagated to `catalogue_sigma_chi.csv/tex` (validated). In passing, the $D_s/B/B_s$ rows of the table were realigned onto the σ_χ catalogue (source of truth; the values 2 037/5 210/5 380 were residues of an earlier calibration); then D_s/B_s were *repaired* by the half-substitution (core$(H_s) = m_Q + (m_\ell{+}m_s)/2$, $-0.5/-0.2$ %, catalogue touched up, §14.3).

MAP selection rule and physical multiplets. Mass alone leaves several configurations per particle. The maximum a posteriori estimator

$$\hat{L}(\text{part.}) = \arg\max_L \left[\ln P(L) - \frac{(\log_{10}(m_L/m_{\rm exp}))^2}{2\sigma^2} \right] \tag{14.12}$$

($P(L)$: P18 composition probability; σ: precision of the formula) makes the assignment *unique* (0 collisions at $\sigma = 0.4$ %). *True* degeneracy (same m, J, Q at the resolution) is then not an ambiguity but a **predicted physical multiplet**: e.g. the "J/ψ" would be a doublet ($3_1{+}6_1{+}7_5$, $5_2{+}7_5{+}7_5$) resolvable by lifetime/decay.

14.13 Composition kinetics: why the spectrum is predictive

The *raw* spectrum of links is too dense to predict: at $n \geq 4$ components, almost any mass becomes "matchable" ($\sim$ 3200 bosonic links between 10 and 300 GeV, spacing $\sim$ 0.1 %). This is the source of the **assignment freedom** (MAP rule above). The **composition probability** restores predictivity: the observed particles are the most *probable* combinations (the simplest ones), not all possible combinations.

Combination probability. The relative abundance of a link L with n components and C crossings is

$$P(L) \;\propto\; \frac{1}{\prod_K m_K!}\, p_{\rm rec}^{n-1}\, \alpha_{\rm cond}^{-C}\, e^{-E_{\rm link}/E_0}, \tag{14.13}$$

where $1/\prod_K m_K!$ is the symmetry factor (identical components); $p_{\rm rec}^{n-1}$ weights the $(n-1)$ **P15 reconnections** needed for assembly ($p_{\rm rec} < 1$ is the *reconnection probability*); $\alpha_{\rm cond}^{-C}$ the condensation penalty per crossing ($\alpha_{\rm cond} = 1.20$, P18); and $E_{\rm link} = \beta_\otimes(\text{Lk} - \text{Lk}_{\rm opt})^2 \langle\sqrt{c_J c_J}\rangle$ ($\beta_\otimes = 0.210$ MeV). The composition **rate** is $\Gamma_{\rm comp} = \Gamma_0\, \rho_K^{n-1}\, \sigma_{\rm geo}\, e^{-E_{\rm link}/E_0}$.

Effect: the spectrum becomes sparse. Near the Z^0, the *effective* number of configurations (participation weighted by P, $(\sum P)^2/\sum P^2$) drops from 10 (raw spectrum) to **2.4**: the 3-knot configuration dominates, the 4-knot ones being suppressed to $\sim 3\,\%$ by $p_{\rm rec}^{n-1}\alpha_{\rm cond}^{-\Delta C}$. **Composition kinetics therefore makes the spectrum *sparse*, hence predictive** — the assignment freedom of the raw spectrum is lifted *dynamically*, with no new parameter ($\alpha_{\rm cond}, \beta_\otimes, E_0$ inherited from P18/P15).

$p_{\rm rec}$ **derived by percolation.** The reconnection probability is itself **derived** from the primordial knot density: at condensation, composition becomes probable at the *3D continuum percolation threshold*, i.e. at the critical filling fraction $\varphi_c = n_K V_{\rm link} = 0.342$, whence

$$p_{\rm rec} = 1 - e^{-\varphi_c} \approx \mathbf{0.29}. \tag{14.14}$$

The corresponding density $n_K \approx 0.08\,\mathrm{fm}^{-3} \approx 0.5\,\rho_{\rm nucl}$ (hadronic scale, consistent with $r_{\rm link} \approx 1$ fm). **No free parameter** (script `simulation/experiments/composition_kinetics_v6.py`).

Two p's not to be confused. The *reconnection probability* $p_{\rm rec} \approx 0.29$ (percolation, eq. (14.13)) and the *mass exponent* $p = q = \frac{3}{4}$ (spin-$\frac{1}{2}$ Casimir $C_2(\frac{1}{2})$, in Ω_{ts}, chap. 9) are **two distinct, unrelated quantities**; likewise, φ_c (critical filling) is not the η of P22 nucleation.

14.14 The proton–neutron chain: what is topological and what is not

Demonstration: the magnetic moment differential. Proton and neutron differ only by their linking (Lk = 3 vs 2, charge $Q = \sum\chi + \mathrm{Lk}$). The magnetic moment *differential* is derived from the linking response at $v = c$, on the peripheral radius of the link:

$$\mu_p - \mu_n = \frac{r_{\rm link}}{\lambda_C}\,\mu_N \approx 4.71\,\mu_N \qquad (r_{\rm link} \approx 1\ \mathrm{fm}), \tag{14.15}$$

against $4.71\,\mu_N$ measured ($\sim 1\,\%$). The mass difference $\Delta m(n{-}p) = 1.29$ MeV comes from the linking tension $\beta_\otimes(\mathrm{Lk} - \mathrm{Lk}_{\rm opt})^2$.

Honest lesson. Only the *differentials* driven by linking (charge $Q = \sum\chi + \mathrm{Lk}$, sign of μ_n by braid parity, $\Delta\mu \propto \Delta\mathrm{Lk}$) are topological *a priori*. The *absolute* values (vev 246 GeV, τ_n via G_F) are not; the *absolute* magnetic moment of the neutron is however now computed from the torsion of the tubes (§14.15, a scale identification). The old "lifetime ladder" in pure topology ($N = 18 - 6|D|$) was an *artefact*: the lifetimes are Q*-driven* (Sargent rule $\tau \sim 1/Q^5$, $Q = \Delta m$), with $\mathrm{corr}(N_{\rm obs}, \log Q) = -0.99$.

14.15 Flux tube torsion: twist, blocked loops and magnetic moments

The exchange law and the derived chain

The flux tube obeys the **Călugăreanu–White–Fuller** theorem:

$$\boxed{\mathrm{Lk}_{\mathrm{framing}} = \mathrm{Tw} + \mathrm{Wr}} \qquad \mathrm{Tw} = \tfrac{1}{2\pi}\oint \tau\,\mathrm{d}s \text{ (torsion)}, \quad \mathrm{Wr} \text{ (writhe)}.$$

Topology is protected; torsion and writhe exchange with each other. *Exact* verification on the valid curves: $\mathrm{Wr} + \mathrm{Tw} = -3.00$ (3_1), 10.00 (5_1), 14.00 (7_1), 18.00 (9_1), 0.00 (4_1) — integers, as the theorem requires. Scripts: `flux_tube_torsion_balance.py`, `neutron_moment_3comp.py`, `three_case_circulation.py`, `chi_flux_quantum.py`.

The fundamental periphery. The Tw of a curve depends on the periphery — the knot-theoretic *framing* — (not topological); the *relaxed* state of the tube chooses the periphery integer closest to its writhe, leaving only the fractional residue:

$$\boxed{\mathrm{Tw}^* = \mathrm{round}(\mathrm{Wr}) - \mathrm{Wr}} \qquad (|\mathrm{Tw}^*| \le \tfrac{1}{2}). \tag{14.16}$$

For the trefoil ($\mathrm{Wr} = -3.354$): $\mathrm{Tw}^* = +0.354$, exactly the Frenet value of the analytic curve. For the 7_2 (repaired curve, $\mathrm{Wr} = 6.06$): $\mathrm{Tw}^* = -0.06$ — *quasi relaxed*.

Twist is a current; the flux quantum is derived. In a thin twisted tube, the internal lines form a helix ($B_\varphi/B_z = qr$); Ampère gives an **axial current** $I = 2q\Phi/\mu_0$ ($q = 2\pi\mathrm{Tw}/L$): an untwisted tube carries no current. The flux of the tube is fixed by the Chern–Simons **flux attachment** — the *same* integer $k+2 = 137$ that quantizes α:

$$\boxed{\Phi_{\chi\to\mathrm{EM}} = \frac{\Phi_0}{k+2} = \alpha\,\frac{h}{e} = \frac{\mu_0\, e\, c}{2}} \tag{14.17}$$

(exact identity; the flux of the tube = the one generated by the elementary charge at c). The twist moment then reads *with no free EM normalization*:

$$\mu_{\mathrm{tw}} = 2\pi\, e\, c\, \frac{\mathrm{Tw}^* S}{L}, \tag{14.18}$$

the only remaining freedom being geometric (S vector area, $L = \ell\, d_{\mathrm{tube}}$, ℓ ropelength).

The absolute moment of the neutron (3 components). The 7_2's being quasi relaxed ($\mathrm{Tw}^* = -0.06$), **the moment is carried by the trefoil core** — the same component that carries the charge ($\chi(3_1) = -2$). Inverting on $|\mu_n| = 1.913\,\mu_N$ gives

$$d_{\mathrm{tube}} = 0.209 \text{ fm} = \lambda\!\!\!^{-}_{C}(p) \quad (1\,\%), \tag{14.19}$$

consistent with the existing CGQ geometry: $L(3_1) = \ell\, d = 3.44$ fm $= 4.1\, r_p$ and, with $r_p = 4\lambda\!\!\!^{-}_{C}$, $\ell(3_1) = 16.37 \approx 16$ ($2.3\,\%$) — three statements, a single scale. *Status*: scale identification (one soft calibration), not an assignment agreement (reference config, geometric invariants, derived quantum).

The three circulation cases: torsion, blocked loop, or both. The charge circulation of a component falls into three cases: **T** (torsion: twist current (14.18), Lk-blind); **B** (**blocked loop**: a loop *threaded* by the linking cannot relax its circulation — the $7_{2,b}/k_b$ concept inherited from the archived Ω_{ts} derivation — an occurrence of 7_2 *blocked* by the linking constraint of the neutron —, which here receives its mechanism); **T+B**. The χ condensate being coherent (cf. superconductivity, §20.16), the blocked loop conserves its *fluxoid* and carries a persistent current:

$$\mu_B = \mathrm{Lk}\,\frac{\alpha\Phi_0\,\pi r^2}{\mu_0 r\,(\ln(8r/a)-2)}\,\cos\theta = 4.5\text{–}6.4\ \mu_N/\mathrm{Lk} \quad (r = 0.5\text{–}1\ \mathrm{fm},\ a = \lambda\!\!\!^{-}_C/2), \tag{14.20}$$

— the scale comes out *with no tuning* (weak logarithmic dependence), with the derived quantum (14.17).

The nucleon decomposed: T for the trefoil, B for the 7_2's

3_1 = case **T** ($\mathrm{Tw}^* = 0.354$): $\mu_T = -1.913\,\mu_N$ — the neutron ($Q = 0$) is core-pure. 7_2 = case **B** (quasi relaxed, threaded): they carry the Lk dependence. The transition $n \to p$ ($\mathrm{Lk}: 2 \to 3$) *blocks one more loop* (aligned): $+4.71\,\mu_N$ — the differential of §14.14, which thereby receives its mechanism (fluxoid) and connects to the $7_{2,b}$: *the blocked loop weighs in the mass (k_b, tension $\beta_\otimes$) as in the moment (μ_B).* $\mu_p = +2.793$ (measured 2.79285). Counting: 2 scales (d_{tube}, r) for 2 data, both identified with pre-existing scales ($\lambda\!\!\!^{-}_C$; $r \approx r_{\mathrm{link}}$, $E_K = \hbar c/r \approx 200$ MeV).

Scope and openings. (i) *Hyperons*: the response per unit of linking fixes the **orientation** of the blocked loop ($\theta_N \approx 2°$, $\theta_\Sigma \approx 67°$, $\theta_\Xi \approx 97°$, quasi perpendicular anti-aligned — whence the sign of Ξ^0–Ξ^-): these angles are *readings*; deriving them from the catalogue embeddings is the open programme. (ii) *External coupling*: the twist couples to B_{ext} via $E = -(4\pi\Phi/\mu_0)(\mathrm{Tw}/L)\,\vec{S}\cdot\vec{B}$, i.e. **neV** at laboratory fields — any mass alteration at the MeV scale (P23) is therefore *not directly magnetic.* (iii) *Lifetime — the Sargent lever (demonstrated) and the locked prediction*: any torsion rebalancing δm hits the weak lifetime with the amplification $5M/Q$ (Sargent, $\tau \propto Q^{-5}$ with $Q = \Delta m$, chap. 15):

$$\frac{\delta\tau}{\tau} = -5\,\frac{\delta m}{Q} = +5\,\frac{\delta\, f_\tau M}{Q} \qquad (\delta = \text{relaxed fraction of the reservoir } f_\tau M).$$

The lever is *hypersensitive* for small Q: for the neutron ($Q = 0.782$ MeV, reservoir $f_\tau m_n = 124$ MeV), a *minuscule* rebalancing $\delta = 1.4\times10^{-5}$ ($\delta m = 1.7$ keV) shifts τ_n by $+1.1\,\%$. *This lever is derived; the existence of an environmental rebalancing is not* — the link with the bottle/beam discrepancy (878.4 vs 888 s) remains **speculative**. The *locked* signature makes it testable: mass, lifetime and moment move *together* (twist→writhe channel), and a $1\,\%$ shift of τ_n would imply $|\delta\mu_n/\mu_n| \sim 7\times10^{-6}$ ($30\times$ the current precision of μ_n). Details: `docs/brouillon_torsion_tube_flux`, `_couplage_B_twist`, `_quantum_flux_chi`, `_neutron_moment_3comp`, `_jonction_mu_p_n`, `_trois_cas_circulation`.

14.16 Quarkonium spectroscopy: the Cornell potential (V5.37b)

NB (2026-07-21): in the spacetime weave reading, quarkonia are the compression *sector ($\tau < 0$, Coulomb-dominated binding — outside the torsion functional on principle, §14.2); $B_c/B_c(2S)$ (half-quarkonium, measured negative binding) belong to the same sector.* Charmonium and bottomonium (J/ψ, $\psi(2S)$, $\Upsilon(nS)$) are strongly bound links; their level spacings follow the Cornell potential $V(r) = -\alpha/r + \sigma r$, whose two constants TQNT fixes:

$$\alpha_{\mathrm{TQNT}} = K_\alpha\,\alpha_{\mathrm{EM}}\,N_c, \qquad \sigma_{\mathrm{TQNT}} = K_\sigma\,\beta_{V5}\,\frac{m_c}{\hbar c}, \tag{14.21}$$

$N_c = 3$ (the $\mathbb{Z}_3$ of the trefoil; derived §22.1). **NB (reference)**: the σ_χ catalogue treatment *supersedes* this V5.37b form: it directly sets $\sigma = \sigma_\chi$ (the same string as the Regge slope, *without* K_σ) and $\kappa = \frac{4}{3}\alpha_s$ (measured running α_s), RMS 0.08 % on the $1S$ — the constants K_α, K_σ below are the *historical* variant. Two constants ($K_\alpha = 18.27$, $K_\sigma = 602.95$) are calibrated *once* on the pair $\psi(2S)/J/\psi$; the three other spacings are *predictions*. Result: $\Sigma|\log| = 0.153$ dex, **4/4 pairs within** ± 0.3 **dex** ($\psi(2S)/J/\psi$ exact, $\psi(3S)/\psi(2S)$ +0.10, $\Upsilon(2S)/\Upsilon(1S)$ −0.04, $\Upsilon(3S)/\Upsilon(2S)$ +0.006). The *universal mode* (α, σ constants of the χ vacuum) works; the "topological" mode ($\alpha, \sigma \propto c_J$) fails — standard Cornell is the correct TQNT reading.

Chapter 15

Lifetimes

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15.1 Master formula: the same backbone as the mass

The lifetime factorizes like the mass: a **scale** × a **geometric factor per category** × a **kinematic dressing**.

Master rate equation (V6)

$$\Gamma = \underbrace{\Gamma_0}_{\text{scale}} \times \underbrace{\mathcal{G}_{\text{class}}}_{\substack{\text{geometry per class} \\ \text{(reconnection core } \Delta\text{Lk)}}} \times \underbrace{\rho(Q)}_{\substack{\text{kinematic} \\ \text{dressing}}} \tag{15.1}$$

$\mathcal{G}_{\text{class}}$ encodes the *reconnection class* (strong ΔLk=0; EM; weak ΔLk=$\pm$1, $\mathcal{G} \sim (G_F \Lambda_\chi^2)^2$); $\rho(Q)$ is the Sargent phase space, $\rho(Q) \propto Q^5$, with $Q = \Delta m$ the released topological energy.

Stability / rate split (not to be conflated).

- *Stability* = **topology** (TQNT's own contribution): a particle at $\mathrm{Lk} = \mathrm{Lk}_{\mathrm{opt}}$ (zero linking tension, "balanced spinning top") does not break up. The proton ($\mathrm{Lk}_{\mathrm{opt}}$=3) and the electron ($3_1$ floor) are the *only* stable fundamentals. *Topology says WHO is stable.*
- *Rate* = **standard kinematics** (*not* a TQNT contribution): $\rho(Q)$ is the Fermi/Sargent phase space, with the *measured* weak constants of the SM. At fixed coupling, $\log\tau$ follows $\log Q$ (Sargent) — reproduced to ~ 2 decades/particle without CKM, but this is standard physics, not a proper prediction.

The rest of this chapter develops $\mathcal{G}_{\mathrm{class}}$ and $\rho(Q)$ channel by channel.

15.2 The four channels: SM Fermi/Sargent theory + topological energy Q

The lifetime of an unstable particle follows the **Fermi/Sargent theory of the Standard Model**, $\Gamma = G_F^2\,|M|^2\,\Phi_n(Q)$. The *proper TQNT contribution* is the released energy $Q = \Delta m$ (topological mass) and the Sargent law $\Gamma \propto Q^5$; the matrix elements $|M|^2$ and the couplings remain electroweak phenomenology. No parameter is *fitted to the lifetimes*, but there are **measured SM constants** (§15.3). The absolute scale G_F is fixed by the **topological relaxation** of the $\Delta\mathrm{Lk} = \pm 1$ reconnection, of energy $E_{\mathrm{relax}} = \mathrm{VEV}$ (246 GeV — *derived*, +28 ppm, §22.4.4; 242 GeV extracted from the W/Z links $+\alpha$, i.e. G_F to $\sim 3\,\%$) — a TQNT topological mass, *not* a mediator — via

$$G_F = \frac{1}{\sqrt{2}\,E_{\mathrm{relax}}^2} = 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2} \quad (\text{measured } 1.166 \times 10^{-5}). \tag{15.2}$$

Each channel then follows its Fermi law, *with the measured weak constants of the SM* (g_A, V_{ud}, V_{us}, f_π, f_K, $\sin^2\theta_W$); none is *fitted to the lifetimes* ($N_{\mathrm{stab}} = |\mathrm{Lk} - \mathrm{Lk}_{\mathrm{opt}}|$ = number of reconnections towards stability):

Channel	**Scale**	**Rate Γ (Fermi, SM constants)**
Weak 3-body	G_F (relaxation)	$G_F^2\,Q^5\,N_{\mathrm{ch}}\,\lvert V\rvert^2/(192\pi^3)$ (Sargent)
Weak 2-body	G_F	$(G_F^2/8\pi)\,f_P^2\lvert V\rvert^2 m_P m_\ell^2(1-m_\ell^2/m_P^2)^2$
Weak non-lept.	G_F	$(G_F^2/8\pi)\,\lvert V_{ud}V_{us}\rvert^2 f_\pi^2 M^2 p_{\mathrm{cm}}$
Strong	tension $\beta_\otimes$	$\beta_\otimes\,\langle\sqrt{c_i c_j}\rangle\,N_{\mathrm{stab}}^3$
EM	α	$\alpha^2 m^3/(64\pi^3 f_\pi^2)$ (chiral anomaly)
EW	G_F (relaxation)	$9\,G_F m_W^3/6\sqrt{2}\pi$; $\sum(g_V^2+g_A^2)$; …

Summary: lifetimes via Fermi/Sargent (SM constants + topological Q), 34 particles

No parameter is fitted *to the lifetimes*; but G_F is anchored on the VEV (or on the muon, which *defines* it — the muon agreement is **tautological**, not a prediction) and $g_A, V_{ud}, f_\pi, f_K, \alpha_s$ are **measured SM inputs**. Only $Q = \Delta m$ and the Sargent law $\Gamma \propto Q^5$ are proper to TQNT. Nearly exact cases (thanks to these SM constants): μ (-0.00), n ($+0.001$, 880.3 s vs 878.4 s), $\pi^\pm$ ($+0.01$), $K^\pm$ ($+0.01$), π^0 ($+0.00$, anomaly), W, Z ($+0.01$). RMS per channel: weak 0.61, strong 0.56 (broad states), EM 0.28, **EW** 0.06; **overall** 0.94 **decade** ($\approx$ factor 9, order of magnitude). The inputs are *measured* constants ($|V_{\mathrm{CKM}}|$, f_π, f_K, g_A, $\sin^2\theta_W$, $\mu_{\Sigma\Lambda}$). The residuals are *known* hadronic effects (charm hierarchy, OZI / $U(1)_A$ suppressions, η-η' mixing). The heavy bosons (W, Z, t, H at 0.06 dec from the *same* G_F) confirm that the relaxation *is* the electroweak scale. Details and derivations: draft *Stability and lifetimes*.

15.3 The weak sector as P15 topological reconnection

In TQNT, weak decay is not mediated by a W boson: it *is* a topological reconnection (postulate P15). Two mechanisms are distinguished.

Two mechanisms. (i) The muon $\mu^- \to e^- \nu\bar{\nu}$ is a $\mathbb{Z}_3$ *mode* transition of the trefoil (generation $2 \to 1$): the trefoil remains a trefoil. (ii) The neutron $n \to p$ is a change of *linking*: n and p are the same link $3_1 + 7_2 + \overline{7}_2$, with Lk = 2 and 3 respectively (charge $Q = \sum \chi + \mathrm{Lk}$). Re-locking the loop costs energy; flipping a mode costs nothing.

Demonstration: the coupling is the vacuum scale. With the amplitude $A = 1/(\sqrt{2}\, v^2)$, the leptonic rate is $\Gamma_\mu = A^2 m_\mu^5 / 192\pi^3$. The *value* of v is now **derived** (open crossing on the anchor, §22.4.4: +28 ppm, conditional). An internal *compatibility* check remains: the scale extracted from the W/Z link masses via the electroweak relation and $\alpha = 1/(k{+}2) = 1/137$ gives $v \simeq 242$ GeV ($-1.6\,\%$ of the measured value). At that scale, $A = 1/(\sqrt{2} \cdot 242^2) = 1.21 \times 10^{-5}$ GeV^{-2}, whence

$$\tau_\mu = \hbar/\Gamma_\mu = 2.04\ \mu\mathrm{s} \quad (-7\,\%\ \text{vs}\ 2.197\ \mu\mathrm{s}),$$

with *no* other parameter (no g_A, no V_{ud}, no calibration): the 7 % deviation is *entirely* carried by the $-1.6\,\%$ on v (a v^4 lever); at the measured v, τ_μ returns to $-0.44\,\%$ (§22.4.4). The muon thus demonstrates that $G_F = 1/(\sqrt{2}v^2)$ is topological in its *structure* — and the scale itself is now derived (§22.4.4).

Demonstration: Cabibbo = reconnection-distance cost. The amplitude of a reconnection D generation steps away follows $|V| = \lambda_{\mathrm{Cab}}^D$, where the cost of one step is

$$\lambda_{\mathrm{Cab}} = \sqrt{m_d/m_s} = \sqrt{4.67/93.4} = 0.224 \tag{15.3}$$

(Gatto–Sartori–Tonin relation, reinterpreted as the cost of a $\mathbb{Z}_3$ mode change). With unitarity $V_{ud} = \sqrt{1 - \lambda_{\mathrm{Cab}}^2}$:

decay	D	prediction	measured
$n \to p$ ($\Delta S = 0$)	0	$V_{ud} = \sqrt{1-\lambda_{\mathrm{Cab}}^2} = 0.975$	0.974 (+0.1 %)
$\Lambda \to p$, $\Sigma \to n$ ($\Delta S = 1$)	1	$V_{us} = \lambda_{\mathrm{Cab}} = 0.224$	0.224 ($-0.3\,\%$)

The "5 % neutron suppression" ($V_{ud}^2 = 0.948$) is the unitarity $1 - \lambda_{\mathrm{Cab}}^2$, not a separate cost. The hierarchy $V_{us} \sim \lambda_{\mathrm{Cab}}$, $V_{cb} \sim \lambda_{\mathrm{Cab}}^2$, $V_{ub} \sim \lambda_{\mathrm{Cab}}^3$ (Wolfenstein) is the hierarchy of re-knotting distances.

CP violation = chirality. The only chiral structure in TQNT is the mirror operation $V_K(t) \to V_K(t^{-1})$ (antimatter = mirror). The amphichiral knots $(4_1, 6_3, 8_3)$ are CP-symmetric; the chiral ones $(3_1, \dots)$ CP-violating. **Testable prediction**: leptonic CP violation (PMNS δ_{CP}) *present* (leptons = chiral trefoil), *suppressed* in the amphichiral sectors — testable at DUNE. The exact magnitude ($\delta_{CP} \approx 65°$, of the order of the $\mathbb{Z}_3$ phase) remains open.

15.4 Particle–antiparticle oscillations: chiral inertia (V5p27–V5p33)

The four neutral-meson systems ($K^0\bar{K}^0$, $D^0\bar{D}^0$, $B^0\bar{B}^0$, $B_s^0\bar{B}_s^0$) oscillate at a frequency Δm. In TQNT, this is neither flavor mixing nor the CKM: it is a *mutual EM torsion* between interlinked knots, endowed with a *chiral inertia* (P15 reconnection).

Mechanism (four steps). (1) The magnetic field of one knot twists its interlinked companion ($\mathrm{Lk}_{\mathrm{eff}}$ coupling, V5p27). (2) The inertia is collective, via the inter-component linkings (V5p28). (3) Complexity asymmetry per pair. (4) Multiplicative three-body combination (V5p29). The frequency is

$$\omega_\chi = \sqrt{\frac{F^{\mathrm{EM}}_{\mathrm{torque}}}{I_\chi}}, \qquad I_\chi = I_{F3}\, A_{\mathrm{cx}} \prod_{i<j<k} \big(1 + \delta\, f_{\mathrm{arith}}\big), \tag{15.4}$$

$A_{\mathrm{cx}} = 1 + \beta_{\mathrm{osc}}\,|c_i - c_j|/\max(c_i, c_j)$, $f_{\mathrm{arith}} = |c_{\min} + c_{\max} - 2c_{\mathrm{mid}}|/c_{\mathrm{mid}}$.

Demonstration: δ from $\mathrm{SU}(2)_{135}$ (V5p31). Closing a 3-link on the modular sphere $\mathrm{SU}(2)_{135}$ amounts to four spin-$\frac{1}{2}$ Wilson loops; the partition function $\propto d_{1/2}^4$, whence

$$\delta = -4 \ln d_{1/2}(k{=}135) = -4 \ln\!\big(2 \cos \tfrac{\pi}{137}\big) = -2.7715. \tag{15.5}$$

The factor 4 (four topological directions $\times\ \mathbb{Z}_2$ chirality) is derived rigorously (Bar-Natan v_4, Reshetikhin–Turaev, V5p32); the explicit weight system on $\mathfrak{sl}_2$ chord diagrams is written out in V5p33. This *eliminated* one parameter at the time (from 11 to 10; current count: 9 calibrated, §27.2).

Result (0.1 % closure, V5p29).

system	Δm V5p29	Δm PDG	log ratio
K^0	3.484×10^{-12}	3.484×10^{-12}	$+0.0000$
D^0	6.252×10^{-12}	6.250×10^{-12}	$+0.0001$
B_d^0	3.333×10^{-10}	3.334×10^{-10}	-0.0001
B_s^0	1.167×10^{-8}	1.169×10^{-8}	-0.0006

Ratio $B_s/B = 35.02$ (PDG 35.05, 0.1 %); $\Sigma|\log| = 0.0008$ dex. **No flavor and no CKM invoked**: the mechanism is purely topological + mutual EM. Honestly, $\beta_{\mathrm{osc}} = -1.30$ remains calibrated (V5p30: ten methods tested for c_J, none below 1 %). *Complement (2026-07-21)*: the *absolute scale* of the flip — the global prefactor calibrated on K^0 — receives a candidate identity: the *double* fusion/splitting pair of the open crossing (counting law n pairs $\leftrightarrow 1/E_{\mathrm{relax}}^{2n}$, boxes reproduced at order, §22.4.4); complementarity: the mutual EM torque carries the internal structure and the *ratios*, the open crossing the absolute scale.

Part IX

Gravitation and emergent spacetime

Chapter 16

Emergent spacetime

Chapter contents

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Postulate P5 — Emergence of spacetime

Spacetime is not a fixed arena. The **bi-conformal** metric emerges from the wave function density :

$$\mathrm{d}s^2 = -c^2\, e^{-\kappa_g|\Psi|^2}\,\mathrm{d}t^2 + \Omega^2\big(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\big), \qquad \Omega^2(\mathbf{r}) = 1+\kappa_g\,|\Psi(\mathbf{r})|^2. \tag{16.1}$$

The constant κ_g is not a free parameter : it is derived from topological exclusion (chapter 17) :

$$\kappa_g = \frac{96\pi}{7}\,\alpha_{\mathrm{EM}}^{18}\,\lambda\!\!\!^{-}_C = \frac{96\pi}{7}\,\alpha_{\mathrm{EM}}^{18}\,\frac{\hbar}{m_p c} = 3.12\times10^{-53}\ \mathrm{m} \tag{16.2}$$

Dimensional convention for κ_g (documented 2026-07-14). κ_g is a **length**. Its operational anchor is the identity

$$\kappa_g \;=\; \frac{8\pi G\, m_{\mathrm{base}}}{c^2} \;=\; 3.1218\times10^{-53}\ \mathrm{m} \qquad\Longleftrightarrow\qquad G \;=\; \frac{\kappa_g\, c^2}{8\pi m_{\mathrm{base}}} \;=\; \frac{12}{7}\,\alpha_{\mathrm{EM}}^{18}\,\frac{\hbar c}{m_{\mathrm{base}}^2}, \tag{16.3}$$

the coincidence of the two forms ($-0.06\,\%$) *being* the historical derivation of G (V5.5 step 6, `kappa_highprecision.py`); the canonical equation of G is *unique* — $G=\hbar c/M_{\mathrm{Pl}}^2$ (§17.8) — and this route is its *nucleon reading* of M_{Pl}. The P5 shorthand $\Omega^2=1+\kappa_g|\Psi|^2$ is the *schematic* form of the operational one (used by the solver and by the derivation of G):

$$h(\mathbf{r}) \;=\; \Omega^2-1 \;=\; \kappa_g\,\tilde\Phi(\mathbf{r}), \qquad \tilde\Phi(\mathbf{r}) = \int \frac{|\Psi(\mathbf{r}')|^2}{4\pi|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}^3r'\ [\mathrm{m}^{-1}], \quad \int|\Psi|^2\,\mathrm{d}^3r = 1, \tag{16.4}$$

i.e. exactly $h = 2\Phi_{\mathrm{Newton}}/c^2$ for a source mass m_{base} distributed according to $|\Psi|^2$: $[\kappa_g] = [\mathrm{m}]\times[\tilde\Phi] = [\mathrm{m}^{-1}] \Rightarrow h$ dimensionless. *Nota bene*: the $\lambda\!\!\!^{-}_C$ of eq. (16.2) is that of the **proton**; the ratio $\alpha_{\kappa_g} = \kappa_g/\lambda\!\!\!^{-}_C(e^-)$ of the dark-energy chapter (eq. 25.2) divides by that of the **electron** (the scale of vacuum fluctuations, consistent with the m_e^4 of the formula): two different Compton wavelengths under the same symbol — now explicit.

Properties of the bi-conformal metric.

1. **Signature.** $g_{00} < 0$ and $g_{ij} > 0$: the signature $(-,+,+,+)$ is always preserved.
2. **Far from the sources.** $|\Psi|^2 \to 0 \Rightarrow$ Minkowski.
3. **Near the knot.** $\Omega > 1$: proper time slowed, space dilated. In the weak field : PPN parameters $\beta_{\mathrm{PPN}} = \gamma_{\mathrm{PPN}} = 1$, identical to GR.
4. **Effective speed of light.** $c_{\mathrm{eff}} = c\, e^{-h/2}/\Omega$. In the weak field : Shapiro delay.

16.1 The quantum gravitational field (QGF)

The P5 metric is not merely a geometric object : it expresses a **physical field** that every particle generates. In the weak field, setting the perturbation

$$h(\mathbf{r}) = \kappa_g\, |\Psi(\mathbf{r})|^2 \qquad \big(g_{00} = -c^2 e^{-h}, \quad \Omega^2 = 1 + h\big),$$

the density $|\Psi|^2$, maximal at the core of the knot and decreasing toward the exterior, generates on every test wave a *centripetal* geodesic acceleration.

Definition 16.1 (Quantum gravitational field (QGF))**.** *Every particle — a knot of the field χ with density $|\Psi|^2$ — generates a vector field directed* ***toward its center****, the* quantum gravitational field *:*

$$\mathbf{g}_q(\mathbf{r}) = -\frac{c^2}{2}\,\nabla h(\mathbf{r}) = -\frac{c^2}{2}\,\kappa_g\,\nabla|\Psi(\mathbf{r})|^2, \tag{16.5}$$

with potential $\Phi_q(\mathbf{r}) = -\frac{c^2}{2}\,\kappa_g|\Psi|^2$ — a well, deepest at the center, whose exterior reach decreases as $1/r$. It is the field that a particle presents to the other *waves ; it must not be confused with the magnetic confinement V_{mag} (P4, 2.10), a* distinct *term of the master equation, which traps the particle's* own *wave (the cause of the knot, of which $|\Psi|^2$ — and thus the QGF — is the consequence).*

The QGF is fundamental ; gravitation emerges from it. Gravitation is not the field of an isolated particle : it is the **large-scale resultant** of many QGFs. Summing the field (16.5) over the N nucleons of a body of mass $M = N m_{\mathrm{base}}$ (chapter 17, eq. 17.4), one obtains $g(r) = GM/r^2$ with $G = 2\hbar c\, h_0/m_{\mathrm{base}}^2$: Newton's constant *emerges* from the sum of the wells. The QGF is therefore the **primary** object ; "gravitation" properly designates its macroscopic limit. At the scale of an isolated nucleon, the gravitational amplitude of the QGF is minute — the surface perturbation is $h_0 = G m_{\mathrm{base}}^2/(2\hbar c) \approx 3 \times 10^{-39}$ ($\Omega^2 - 1 \sim 10^{-39}$). What dominates *locally* is therefore not this gravitational tail, but the magnetic confinement (P4) that structures the knot.

QGF and P22 nucleation. The dimensionless strength of the QGF is exactly the metric gradient $\xi = |\nabla\Omega^2|\,\lambda_C/\Omega^2$ that drives nucleation at the boundaries (P22, §25.7) : at the nucleon $\xi \sim 10^{-39} \ll 1$ (purely attractive QGF, no production of particles) ; at $\xi \sim 1$ (horizons, critical fields) the gradient of the QGF materializes pairs (Schwinger), and $\xi \gg 1$ gives Hawking radiation. The QGF thus unifies attraction (large-scale resultant → gravity) and nucleation (strong-gradient limit → P22).

16.2 Modified variational formulation

Proposition 16.1 (Weak form in conformal metric)**.**

$$\boxed{\int_\Omega \frac{\hbar^2}{2m}\,\Omega\,\nabla\Psi\cdot\nabla\varphi\;\mathrm{d}^3x \;+\; \int_\Omega V_{\mathrm{mag}}\,\Omega^3\,\Psi\,\varphi\;\mathrm{d}^3x \;=\; E\int_\Omega \Omega^3\,\Psi\,\varphi\;\mathrm{d}^3x} \tag{16.6}$$

Origin of the Ω factors. In three dimensions, with $g_{ij} = \Omega^2 \delta_{ij}$:

- The Riemannian volume element is $\sqrt{g}\,\mathrm{d}^3x = \Omega^3\,\mathrm{d}^3x$.
- The contravariant gradient gives the factor Ω^{-2} in the kinetic term.
- The kinetic term integrates $\Omega^{-2} \cdot \Omega^3 = \Omega$.

16.3 FEM modifications for the conformal metric

16.3.1 Modified stiffness matrix

$$\widetilde{K}_{ij}^{(e)} = \bar{\Omega}_e \cdot K_{ij}^{(e)} \tag{16.7}$$

16.3.2 Modified potential matrix

$$\widetilde{P}_{ij}^{(e)} = \sum_{q=1}^{4} \frac{V_e}{4}\, V_{\mathrm{mag}}(\mathbf{r}_q)\, \Omega^3(\mathbf{r}_q)\, N_i(\mathbf{r}_q)\, N_j(\mathbf{r}_q) \tag{16.8}$$

16.3.3 Modified mass matrix

$$\widetilde{M}_{ij}^{(e)} = \sum_{q=1}^{4} \frac{V_e}{4}\, \Omega^3(\mathbf{r}_q)\, N_i(\mathbf{r}_q)\, N_j(\mathbf{r}_q) \tag{16.9}$$

In the three expressions above, $K_{ij}^{(e)}$, $P_{ij}^{(e)}$ and $M_{ij}^{(e)}$ denote respectively the elementary stiffness, potential and mass matrices (P1 finite elements) for the nodes i, j of the tetrahedron e ; the tilde marks their version modified by the conformal metric. $\bar{\Omega}_e$ is the average of the conformal factor $\Omega^2 = 1 + \kappa_g |\Psi|^2$ over the element, V_e its volume, $N_i(\mathbf{r})$ the P1 (linear) shape functions and $\{\mathbf{r}_q\}_{q=1}^4$ the four Gauss quadrature points. The factor Ω^3 comes from the volume element $\sqrt{g}\,\mathrm{d}^3x$ of the emergent metric.

16.4 Self-consistent algorithm

The wave Ψ determines the metric ($\Omega^2 = 1 + \kappa_g|\Psi|^2$), and the metric influences the wave. This circularity requires a self-consistent iteration.

Algorithm 2 Self-consistent iteration with metric backreaction

Input : knot K, mesh, $\tilde{\kappa}_g = 0.1$ (mesh units[1]), $\alpha_d = 0.5$, $\texttt{tol} = 10^{-4}$
$E^{(0)} \leftarrow 0$, $\Omega^{2\,(0)}(\mathbf{r}) \leftarrow 1$ for all $\mathbf{r}$
for $i = 0, 1, 2, \ldots, i_{\max}$ **do**
    Assemble $\widetilde{\mathbf{H}}^{(i)}$ and $\widetilde{\mathbf{M}}^{(i)}$ with $\rho(K, E^{(i)})$ and $\Omega^{2\,(i)}$
    Solve $\widetilde{\mathbf{H}}^{(i)}\boldsymbol{\phi} = E_{\mathrm{new}}\,\widetilde{\mathbf{M}}^{(i)}\boldsymbol{\phi}$
    Extract Ψ_{new} (ground state)
    $\Omega^2_{\mathrm{new}}(\mathbf{r}) \leftarrow 1 + \kappa_g\,|\Psi_{\mathrm{new}}(\mathbf{r})|^2$
    $E^{(i+1)} \leftarrow \alpha_d\,|E_{\mathrm{new}}| + (1-\alpha_d)\,E^{(i)}$
    $\Omega^{2\,(i+1)} \leftarrow \alpha_d\,\Omega^2_{\mathrm{new}} + (1-\alpha_d)\,\Omega^{2\,(i)}$
    if $\Delta E < \texttt{tol}$ **and** $\Delta\Omega < \texttt{tol}$ **then**
        Convergence reached
    end if
end for

Physical interpretation. The self-consistent loop realizes the *wave–spacetime backreaction* :

1. The wave Ψ creates spacetime via $\Omega^2 = 1 + \kappa_g|\Psi|^2$.
2. Spacetime modifies the Schrödinger equation.
3. The new wave modifies spacetime again, until equilibrium.

This mechanism is the discrete analogue of the Einstein equations coupled to matter.

16.5 No Higgs mechanism, no graviton

Postulate P9

The Higgs mechanism is replaced by magnetic confinement (P4). The graviton is replaced by the emergent metric (P5). The H boson observed at the LHC exists, but as a composite state.

The complete argument refuting the Higgs boson — topological mass, relaxation scale, degeneracy of the couplings $\propto$ mass, WW unitarity — is developed in chapter 23.

[1]The value $\tilde{\kappa}_g = 0.1$ is the dimensionless coupling in the FEM mesh units ($L = 1$, $\hbar = 1$). The physical constant $\kappa_g = 3.12 \times 10^{-53}$ m is derived from topological exclusion : $\kappa_g = 96\pi\,\alpha_{\text{EM}}^{18}\,\lambda\!\!\!^{-}_C/7$ (eq. 16.2).

Part X

Emergent gravitation

Chapter 17

Gravitation by topological exclusion

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This chapter derives the Newton constant G from a single hypothesis : matter repels spacetime. No external gravitational constant is introduced.

P20 — Topological exclusion

Spacetime cannot penetrate matter. A topological knot of the field χ generates a **minimal distance** between it and the surrounding flat spacetime ; this exclusion zone forces the metric to deform around the knot. This deformation is the **quantum gravitational field** (QGF) of the knot (§16.1) ; the **gravitation** is *not* the field of an isolated knot, but the **large-scale resultant** of many QGFs — the sum of their deformations.

Physical picture. The proton is a link $3_1+7_2+\bar{7}_2$ of the field χ. Where the field is knotted, its density $|\Psi|^2$ is high and the metric *cannot remain flat*. Spacetime is repelled : it must go around the knot, creating a "bump" in the metric that propagates to infinity as $1/r$ — the **quantum gravitational field** (QGF, def. 16.1) of the knot. A second knot placed in this field is deflected toward the first. The **gravitation** is its *large-scale resultant*, obtained by summing the QGF over the N nucleons of a body.

17.1 Boundary-value problem

17.1.1 Geometry of the exclusion

We model the proton as a sphere of radius $a = r_p$ that excludes flat spacetime. Outside ($r > a$), the metric perturbation $h(\mathbf{r})$ satisfies the Laplace equation :

$$\nabla^2 h = 0 \qquad (r > a) \tag{17.1}$$

17.1.2 Spherical solution

The spherically symmetric solution that vanishes at infinity is :

$$h(r) = h_0 \times \frac{a}{r} \qquad (r \geq a) \tag{17.2}$$

where $h_0 = h(a)$ is the **metric perturbation at the exclusion surface**, to be determined.

17.1.3 Geodesic acceleration

In the perturbed metric $g_{00} = -(1-h)\,c^2$, the geodesic acceleration of a test particle is :

$$g(r) = -\frac{c^2}{2}\,\frac{\partial h}{\partial r} = \frac{c^2\,h_0\,a}{2\,r^2} \tag{17.3}$$

For a source mass $M = N\,m_{\text{base}}$ (body of N nucleons) :

$$g(r) = \frac{N \times c^2\,h_0\,a}{2\,r^2} = \underbrace{\frac{c^2\,h_0\,a}{2\,m_{\text{base}}}}_{G} \times \frac{M}{r^2} \tag{17.4}$$

The identification with $g = GM/r^2$ gives :

$$\boxed{G = \frac{c^2\,h_0\,a}{2\,m_{\text{base}}} = \frac{2\,\hbar c\,h_0}{m_{\text{base}}^2}} \tag{17.5}$$

where we have used $a = r_p = 4\hbar/(m_{\text{base}}c)$. **The whole problem reduces to determining h_0.**

17.2 Determination of h_0

17.2.1 Topological suppression by crossing

The proton is a link $3_1 + 7_2 + \bar{7}_2$ with :

$$c_n(\text{nucleon}) = c_n(3_1) + 2 \times c_n(7_2) = 3 + 7 + 7 = 17 \tag{17.6}$$

The metric perturbation h_0 at the edge of the proton is generated by the quantum fluctuations of the field χ inside the knot. In Chern-Simons theory at level k, the perturbative Kontsevich expansion gives :

$$\langle W_K \rangle - 1 = \sum_{n=1}^{\infty} v_n(K) \times \left(\frac{1}{k+2}\right)^n \tag{17.7}$$

where the $v_n(K)$ are the Vassiliev invariants of order n, and the first nontrivial invariant appears at order $n = c_n(K)$. The metric perturbation is proportional to $|\langle W_K \rangle - 1|^2$; for the nucleon ($c_n = 17$), the dominant term is of order α_{EM}^{17}, plus a factor α_{EM} coming from the Gauss phase of the linking $\text{Lk} = +3$ (odd). Hence :

Proposition 17.1 (Suppression by crossing)**.**

$$h_0 = C \times \alpha_{\text{EM}}^{c_n^{\text{eff}}} \tag{17.8}$$

with $c_n^{\text{eff}} = c_n(\text{nucleon}) + 1 = 18$ and C a prefactor determined by the internal structure of the nucleon.

17.2.2 The prefactor $C = 6/7$

The prefactor encodes the composite structure of the nucleon via two ratios :

1. $r_p/\lambda\!\!\!^{-}_C = 4 = 2\,N_\Psi$, where $N_\Psi = 2$ is the number of active knots (7_2 and $\bar{7}_2$) ;
2. $c_n(3_1)/c_n(7_2) = 3/7$ (fermionic core 3_1 / bosonic companions 7_2).

The product gives :

$$2C = \frac{12}{7} = \underbrace{4}_{r_p/\lambda\!\!\!^{-}_C} \times \underbrace{\frac{3}{7}}_{c_n(3_1)/c_n(7_2)} \tag{17.9}$$

17.3 G derived : the formula

Theorem 17.1 (Emergent G — canonical equation and reading)**.** *The equation of G is **unique**: that of the spacetime weave (§17.8),*

$$\boxed{G = \frac{\hbar c}{M_{\mathrm{Pl}}^2}} \tag{17.10}$$

(G = the amplitude of the weave, inverse square of its rigidity M_{Pl}). What this chapter derives is the reading channel *of the rigidity by matter — the 18-crossing ladder:*

$$m_{\mathrm{base}} = M_{\mathrm{Pl}}\sqrt{\tfrac{12}{7}\,\alpha_{\mathrm{EM}}^{18}} \qquad \Longrightarrow \qquad G = \frac{12}{7}\,\alpha_{\mathrm{EM}}^{18}\,\frac{\hbar c}{m_{\mathrm{base}}^2} \tag{17.11}$$

(composition definition × reading — not *a second equation of G); that is, in terms of the topological invariants of the nucleon :*

$$G = \frac{4\,c_n(3_1)}{c_n(7_2)} \times \left(\frac{1}{k+2}\right)^{c_n(\mathrm{nucleon})+1} \times \frac{\hbar c}{m_{\mathrm{base}}^2} \tag{17.12}$$

with $k = 135$, $c_n(3_1) = 3$, $c_n(7_2) = 7$, $c_n(\mathrm{nucleon}) = 17$, and $m_{\mathrm{base}} = 938.2$ MeV — the mass of the base nucleon *(*V6 base $\sqrt{\sigma_\chi}\mathcal{S}$*, recalibrated 2026-07-18, equal to m_p^{exp} at 0.003 %): the derivation is* self-contained *(the old* 940.2 *was a V4 remnant, inconsistent with the numerical check below).* Convention (2026-07-19, revised)*: the reference mass is that of the nucleon* as an object *— the link $3_1 + 7_2 + \bar{7}_2$* before Lk *selection, which encompasses proton and neutron as linking states of the same object. The proton occupies it at the floor (*$\mathrm{Lk}_{\mathrm{opt}}$*, zero tension), whence $m_{\mathrm{base}} = m_p$; the neutron adds* 1.29 MeV *of linking tension —* state *energy, which gravitates in the source mass but does not enter the coupling (including it there would count it twice).* Notation NB*: the historical symbols of the chain ($\lambda\!\!\!^{-}_C$, r_p, labelled "proton") refer to the floor state that realizes m_{base}; all values are unchanged. The* cosmic-average *reading (G tracking the weighted nucleon abundance) was tested and discarded (2026-07-19): mixed summation — the local architecture is excluded ×423 by the triple pulsar, and cosmic averaging has no physical channel (deficits ≥41.6 orders); cf. the G-abundance draft.*

No external gravitational constant intervenes. *The old constant κ_g of postulate P5 is eliminated : $\kappa_g = 96\pi\,\alpha_{\mathrm{EM}}^{18}\,\lambda\!\!\!^{-}_C/7$.*

Numerical verification (via the nucleon reading).

$$G_{\mathrm{predicted}} = \frac{12}{7} \times (7.2974 \times 10^{-3})^{18} \times \frac{3.1615 \times 10^{-26}}{(1.6726 \times 10^{-27})^2} = 6.6710 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{17.13}$$

($m_{\mathrm{base}} = 938.24$ MeV $= 1.6726 \times 10^{-27}$ kg.)

	Value	
$G_{\mathrm{predicted}}$	6.6710×10^{-11}	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$
$G_{\mathrm{CODATA\ 2018}}$	6.6743×10^{-11}	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Deviation	**0.05 %**	

The residual of 0.06 % was partly attributable to the historical deviation on m_p. *NB 2026-07-18* : the $\sqrt{\sigma_\chi}$ recalibration brings the base to 938.2 (m_p^{exp} at 0.003 %); the formula, now *self-contained* (V6 base in $\hbar c/m_{\mathrm{base}}^2$), gives $G = 6.671 \times 10^{-11}$, i.e. −0.05 % vs CODATA — a clean, unattributed residual.

Significance NB (2026-07-19). The *relative* CODATA 2018 uncertainty is 2.2×10^{-5} ($G = 6.67430(15) \times 10^{-11}$): the -4.9×10^{-4} residual is therefore *significant* ($\sim$22 formal standard deviations), not measurement noise. Metrological context (the "G puzzle"): individual G measurements are mutually inconsistent at the $\sim 5\times10^{-4}$ level — BIPM-14 at 6.67554(16) ($+1.9\times10^{-4}$), atom interferometry Rosi-14 at 6.67191(99) (-3.6×10^{-4}, relative uncertainty 1.5×10^{-4}) — and the CODATA mean carries an expansion factor for that reason. The prediction 6.6710 sits at the *low* edge of the experimental cloud, within 1σ of Rosi-14. Attached prediction: metrological convergence must pull G downward; a consolidated G at 6.674 with the dispersion resolved would refute the formula at the 5×10^{-4} level.

Computation convention (2026-07-19). All applications of emergent gravitation (light bending, precession, Schwarzschild, gravitational waves, cosmology) use $G_{\text{predicted}} = 6.6710 \times 10^{-11}$ — the chain is *self-contained*; G_{CODATA} only enters as a metrological comparison point. At the displayed precision of the classical tests, the -4.9×10^{-4} deviation is invisible (1.750″ unchanged; Mercury 42.98″/century). Cosmological chains: G only enters through $H(T)$ (Friedmann, $\propto \sqrt{G}$), i.e. -2.5×10^{-4} on the Γ/H links ($c_{\min}$ decoupling, survival) — invisible against the Vassiliev jumps ($\times$137 per crossing: $c_{\min} = 23$ unchanged) and the T_c envelope; the $\Omega_{\text{DM}}/\Omega_b$ ratio is moreover *G-independent by construction*, on both sides ($\rho_{\text{crit}} \propto 1/G$ cancels from the predicted ratio as from the measured $\omega_{\text{DM}}/\omega_b$). The H_0 uncertainty (tension) does not enter either: the chain lives in the radiation era ($H(T)$ carries no H_0), η_B and $\omega_{\text{DM}}/\omega_b$ are h-free; H_0 only bites the ρ_Λ audit, already on record ($+5.9/-9.9\,\%$ band).

17.4 Consistency with the P5 metric : the QGF profile

The perturbation h_0 was just obtained through the topological suppression (eq. 17.8). But Postulate P5 also gives it *directly*, through the metric $\Omega^2 = 1 + \kappa_g|\Psi|^2$, that is $h(\mathbf{r}) = \kappa_g|\Psi(\mathbf{r})|^2$. These two determinations must coincide at the exclusion surface : this requirement fixes the normalization of the quantum gravitational field (def. 16.1) and reveals where the smallness of gravity resides.

$|\Psi|^2$ is a linear density. Since $\kappa_g = \frac{96\pi}{7}\,\alpha_{\text{EM}}^{18}\,\lambda\!\!\!^{-}_C$ is a *length*, the product $\kappa_g|\Psi|^2$ is dimensionless only if $|\Psi|^2$ has the dimension of a **linear** density (length^{-1}) — the density of the field χ integrated over the cross-section of the knot tube (a 1D curve in S^3).

The cancellation of α_{EM}^{18}. The two forms coincide, $h_0 = \kappa_g\,|\Psi(a)|^2 = \frac{6}{7}\,\alpha_{\text{EM}}^{18}$, if and only if

$$\boxed{|\Psi(a)|^2 = \frac{h_0}{\kappa_g} = \frac{6/7}{(96\pi/7)\,\lambda\!\!\!^{-}_C} = \frac{1}{16\pi\,\lambda\!\!\!^{-}_C} \approx 0.095\ \text{fm}^{-1}} \tag{17.14}$$

The factor α_{EM}^{18} cancels. All the smallness of gravity ($\sim 10^{-39}$) is thus carried by the single constant κ_g ; the wave density $|\Psi(a)|^2$ is a purely *geometric* quantity, of order $0.1\,\text{fm}^{-1}$ (in dimensionless form, $|\Psi(a)|^2\,\lambda\!\!\!^{-}_C = 1/16\pi \approx 0.020$). The QGF is thus a field with **a single scale**, uniformly weak because κ_g contains α_{EM}^{18} : it carries *no* "deep well", the confinement $\sim mc^2$ pertaining to the magnetic potential V_{mag} (P4), a distinct field.

Cross-check of the field. The quantum gravitational field at the surface of the proton is

$$g_q(r_p) = \frac{c^2 h_0}{2\,r_p} = \frac{G\,m_p}{r_p^2} \approx 1.58 \times 10^{-7}\ \text{m}\,\text{s}^{-2}, \tag{17.15}$$

that is *exactly* the Newtonian self-field of the proton : the QGF of an isolated nucleon *is* its own gravity field. The full radial profile ($h(r) = \kappa_g|\Psi|^2$ inside, tail $h_0\,a/r$ outside, eq. 17.2) as well as the round-trip $\kappa_g \to G = \frac{12}{7}\alpha_{\text{EM}}^{18}\hbar c/m_{\text{base}}^2$ ($-0.1\,\%$) are verified in `cpq_profile_consistency.py`.

Non-circular geometric test. In scale-free form (since $r_p = 4\lambda\!\!\!^{-}_C$), the condition (17.14) is equivalent to $L_{\text{eff}} = 16\pi\lambda\!\!\!^{-}_C = 4\pi\, r_p$, that is an arc-length to radius ratio equal to 4π. Measured on the catalogue curves alone — without any physical input (no fm, no G, no α_{EM}) — this ratio equals, per component, $L/R_{\text{gyr}} = 12.3$ (3_1) and 12.9 (7_2), to be compared with $4\pi \approx 12.57$: agreement to 2–3 % on two *independent* knots (about twice the ratio of a circle, 2π, owing to the winding). The length imposed by the linear normalization of the QGF thus coincides with a geometric regularity of the knots (`cpq_length_test.py`).

17.5 Universality of G

All baryonic matter is composed of nucleons. The electron contributes $(m_e/m_p)^2 \approx 3 \times 10^{-7}$ of the metric perturbation : negligible. The effective G is entirely determined by the nucleon.

Dark matter ($c_n > 18$) generates a negligible self-curvature ($G_{\text{DM}}^{\text{self}} < 10^{-5}\, G$) but *responds* to the field created by baryonic matter : the equivalence principle is respected (geodesics of $g_{\mu\nu}$). Consequences :

- **Spherical halos** : without self-gravity nor Van der Waals ($C_6 = 0$), the DM does not collapse into a disk.
- **No dark stars** : $G_{\text{self}} \approx 0$ forbids the formation of bound structures.
- **Rotation curves** : the DM responds to the baryonic well and contributes to the total felt mass.

17.6 Immediate consequences

17.6.1 The hierarchy problem solved

$$\frac{\alpha_G}{\alpha_{\text{EM}}} = \frac{12}{7} \times \alpha_{\text{EM}}^{17} \approx 10^{-36} \tag{17.16}$$

Gravity is weak because the nucleon has 18 effective crossings : each one suppresses the coupling by a factor 1/137.

17.6.2 Unification gravity / dark matter / halos

The exponent $18 = c_n(\text{nucleon}) + 1 = c_n^*$ is the threshold that separates visible matter ($c_n \leq 18$) and dark matter ($c_n > 18$). The three phenomena share the same suppression α_{EM}^{18} :

Phenomenon	**Factor**	**Mechanism**
Weakness of gravity	$\alpha_{\text{EM}}^{18} \approx 10^{-38.5}$	18 effective crossings
Dark matter threshold	$c_n^* = 18$	same suppression
Spherical halos	$G_{\text{DM}}^{\text{self}} \approx 0$	over-suppression ($c_n > 18$)

17.6.3 Derived constants

The old constant κ_g and the Planck mass are now derived :

$$\kappa_g = \frac{8\pi\, G\, m_p}{c^2} = \frac{96\pi}{7}\, \alpha_{\text{EM}}^{18}\, \lambda\!\!\!^{-}_C \tag{17.17}$$

$$m_P = \sqrt{\frac{\hbar c}{G}} = m_p \times \sqrt{\frac{7}{12}}\, \alpha_{\text{EM}}^{-9} \tag{17.18}$$

17.6.4 Verification : the Earth

$$g = G_{\text{predicted}} \times M_\oplus / R_\oplus^2 = 6.671 \times 10^{-11} \times 5.972 \times 10^{24} / (6.371 \times 10^6)^2 = 9.81\ \text{m s}^{-2}\ \checkmark \tag{17.19}$$

17.6.5 Inventory of constants

Constant	Status	Origin
$\hbar$	fundamental	quantum mechanics
c	fundamental	special relativity
$k = 135$	**derived** (integer lock, conditional)	$k{+}2 = \text{round}(\frac{8\pi}{3}\ell(3_1)) = 137$ (3.4)
m_p	derived	topology of $3_1 + 7_2 + \bar{7}_2$
$G = 6.671 \times 10^{-11}$	**derived**	eq. (17.10)
$\kappa_g = 3.12 \times 10^{-53}$ m	**derived**	eq. (17.17)
$m_P = 1.22 \times 10^{22}$ MeV	**anchor** $\leftrightarrow$ derived	exchange relation (§17.7)

17.7 The metric matching : rigidity across T_c

Theorem 17.1 is anchored on the nucleon — which does not exist before the χ condensation. What is the metric response before T_c, and what fixes the anchor? (Consolidated from the matching draft, 2026-07-19; scripts `raccord_fond_ondes_chi.py`, `derive_rigidite_mpl.py`, `etude_v_mpl_mur.py`.)

The $G(t)$ step. Before T_c (62 ± 3 MeV *derived*, §26.7.8; $t_c \approx 1.6 \times 10^{-4}$ s, $g_* = 15.2$): no knots, no QGF wells, no operational definition of G — gravitation exists only as the global metric response of the χ wave background (expansion). At T_c, G is born in *a single step*, directly at its final value: its ingredients ($12/7$, α^{18}, $\sqrt{\sigma_\chi}\,\mathcal{S}$) are fixed by topology and by the condensation itself. No kinetics of value — neither by epoch nor by particle (§ theorem convention).

Inverted reading: the exchange relation. The theorem is equivalent to the hierarchy identity $m_{\text{base}}/M_{\text{Pl}} = \sqrt{(12/7)\,\alpha^{18}}$. The only *fixed* object of the sector is the **rigidity** of the χ vacuum ($S_{\text{vac}} = c^4/8\pi G$, i.e. M_{Pl}), a property of the $k = 135$ algebra which pre-exists T_c. Read backwards, the identity *predicts* the condensate scale: $\sqrt{\sigma_\chi} = (M_{\text{Pl}}/\mathcal{S}_{\text{octet}})\sqrt{(12/7)\,\alpha^{18}} = 418.38$ MeV (recalibrated: 418.47, $-0.022\,\%$ — the same residual as G). The theorem is the **exchange relation** between the two writings of the anchor, not an oriented derivation.

The per-crossing ladder. $h_0 = \frac{6}{7}\alpha^{c_n+1}$ reads per crossing: $(\frac{7}{6}h_0)^{1/18} = 7.2974 \times 10^{-3} = \alpha_{\text{EM}}$ ($-0.03\,\%$ from $1/137$). Top of the ladder: α^0 = the wave background itself (un-knotted χ density *is* the metric, full leakage); α^1 = the interface step (the $+1$ in the exponent); α^{18} = the nucleon — the 10^{-36} hierarchy is born *with* matter (closed flux). Guard-rail: this is the *construction* of the universal coupling, not a per-particle G ($\eta(\text{H}, \text{Pt}) \sim 0.35$, excluded $\times 10^{14}$; per-species ladder $\Rightarrow$ electrons dominate $\times 10^{33}$).

Exact κ_g. Two equivalent closed forms: $\kappa_g = \frac{96\pi}{7}\alpha^{18}\lambda\!\!\!^{-}_C = 8\pi\sqrt{12/7}\;\alpha^9\ell_{\text{Pl}} = 3.1201 \times 10^{-53}$ m (self-contained; the historical 3.1218 was CODATA-anchored, $+0.54$ per mille = the G residual). A factor $\sqrt{\alpha}$ per crossing. Per species, $\kappa_g(K) = 8\pi G m_K/c^2 \propto m_K$ (electron: 1.699×10^{-56} m): the *length* varies, the rigidity (G) is the invariant.

Honest pinning and the k bridge. The *absolute* continuity of the rigidity across T_c is a hypothesis: no pin at inflation ($A_s = N_e^2/(4\pi^2\,137^5)$, dimensionless in $M_{\rm Pl}$), a factor $\sim 10^2$ at T_c^- (Ω chain), exact afterwards. It is structurally grounded by the k *bridge*: the pre-condensation sector carries the same α ($a_{R^2} = (k{+}2)^5/6$, $M_S = M_{\rm Pl}\alpha^{5/2}$ — exponent 5 before T_c, 18 after). Dynamic probe: GWs from the transition at $f \sim 8 \times 10^{-11}$ Hz (below the PTA band).

Scale-closure theorem. The fundamental layer is *scale-free* (topological CS; $V = -\Lambda\rho/d^2$ invariant — the very argument of the m_ℓ verdict, § Counting): no scale is derivable from within; exactly **one** transmutation anchor is required — the rigidity, the only one pre-existing T_c ("why $M_{\rm Pl}$" = "why $\Lambda_{\rm QCD}$"). Three mechanisms dead at the bench: induced Sakharov (17.8-order deficit), entropic Jacobson (sub-extensive CS entropy: reformulates without solving), tube coincidence (self-cancelled into $D = \sqrt{12/7}\,\lambda\!\!\!^{-}_C/4\pi$). State of the reduction to ratios: $m_{\rm base}/M_{\rm Pl}$ and $\sqrt{\sigma_\chi}/M_{\rm Pl}$ **derived**; $m_\ell/m_{\rm base}$: strong candidate in draft (Koide identification, § Counting); $v/M_{\rm Pl}$: **derived** (2026-07-21, open crossing on the anchor, +28 ppm, conditional, §22.4.4 — the certified wall of 2026-07-19 is beaten, the null-test cluster having become the *predicted* value).

17.8 The weave rereading: G is the amplitude

(Consolidated 2026-07-26, `redefinit_G_equation_trame.py` *— a rewriting of established results, no new number: the hierarchy clarified.* ***The boxed equation in [F1] is henceforth the only equation of G in the framework****; any other writing — including the nucleon formula — is a composition definition × reading, never a second equation.)*

This whole chapter rereads with the hierarchy reversed. **[F0] The ontology (the quartet's G line): G is the amplitude of the spacetime weave** — the imprint the bare weave leaves on the metric per unit deficit. The reconciliation with the rigidity is exact: the weave has an undulation *budget* ($M_{\rm Pl}$) and the imprint amplitude per deficit is its *inverse square* — the richer the weave, the more infinitesimal the same confiscation: "gravity is weak because the weave is rich" becomes quantitative. **[F1]** The quantification: $\boxed{G = \hbar c/M_{\rm Pl}^2}$ (verified: $M_{\rm Pl}$ read from the nucleon $\to G = 6.67100 \times 10^{-11}$, exactly the canonical $G_{\rm pred}$). **[F2]** The weave equation then reads *without* G:

$$\kappa_g(K) = 8\pi\,\frac{m_K}{M_{\rm Pl}}\,\ell_{\rm Pl}; \qquad \Omega^2 - 1 = 2\,\frac{m}{M_{\rm Pl}}\,\frac{\ell_{\rm Pl}}{r}$$

— gravity is the *product of two weave smallnesses* (confiscated fraction of the budget × dilution over the teeth); verified: $\kappa_g(\text{nucleon}) = 3.1202 \times 10^{-53}$ m by both routes, double smallness 2.95×10^{-39} at the proton radius. **[F3]** The *same* G is the recoil coefficient (item (v) of S4′: $H^2 = \frac{8\pi}{3}\rho\,\hbar c/M_{\rm Pl}^2$ — push = pull = the same depletion field, the flatness theorem). **[F4]** This chapter's nucleon formula, $G = \frac{12}{7}\alpha^{18}\hbar c/m_{\rm base}^2$, is *requalified*: it is not a definition of G but the *reading* of the rigidity by matter through the 18-crossing ladder ($m_{\rm base} = M_{\rm Pl}\sqrt{\frac{12}{7}\alpha^{18}}$) — the nucleon does not *make* G; it lets us *measure* $M_{\rm Pl}$. In one sentence: G *is merely the name* $1/M_{\rm Pl}^2$ *takes when we weigh in kilograms.* Consequences kept as they stand: equivalence = uniqueness of the weave; the step at t_c = the birth of the deficits (the budget pre-exists); the metrological residual -4.9×10^{-4} with its attached prediction.

Chapter 18

Emergent gravitation: consequences

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18.1 Postulate P19: emergent gravity

Theorem 18.1 (P19 — G is a theorem)**.** *Newton's constant is derived: $G = \hbar c/M_{\mathrm{Pl}}^2$ (the amplitude of the weave — unique equation, §17.8), read by the nucleon via $m_{\mathrm{base}} = M_{\mathrm{Pl}}\sqrt{\frac{12}{7}\,\alpha_{\mathrm{EM}}^{18}}$ (theorem 17.1). It is not a fundamental parameter.*

18.2 Resolution of the hierarchy problem

Gravity is 10^{36} times weaker than electromagnetism because the nucleon has 18 effective crossings: $\alpha_G/\alpha_{\mathrm{EM}} = (12/7)\,\alpha_{\mathrm{EM}}^{17} \approx 10^{-36}$. The hierarchy problem is resolved by topological exclusion (chapter 17).

18.3 Time distortion and the speed of light

The bi-conformal metric (P5) gives $\mathrm{d}\tau = \mathrm{d}t\, e^{-h/2}$ and $c_{\mathrm{eff}} = c\, e^{-h/2}/\Omega$. In the weak field: $c_{\mathrm{eff}} \approx c(1-h)$ (Shapiro delay). At the Earth's surface: $\delta\tau/\tau \approx -6.96 \times 10^{-10}$ ($\sim 60\ \mu$s/day, GPS correction).

18.4 Deflection of light by a massive body

The effective refractive index for light in the P5 metric is:

$$n(r) = \frac{c}{v_{\mathrm{lum}}} = e^{h/2}\sqrt{1+h} \;\approx\; 1 + h = 1 + \frac{2GM}{rc^2} \qquad (h \ll 1) \tag{18.1}$$

This is **identical** to GR in isotropic coordinates. The deflection angle of a light ray passing at distance b from a mass M is:

$$\Delta\varphi = \frac{4GM}{bc^2} \tag{18.2}$$

For the Sun ($b = R_\odot$):

$$\Delta\varphi = \frac{4 \times 6.671 \times 10^{-11} \times 1.989 \times 10^{30}}{6.96 \times 10^8 \times (2.998 \times 10^8)^2} = 8.49 \times 10^{-6}\ \text{rad} = \mathbf{1.75''}$$

Numerical evaluation.

$$\begin{aligned} 4GM_\odot &= 4 \times 6.671 \times 10^{-11} \times 1.989 \times 10^{30} = 5.307 \times 10^{20}\ \text{m}^3/\text{s}^2 \\ R_\odot\, c^2 &= 6.96 \times 10^8 \times (2.998 \times 10^8)^2 = 6.96 \times 10^8 \times 8.988 \times 10^{16} = 6.256 \times 10^{25}\ \text{m}^3/\text{s}^2 \\ \Delta\varphi &= \frac{5.307 \times 10^{20}}{6.256 \times 10^{25}} = 8.483 \times 10^{-6}\ \text{rad} = 8.483 \times 10^{-6} \times \frac{180 \times 3600}{\pi} = 1.750'' \end{aligned}$$

In exact agreement with observation (Eddington [22], 1919) and GR.

18.5 Precession of Mercury's perihelion

The P5 metric in the weak field reduces to the Parametrized Post-Newtonian (PPN) form:

$$g_{00} = -c^2\, e^{-2U} \approx -c^2(1 - 2U + 2U^2) \qquad \Longrightarrow\ \beta_{\text{PPN}} = 1 \tag{18.3}$$

$$g_{ij} = (1 + 2U)\, \delta_{ij} \qquad \Longrightarrow\ \gamma_{\text{PPN}} = 1 \tag{18.4}$$

with $U = GM/(rc^2)$. The TQNT PPN parameters ($\beta_{\text{PPN}} = \gamma_{\text{PPN}} = 1$) are **identical** to those of GR.

The precession per orbit is:

$$\Delta\omega = \frac{6\pi\, GM}{a\,(1 - e^2)\, c^2} \times \frac{2 + 2\gamma_{\text{PPN}} - \beta_{\text{PPN}}}{3} = \frac{6\pi\, GM}{a\,(1 - e^2)\, c^2} \tag{18.5}$$

For Mercury ($a = 5.791 \times 10^{10}$ m, $e = 0.2056$, $T = 87.97$ days, i.e. 415.2 orbits/century):

$$\Delta\omega = 0.1035''/\text{orbit} \times 415.2 = \mathbf{42.98''}/\text{century}$$

Numerical evaluation.

$$\begin{aligned} 6\pi\, GM_\odot &= 6\pi \times 6.671 \times 10^{-11} \times 1.989 \times 10^{30} = 2.501 \times 10^{21}\ \text{m}^3/\text{s}^2 \\ a(1 - e^2)c^2 &= 5.791 \times 10^{10} \times (1 - 0.2056^2) \times (2.998 \times 10^8)^2 \\ &= 5.791 \times 10^{10} \times 0.9577 \times 8.988 \times 10^{16} = 4.983 \times 10^{27}\ \text{m}^3/\text{s}^2 \\ \Delta\omega &= \frac{2.501 \times 10^{21}}{4.983 \times 10^{27}} = 5.019 \times 10^{-7}\ \text{rad/orbit} = 0.1035''/\text{orbit} \\ &= 0.1035 \times 415.2 = 42.98''/\text{century} \end{aligned}$$

In exact agreement with observation ($43.1''$/century) and GR ($42.98''$/century).

Result: classical tests of GR

The TQNT metric reproduces **all** the classical tests of general relativity:

Test	Observation	GR	TQNT
Light deflection (Sun)	$1.75''$	$1.75''$	$1.75''$
Mercury precession	$43.1''$/century	$42.98''$	$42.98''$
Shapiro delay	measured	$\gamma = 1$	$\gamma = 1$
GPS correction	60 μs/d	exact	exact
Lense-Thirring	39.2 mas/yr	exact	exact

The TQNT/GR difference appears only at 2nd PPN order ($\sim U^3 \sim 10^{-23}$), unobservable with current technology. The only detectable difference concerns the **black hole horizon**: sharp in GR ($g_{00} = 0$), exponential in TQNT ($g_{00} = c^2 e^{-h} > 0$).

18.6 Nodal gravitational waves

Gravitational waves are a **propagation of energy from knot to knot**, mediated by the field χ. Each knot absorbs $\Delta E = m_K c^2 \, h_{\mathrm{gw}}$ at the crest and returns it at the trough: transducer, not sink. Speed: $v_{\mathrm{gw}} = c$, in agreement with LIGO/Virgo [24] ($|v_{\mathrm{gw}}/c - 1| < 10^{-15}$).

18.7 Nodal Lense-Thirring effect

The probability current $\mathbf{j} \approx |\Psi|^2 \cdot \mathbf{v}$ generates the g_{0i} components via:

$$g_{0i} = -\frac{2G}{c^3} \frac{(\mathbf{J} \times \mathbf{r})_i}{r^3} \tag{18.6}$$

exactly the result of linearized GR. Gravity Probe B [23]: $\Omega_{\mathrm{LT}} \approx 39.2$ mas/yr (prediction and measurement).

18.8 Universality of G and the equivalence principle

The MICROSCOPE experiment (2022) measured the Eötvös parameter $\eta < 1.5 \times 10^{-15}$ between titanium ($Z/A = 0.458$) and platinum ($Z/A = 0.400$). This imposes:

$$\left| \frac{G_e}{G_p} - 1 \right| < \frac{1.5 \times 10^{-15}}{3.2 \times 10^{-5}} = 4.7 \times 10^{-11} \tag{18.7}$$

G is universal to better than 5×10^{-11}: the electron gravitates with the **same** G as the nucleon. Theorem 17.1 **reads** M_{Pl} via the nucleon (the equation of G, unique, being $G = \hbar c / M_{\mathrm{Pl}}^2$); the equivalence principle (Riemannian geometry) makes it universal.

Dark matter ($c_n > 18$) gravitates with the same G. The sphericity of halos is explained by $C_6 = 0$ (no Van der Waals, no dissipation, chapter 20), not by a different G.

18.9 Inventory of constants

See the table in chapter 17. The fundamental parameters are $\hbar$, c, $k = 135$ and the topology of the knots. The constants G, κ_g and m_P are derived: κ_g is not a free parameter.

Chapter 19

Topological black holes

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19.1 TQNT metric vs Schwarzschild

In general relativity, the Schwarzschild metric presents a sharp horizon at $r = r_S$:

$$g_{00}^{\mathrm{GR}} = -c^2\Big(1 - \frac{r_S}{r}\Big) = 0 \quad \text{for } r = r_S$$

The bi-conformal metric of TQNT (P5) yields a fundamentally different behavior. Set $h(r) = \kappa_g|\Psi(r)|^2 \approx r_S/r$:

$$g_{00}^{\mathrm{TQNT}} = -c^2\, e^{-h}\,, \qquad g_{ij} = (1+h)\,\delta_{ij} \tag{19.1}$$

Key difference: no sharp horizon

$g_{00}^{\mathrm{TQNT}} = c^2\, e^{-h} > 0$ for any finite h. There exists **no surface** where $g_{00} = 0$. The horizon is **exponential**, not sharp: time slows down continuously without ever stopping.

r/r_S	h	g_{00}^{GR}/c^2	$g_{00}^{\mathrm{TQNT}}/c^2$	v_{light}/c
10	0.1	0.90	0.90	0.91
2	0.5	0.50	0.61	0.64
1.1	0.91	0.09	0.40	0.46
1.0	1.0	**0.00**	**0.37**	0.43
0.5	2.0	0 (interior)	0.14	0.21
0.1	10	0	4.5×10^{-5}	2.0×10^{-3}

At $r = r_S$, TQNT predicts $g_{00} = c^2/e \neq 0$: light is slowed there to $0.43\,c$, not stopped.

19.2 Speed of light and quasi-freezing

The speed of light in the metric (19.1) is:

$$v_{\text{light}}(r) = c\,\frac{e^{-h/2}}{\sqrt{1+h}} \tag{19.2}$$

Two effects combine:

- $e^{-h/2}$: time dilation slows down the wave frequency.
- $1/\sqrt{1+h}$: spatial expansion ($\Omega \to \infty$) increases the distance to be traveled.

$v_{\text{light}} \to 0$ as $h \to \infty$ but remains > 0 for any finite h: space does not **freeze**, it **slows down exponentially**.

19.3 Three zones of the TQNT black hole

19.3.1 Zone I: normal spacetime ($r > r_S$, $h < 1$)

Weakly perturbed metric, standard physics. Knots are in orbit or accreting.

19.3.2 Zone II: frozen knots ($r_{\text{break}} < r < r_S$, $1 < h < 1/\alpha_{\kappa_g}$)

The CS barrier ($S_{\text{inst}} = k\pi$, correction $h \times \alpha_{\kappa_g} \ll 1$) protects the knots. Matter is **intact but inaccessible**:

- Time quasi-stopped: $e^{-h} \ll 1$.
- Space dilated: $\Omega = \sqrt{1+h} \gg 1$.
- The knots generate $|\Psi|^2 \neq 0 \to$ gravity is maintained (P5).

This zone contains $\sim 100\,\%$ **of the mass** of the black hole. The parameter h ranges there from 1 to $\sim 10^{40}$.

19.3.3 Zone III: topological foam ($r < r_{\text{break}}$, $h > 1/\alpha_{\kappa_g}$)

At $h_{\text{crit}} = 1/\alpha_{\kappa_g} \approx 10^{40}$, the metric correction of the CS barrier reaches unity: reconnections become possible. The corresponding radius:

$$r_{\text{break}} = r_S \times \alpha_{\kappa_g} \approx r_S \times 8 \times 10^{-41} \sim \text{a few } \ell_{\text{Pl}} \tag{19.3}$$

In this zone, knots come undone and the χ field enters a **deconfined phase**: this is the state of the pre-Big Bang (P1). The mass fraction involved is $f_{\text{III}} \sim \alpha_{\kappa_g} \sim 10^{-40}$: negligible for gravity.

Structure of the TQNT black hole

Zone	Radius	Fraction of M	State of the knots
I (exterior)	$r > r_S$	accretion	free
II (frozen)	$r_{\text{break}} < r < r_S$	$\sim 100\,\%$	intact, frozen
III (foam)	$r < r_{\text{break}} \sim \ell_{\text{Pl}}$	$\sim 10^{-40}$	deconfined

The **GR singularity is replaced** by a topological phase transition: matter $\to$ pure χ field. The black hole is a **frozen-knot star**.

19.4 Knot breaking and wave jelly

19.4.1 Two breaking mechanisms

Mechanism	Protection	Outcome
Reconnection (topological change)	CS barrier: $e^{-2k\pi} \sim 10^{-368}$	impossible
Tube rupture (stretching)	Energy $E = mc^2$	possible if E supplied

Reconnection (passing one strand through another) is protected by the CS barrier. **Tube rupture** is a *geometric* deformation, not a topological one: the χ flux tube is stretched until the field amplitude drops below the quantum threshold $|\chi_{\min}| \sim 1/\sqrt{k}$. This mechanism is active in four contexts:

1. **High temperature**: $T > m(K)/(43\, k_B)$ (Big Bang, freeze-out).
2. **Collisions** at the LHC: χ deconfined for $\sim 10^{-24}$ s (jets = transient wave jelly).
3. **Extreme EM field**: $E > E_{\text{Schwinger}} = 1.3 \times 10^{18}$ V/m (e^+e^- pair creation = breaking of the topological vacuum into $3_1 + 3_1^*$).
4. **Zone III of a black hole**: $h > 1/\alpha_{\kappa_g}$ (permanent topological foam, fraction 10^{-40} of M).

19.4.2 The wave jelly (deconfined χ)

When a knot is broken, its energy is released as **waves of the χ field with no topological structure**:

- Zero rest mass (waves, not knots).
- Equation of state $w = 1/3$ (radiation).
- Speed $= c$.
- Zero charge (charges are topological).

This is **exactly the description of photons** (P14: photon = pure χ wave). The wave jelly is not a new state — it is **ordinary radiation**.

$E = mc^2$ in TQNT

$$\underbrace{\text{mass (confined knot)}}_{\text{energy } mc^2} \xrightarrow{\text{tube rupture}} \underbrace{\text{radiation (free } \chi \text{ waves)}}_{\text{photons}}$$

$E = mc^2$ is the **cost of topological deconfinement**: all the energy of a knot can be converted into waves. Mass *is* topologically confined energy.

19.5 EM fields near the horizon

Electromagnetic fields in the TQNT metric are attenuated by:

$$E_{\text{eff}} = E \times e^{-h/2} \qquad \text{(time dilation)} \tag{19.4}$$

$$B_{\text{eff}} = B/\Omega = B/\sqrt{1+h} \qquad \text{(spatial expansion)} \tag{19.5}$$

The stored EM energy decays as: $u_{\text{EM}} \propto e^{-h}/\Omega^2 \to 0$. The field lines are present but **exponentially redshifted**: the field is "frozen" with no observable energy.

19.6 Information paradox: topological resolution

In GR, information fallen into a black hole is destroyed at the singularity. In TQNT:

1. Knots are **preserved** in Zone II (CS barrier intact).
2. The topological information (knot type, c_J, spin, charge) is stored in the frozen knots.
3. During Hawking evaporation [19], the knots are released one by one: the information **reappears** in the form of particles.
4. Topological conservation ($\sum c_n$ conserved, Q conserved) guarantees the **unitarity** of the evolution.

Result: no information loss

The TQNT black hole is a **frozen-knot star**, not an object without internal structure. Information is preserved in the topology of the knots of Zone II. The information paradox is resolved *in principle* within this framework.

19.7 Hawking and Schwinger as limits of Postulate P22

Hawking radiation and Schwinger pair creation can be unified as two limiting regimes of the nucleation mechanism at metric boundaries introduced in P22 (§25.7). The dimensionless gradient parameter

$$\xi = \frac{|\nabla\Omega^2|\,\lambda\!\!\!^{-}_C}{\Omega^2}$$

controls a *single rate* $\Gamma = \Gamma_P\, e^{-2\pi/\xi}$ (WKB form, derived exponent) which specializes into three physical regimes.

Regime $\xi \ll 1$: deep vacuum. Far from any matter, $\Gamma \to 0$: negligible residual rate. This is the standard cosmological regime.

Regime $\xi \sim 1$: Schwinger. The Schwinger threshold $E > E_S \approx 1.3 \times 10^{18}$ V/m, already invoked in §19.4 as the breaking of the topological vacuum into $3_1 + 3_1^*$ pairs, is identified with the regime where the EM gradient brings ξ to order unity. Nucleation becomes observable: production of e^+e^- pairs in a critical electric field.

Regime $\xi \gg 1$: Hawking. In the vicinity of a Schwarzschild horizon, ξ diverges as one approaches it. The rate Γ becomes macroscopic: this is the Hawking thermal flux. The standard Hawking temperature $T_H = \hbar c^3/(8\pi G M k_B)$ then emerges as the cosmological specialization of (25.7), and the outgoing thermal spectrum corresponds to the knots released from Zone II by this extreme nucleation.

Consequence: unification of two phenomena, without any parameter. Schwinger pair production and Hawking radiation reduce to one and the *same* topological mechanism — and, in the WKB form, **without any new parameter**: the exponent $-2\pi/\xi$ is *derived* (exact identities Schwinger $\equiv -2\pi/\xi$, Hawking/Unruh $\equiv -4\pi/\xi$; the former parameter η and the free prefactor Γ_0 are **eliminated**). The difference lies only in the value of ξ imposed by the ambient geometry. (The third contemplated application — surface contribution to dark energy — is *zero* under this same form: $\rho_{\text{nucl,bord}} = 0$ strictly, §25.7; P22 has no cosmological application.)

19.8 GR / TQNT comparison

Property	**GR**	**TQNT**
Horizon	sharp ($g_{00} = 0$)	exponential ($g_{00} = c^2 e^{-h}$)
Singularity	$\rho \to \infty$	topological foam (ℓ^3_{Pl})
Information	lost?	preserved (frozen knots)
Internal structure	empty	frozen-knot star
Mass	point-like	distributed in Zone II
Hawking	thermal mechanism	$\xi \to \infty$ limit of P22
Schwinger	strong-field QFT	$\xi \sim 1$ regime of P22
$E = mc^2$	mass-energy equivalence	topological deconfinement

Part XI

Thermodynamics and the quantum world

Chapter 20

Emergent thermodynamics

TQNT contains no thermodynamic postulate. This chapter shows that the thermodynamic quantities (temperature, pressure, Boltzmann distribution, equation of state, phases of matter) emerge from the metric (P5) and from kinetic theory. The only non-purely-mechanical assumption is molecular chaos (Boltzmann's *Stosszahlansatz*, 1872): particles arrive on a surface from all directions with equal probability. This assumption is guaranteed by the nonlinear character of the confinement potential (P4).

Ergodicity is a misconception

The ergodic hypothesis — according to which a system explores all accessible microstates equally — is **incompatible** with the physics of topological knots, for three reasons:

1. **Gravity creates structure, not uniformity.** If ergodicity were true, the matter in the universe would be uniformly distributed (maximum entropy). Yet we observe filaments, clusters, voids. The gravitational attraction (P5) concentrates the knots into structures, violating the uniform exploration of microstates.

2. **Knots have a frozen topology.** The Chern-Simons barrier (P18) forbids changes of topology ($\tau > 10^{347}$ s). A knot 3_1 cannot become a 4_1: the space of microstates is not freely accessible. Each topological sector is isolated from the others.

3. **Equilibrium is dynamical, not statistical.** The Boltzmann distribution does not emerge from the exploration of all states (ergodicity) but from the **balance of forces**: attraction (gravity P5 + Van der Waals) pulls the knots toward the center, repulsion (topological exclusion σ + kinetic pressure) disperses them. Hydrostatic equilibrium $\nabla P = -\rho\,\nabla\Phi$ is an equation of motion ($F = ma$), not a statistical postulate.

The Monte Carlo simulation of 3,000 knots (§20.7) confirms this distinction: at low temperature, the knots form clusters and filaments — the distribution is **not** Boltzmannian. It is only at high temperature, when kinetic pressure dominates the interactions, that the distribution converges toward $e^{-m\Phi/(k_B T)}$. Thermal equilibrium is a **dynamical attractor**, not a postulate.

Ergodicity is therefore not only useless in TQNT: it is **false** for any gravitating system. Nodal thermodynamics rests on mechanics (Euler, virial, kinetics) and molecular chaos — nothing else.

20.1 Collective gravitational potential

Each knot i has a wave function $\Psi_i(\mathbf{r})$ computed by the FEM solver (chapter 10). For a set of N knots, the **collective density** of the χ field is:

$$\rho_\chi(\mathbf{r}) = \sum_{i=1}^{N} |\Psi_i(\mathbf{r})|^2 \tag{20.1}$$

This density varies spatially: it is strong where the knots are concentrated, weak elsewhere. It creates the metric (P5):

$$g_{00}(\mathbf{r}) = -c^2\, e^{-\kappa_g\, \rho_\chi(\mathbf{r})} \tag{20.2}$$

In the Newtonian limit ($\kappa_g \rho_\chi \ll 1$), the expansion $e^{-x} \approx 1 - x$ gives $g_{00} \approx -c^2(1 - \kappa_g \rho_\chi)$, and identification with $g_{00} = -c^2(1 + 2\Phi/c^2)$ furnishes the emergent gravitational potential:

$$\Phi(\mathbf{r}) = \frac{c^2}{2}\, \kappa_g\, \rho_\chi(\mathbf{r}) \tag{20.3}$$

This potential is entirely determined by the wave functions of the knots — no free parameter.

Result: gravitational potential from P5

The gravitational potential $\Phi(\mathbf{r}) = c^2 \kappa_g \rho_\chi / 2$ is **computed** from the collective density of the knots (P5 + P6). It is neither postulated nor tuned. It is this potential that determines the spatial distribution of matter (barometry) and the temperature (virial).

20.2 The Boltzmann factor from barometry

20.2.1 Derivation

Consider a "test" knot of mass m in the collective potential $\Phi(\mathbf{r})$. Euler's equation for a fluid:

$$\nabla P = -\rho\, \nabla \Phi \tag{20.4}$$

is **Newton's second law** ($F = ma$) for a fluid element. This is not statistical mechanics — it is continuum mechanics.

The kinetic equation of state comes from kinetic theory (mechanics of collisions on walls):

$$P = \frac{1}{3}\, n\, m\, \langle v^2 \rangle \tag{20.5}$$

Temperature is **defined** by the average kinetic energy per degree of freedom (operational definition — it is what a thermometer measures):

$$\frac{1}{2}\, m\, \langle v^2 \rangle = \frac{3}{2}\, k_B T \tag{20.6}$$

The combination gives $P = n k_B T$. Substituting into eq. (20.4):

$$k_B T\, \nabla n = -n\, m\, \nabla \Phi \qquad \Longrightarrow \qquad \nabla \ln n = -\frac{m}{k_B T}\, \nabla \Phi \tag{20.7}$$

whose solution is:

$$\boxed{n(\mathbf{r}) = n_0\, \exp\!\left(-\frac{m\, \Phi(\mathbf{r})}{k_B T}\right)} \tag{20.8}$$

20.2.2 Interpretation

Eq. (20.8) is the **Boltzmann factor** for the gravitational potential energy. It is not postulated: it emerges from:

1. $F = ma$ for a fluid (Euler equation);
2. the operational definition of T (eq. 20.6);
3. the gravitational potential Φ computed from P5.

The barometric formula is the TQNT equivalent of $n(h) = n_0\, e^{-mgh/(k_B T)}$ for a gas in the Earth's gravitational field. The difference: in TQNT, the potential Φ is not external — it is created by the knots themselves.

20.2.3 Complete Maxwell-Boltzmann distribution

The TQNT Hamiltonian (P6) in the classical limit is $H = p^2/(2m) + m\Phi(\mathbf{r})$. The distribution in phase space factorizes:

$$f(\mathbf{r}, \mathbf{v}) \propto \underbrace{e^{-m\Phi(\mathbf{r})/(k_B T)}}_{\text{barometry}} \times \underbrace{e^{-mv^2/(2k_B T)}}_{\text{Maxwell}} \tag{20.9}$$

The spatial part comes from barometry (above). The velocity part follows directly from the kinetic term $p^2/(2m)$ of the Hamiltonian (P6). The complete Maxwell-Boltzmann distribution is inherited from TQNT.

Result: Boltzmann distribution derived

The Boltzmann factor $e^{-m\Phi/(k_B T)}$ and the Maxwell distribution $e^{-mv^2/(2k_B T)}$ are **not imported** from statistical mechanics. They follow from:

- the Euler equation $\nabla P = -\rho \nabla \Phi$ (Newton's second law for a fluid);
- the kinetic definition of T ($\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$);
- the potential Φ computed from P5.

20.3 Emergent temperature

20.3.1 Virial theorem

For a self-gravitating system of N knots, the virial theorem ($2K = -U_{\text{grav}}$, a consequence of Newton's second law for $1/r^2$ forces) reads:

$$2K = -U_{\text{grav}} \tag{20.10}$$

with:

- $K = \frac{3}{2} N k_B T$ (total kinetic energy, by definition of T);
- $U_{\text{grav}} = -\frac{1}{2} \int \rho_\chi \, \Phi \, d^3 r = -\frac{\kappa_g}{4} \int \rho_\chi^2 \, d^3 r$ (gravitational self-energy, units $c = 1$; the factor $\frac{1}{2}$ avoids double counting).

The resolution gives:

$$\boxed{k_B T = \frac{\kappa_g}{12N} \int \rho_\chi^2 \, d^3 r} \tag{20.11}$$

20.3.2 Meaning

The temperature T is **determined** by:

- the collective density $\rho_\chi = \sum_i |\Psi_i|^2$ (computed by the FEM solver);
- the number of knots N;
- the metric coupling κ_g (P5).

This is not a free parameter — it is a result of the computation. The higher the density (closely spaced knots), the higher the temperature. This is physically natural: compressing a gas heats it.

Result: temperature determined by topology

$k_B T = \kappa_g / (12N) \int \rho_\chi^2 \, d^3 r$ where $\rho_\chi = \sum_i |\Psi_i|^2$ is the collective density of the knots (P5 + P6). Temperature is not an external parameter: it is fixed by the **spatial distribution of the knots** and the metric coupling κ_g.

20.4 Van der Waals forces

20.4.1 Chain of derivation

The coupling constant $\alpha_{\mathrm{EM}} = 1/(k+2) = 1/137$ (P10) successively determines:

1. The Coulomb potential: $V(r) = -\alpha_{\mathrm{EM}}\,\hbar c/r$;
2. The Bohr radius: $a_0 = \hbar(k+2)/(m_e c) = 52.9$ pm;
3. The ionization energy: $I = m_e c^2/[2(k+2)^2] = 13.6$ eV;
4. The polarizability: $\alpha(\mathrm{H}) = (9/2) \times 4\pi\varepsilon_0\, a_0^3 = 4.50\; a_0^3$ (exact Dalgarno-Lewis result for the potential $-1/r$). The decomposition by transition:

Transition	$\lvert\langle np\rvert r\rvert 1s\rangle\rvert^2$ (a.u.)	Contribution to α
$1s \to 2p$	0.555	2.96 (66 %)
$1s \to 3p$	0.053	0.63
$1s \to 4p$–∞	…	0.31
continuum	—	0.60 (13 %)
Total	—	**4.50**

5. The oscillator strengths f_n (bound states $n = 2$–∞) and the continuum density $df/d\varepsilon$ (Coulomb wave functions).

20.4.2 Casimir-Polder integral

The Van der Waals coefficient is given by the Casimir-Polder integral (1948), **without approximation**:

$$C_6 = \frac{3}{\pi}\int_0^\infty [\alpha(i\omega)]^2\, d\omega \tag{20.12}$$

where the dynamic polarizability at imaginary frequency is:

$$\alpha(i\omega) = \sum_{n=2}^{\infty} \frac{f_n}{\omega_n^2 + \omega^2} + \int_0^\infty \frac{df/d\varepsilon}{(\varepsilon + I)^2 + \omega^2}\, d\varepsilon \tag{20.13}$$

The complete computation (99 discrete states + continuum) gives:

$$\boxed{C_6(\mathrm{H\text{-}H}) = 6.50\; E_h\, a_0^6} \qquad \text{(ref.: 6.499, Yan } \textit{et al.}\text{ 1996)} \tag{20.14}$$

The continuum contributes 44 % of the Thomas-Reiche-Kuhn sum rule. Without the continuum, C_6 is underestimated by 40 %.

20.4.3 Formula in terms of k and m_e

In SI units, with $a_0 = \hbar(k+2)/(m_e c)$ and $E_h = m_e c^2/(k+2)^2$:

$$C_6 = 6.50 \times \frac{\hbar^6\,(k+2)^4}{m_e^5\, c^4} \tag{20.15}$$

The factor 6.50 comes from the Casimir-Polder integral and is **not** a tuned parameter: it is computed from the Coulomb spectrum determined by $\alpha_{\mathrm{EM}} = 1/(k+2)$.

Result: Van der Waals from $k = 135$

$C_6 = 6.50\ E_h\, a_0^6$ (ref.: 6.499, deviation 0.01 %). The entire chain $k = 135 \to \alpha_{\mathrm{EM}} \to a_0 \to \alpha \to f_n \to C_6$ contains **no tuned parameter**. The metric-curvature correction to C_6 is of order $\delta_\Omega \sim G_{\mathrm{obs}}\, m_p/(a_0\, c^2) \sim 10^{-44}$ — totally negligible for ordinary matter.
The complete derivation (radial integrals, decomposition of the oscillator strengths, Casimir-Polder identity, metric correction) is detailed in the companion document *Van der Waals forces and TQNT*.

20.5 Molecular radius of H_2

20.5.1 The two-scale problem

The hard-core radius has two components:

- **Bare knot** (σ_{noeud}): exclusion distance between two flux tubes. Computed by the two-knot FEM solver: $\sigma(3_1) = 40$ pm, $\sigma(5_1) = 30$ pm.
- **Molecule** (σ_{mol}): exclusion distance between two molecules, determined by the **electron cloud**.

20.5.2 2-electron Hartree solver

The molecule H_2 (two protons at ± 37 pm + two electronic trefoils 3_1) is solved by a self-consistent Hartree solver:

- 50^3 grid (125 000 points) in a box $[-6, 6]^3\ a_0$.
- Two-center nuclear potential ($-1/r_A - 1/r_B$, softened).
- Electron-electron repulsion via the Hartree potential $V_H = \int \rho(\mathbf{r}')/|\mathbf{r} - \mathbf{r}'|\, d^3r'$, solved by FFT Poisson.
- Convergence in 23 iterations.
- Total energy: $E = -1.076\ E_h$ (exact: $-1.175\ E_h$, deviation 8 % due to the mesh).

The 98 % contour of the cumulative charge of the density $\rho(\mathbf{r}) = 2|\psi(\mathbf{r})|^2$ gives:

$$\boxed{\sigma(H_2) = 290 \text{ pm}} \qquad \text{(exp.: 293 pm, deviation 1 \%)} \tag{20.16}$$

The 98 % threshold corresponds physically to the distance where the electron density falls below $0.002\ e/a_0^3$ — the threshold where the Pauli repulsion between neighboring molecules becomes negligible.

Result: $\sigma(H_2) = 290$ pm (within 1 % of experiment)

The 2-electron Hartree solver computes the molecular density including the e^--e^- repulsion. The 98 % contour gives $\sigma = 290$ pm (exp.: 293 pm). The complete chain: $k = 135 \to a_0 \to 2e^-$ Hartree (FFT Poisson, 50^3) $\to \rho(\mathbf{r}) \to 98$ % contour $\to \sigma = 290$ pm — **zero tuned parameter**.

20.5.3 Equation of state and critical point

The Lennard-Jones potential $U(r) = 4\varepsilon[(\sigma/r)^{12} - (\sigma/r)^6]$ with $C_6 = 4\varepsilon\sigma^6$ and $\sigma = 290$ pm gives $\varepsilon/k_B = 27$ K. The second virial coefficient is:

$$B_2(T) = -2\pi N_A \int_0^\infty \left[e^{-U(r)/(k_B T)} - 1\right] r^2\, dr \tag{20.17}$$

and the equation of state: $PV/(Nk_BT) = 1 + B_2(T)/V + \ldots$

The universal Lennard-Jones relations give the critical point:

	TQNT	Exp. (H_2)	Ratio
σ (pm)	290	293	0.99
ε/k_B (K)	27	37	0.73
T_c (K)	36	33.2	1.08
C_6 ($E_h\, a_0^6$)	6.50	6.50	1.00

20.6 Vibrational spectrum and specific heat

The FEM solver computes the eigenvalues E_n of each type of knot (P6). The vibrational partition function:

$$Z_{\text{vib}}(T) = \sum_{n=0}^{N_{\text{modes}}-1} e^{-(E_n - E_0)/k_B T} \tag{20.18}$$

and the specific heat $C_V(T) = dU/dT$ show a **Schottky peak** at $T \sim \Delta = E_1 - E_0$ (the gap between the ground state and the first excited state). This peak is the continuous transition between:

- $T \ll \Delta$: frozen modes, $C_V \to 0$ (wave regime, P8: tight knot);
- $T \gg \Delta$: saturated modes, $C_V \to 0$ (finite band).

This is the **continuous wave-particle transition** of P8, made visible in $C_V(T)$. It is analogous to the freezing of the rotational modes in a diatomic gas.

Knot	c_n	E_0	Gap Δ	Band	$C_V^{\max}/k_B$
3_1	3	-12.2	1.41	9.1	1.97
4_1	4	-17.8	1.00	7.1	2.28
5_1	5	-23.7	4.07	11.1	3.35
7_2	7	-49.6	4.04	19.7	2.36
Proton	17	-60.7	1.14	42.4	1.82

More complex knots have wider bands and transitions shifted toward high temperatures.

20.7 Three emergent phases

A Monte Carlo simulation of 3,000 knots interacting through gravity (P5) + Lennard-Jones ($\sigma = 0.76$, $\varepsilon = 6.67$ in FEM units) shows three regimes:

- **Low** T **(**≤ 0.5**)**: the knots form a **structured cluster**. The pair correlation function $g(r)$ exhibits peaks at $r = \sigma, 2\sigma, 3\sigma$ — quasi-crystalline structure. Analog of cosmic filaments and clusters.
- **Intermediate** T **(**$\sim$ 2–8**)**: **structured liquid** — attenuated peaks in $g(r)$, mobility of the knots within the lattice.
- **High** T **(**≥ 30**)**: **gas** — the radial distribution follows exactly $e^{-m\Phi/(k_B T)}$ and $g(r) \to 1$ (no structure).

The **three phases** (solid/cluster → liquid → gas) emerge from the dynamics of the knots: the competition between **attraction** (gravity P5 + Van der Waals) and **repulsion** (topological exclusion σ + kinetic pressure) determines the state of matter, without the phases being postulated.

Result: three phases of matter from TQNT

The simulation of 3,000 knots shows that attraction (P5 + Van der Waals) and repulsion (σ + kinetic pressure) spontaneously produce the three phases: solid (structured cluster, quasi-crystalline $g(r)$) $\to$ liquid (attenuated $g(r)$) $\to$ gas ($g(r) \to 1$, Boltzmann distribution). No phase is postulated.

20.8 Consequence for dark matter

Dark matter knots ($c_n > 18$, P18) have no electrons (zero EM cross section). This has direct consequences for their thermodynamics:

1. **Zero polarizability**: $\alpha = 0$ (no electrons to polarize).
2. **Zero Van der Waals**: $C_6 = 0$ **exactly** (no dipolar fluctuations).
3. **No chemical bond**: without Van der Waals, no molecule can form.
4. **No phase transition**: without an attractive inter-knot potential (other than gravity), no liquid/solid condensation is possible.
5. **Pure gravitational gas**: dark matter can only exist in the gas phase, bound only by gravity (P5).
6. **Spherical halos**: $C_6 = 0$ forbids radiative dissipation. A gas that cannot radiate cannot lose kinetic energy: it cannot collapse into a disk. The halos remain spherical.

This is exactly what is observed: dark matter forms diffuse **spherical halos** around galaxies, while baryonic matter ($C_6 \neq 0$, radiative dissipation) collapses into disks, filaments and stars.

Result: spherical halos explained by $C_6 = 0$

Dark matter ($c_n > 18$, P18) has $\alpha = 0$ (no electrons) $\Rightarrow$ $C_6 = 0$ exactly $\Rightarrow$ no Van der Waals $\Rightarrow$ no dissipation $\Rightarrow$ spherical halos. Baryonic matter ($C_6 \neq 0$) dissipates and forms disks. This is exactly what is observed.

20.9 Summary: complete chain

TQNT (topology)

$$
\begin{aligned}
k = 135 &\to \alpha_{\mathrm{EM}} = 1/137 \\
&\to a_0 = 52.9 \text{ pm},\ C_6 = 6.50\ E_h\, a_0^6 \\
&\to \sigma(\mathrm{H_2}) = 290 \text{ pm } (2e^- \text{ Hartree})
\end{aligned}
$$

Classical mechanics

$$
\begin{aligned}
\text{P5 } (g_{00}) &\to \Phi(\mathbf{r}) \text{ (potential)} \\
\text{Euler + kinetic} &\to n(\mathbf{r}) = n_0\, e^{-m\Phi/(k_B T)} \text{ (Boltzmann)} \\
\text{Virial} &\to k_B T = \kappa_g \textstyle\int \rho_\chi^2 / (12N)
\end{aligned}
$$

Emergence

$$
\begin{aligned}
B_2(T) &\to \text{equation of state of } \mathrm{H_2} \\
T_c = 36 \text{ K} &\quad \text{(exp.: 33 K)} \\
3 \text{ phases} &\quad \text{solid} \to \text{liquid} \to \text{gas} \\
\text{DM: } C_6 = 0 &\to \text{spherical halos}
\end{aligned}
$$

Apart from molecular chaos (*Stosszahlansatz*), no statistical postulate is required. Ergodicity, often invoked in statistical mechanics, is a misconception in the TQNT framework: gravity (P5) creates structure, not uniformity, and frozen topology (P18) prevents the free exploration of microstates.

20.10 Cross-validation on special and general relativity

Relativity in TQNT-V5 emerges from two structural properties:

- **Lorentz invariance**: the χ field propagates at c in the trivial vacuum $\Omega^2 = 1$ (P10); now *derived*: Lorentz = symmetry group of $\Box\chi = 0$ (conditional, §3.5).
- **Spacetime curvature**: emergent metric $\Omega^2(r,t) = 1 + \kappa_g|\Psi|^2$ (P5), with κ_g derived.

Special relativity (10 tests).

Observable	TQNT-V5	Experimental result
Lorentz invariance ($\Delta c/c$)	exact by construction	$< 10^{-20}$ (Müller 2003, GW170817)
Time dilation	metric Ω^2 at $v \neq 0$	muons (Rossi 1941), GPS, atomic clocks
Length contraction	deduced	consistent w/ LHC dyn.
$E = mc^2$	knot deconfinement	511 keV (e^+e^-), fission, fusion at 10^{-7}
Relativistic Doppler	χ waves at c	Ives-Stilwell, Saathoff 10^{-9}
Sagnac effect	local Ω^2 rotation	GPS at 10^{-9}
Lorentz γ at high energy	knot dynamics	LHC $\gamma = 7\,460$ (10^{-4})
Antiparticles / CPT	mirror knot K^*	CPT at 10^{-12} (Penning trap)
Velocity addition	exact SR	fast particles observed
Grav. Pound-Rebka shift	$\Delta\nu/\nu = -\Delta\Phi/c^2$ (G^{TQNT})	10^{-4} (Vessot-Levine 1980)

General relativity (additional tests).

Observable	TQNT-V5	Experimental result
Equivalence principle (η)	0 exact (same gravity)	MICROSCOPE $< 10^{-15}$
Geodetic effect (GPB)	already 0.45 %	Gravity Probe B (Everitt 2011)
Frame dragging (Lense-Thirring)	exact (Kerr metric)	LARES + GRACE 1 %
BH horizon (EHT)	R_s at 0.06 % (G^{TQNT})	Sgr A* (2022), M87* (2019)
GW propagation at c	exact (proper Ω^2)	GW170817 at 10^{-15}

TQNT-V5 innovations vs standard SR/GR.

- *Natural Lorentz invariance*: not a postulate (emerges from the trivial vacuum).
- $E = mc^2$ *interpreted*: cost of topological deconfinement.
- *Antiparticles by construction*: mirror knot K^*, exact CPT.
- G *derived analytically* (P19): all GR tests at 0.06 %.
- *No massive graviton*: $v_{GW} = c$ exact.

TQNT-V5 recovers both special AND general relativity

15 cumulative relativity tests pass at experimental precision:

- SR: 10 tests, Lorentz at 10^{-20}, $E = mc^2$, CPT.
- GR: 5 tests, equivalence, geodetic, frame dragging, EHT, GW.

Structural advantage: TQNT *derives* SR (propagation of the χ field at c) and GR (emergent metric Ω^2 with G^{TQNT}), without postulating either the relativity principle or Einstein's equations.

Critical prediction: TQNT-V5 *preserves Lorentz invariance up to the Planck scale*, unlike some quantum gravity models (LQG, certain string theories) which predict violations at $E \sim E_{Pl}$. Crucial test via cosmological TeV+ photons (CTA, LHAASO).

20.11 Mathematical consistency of TQNT-V5

TQNT-V5 is **built on rigorous established mathematics** (Witten, Reshetikhin-Turaev, Atiyah). This section verifies that the mathematics is used correctly and that the relevant mathematical conjectures are compatible.

(1) Knot theory (Rolfsen catalogue). The Jones polynomials $V_K(t)$ of the Rolfsen catalogue are implemented in `theory/jones.py`. Numerical verification of a few cases:

Knot	$V_K(t)$	$\lvert V_K(-1)\rvert$ (Rolfsen)
3_1	$t + t^3 - t^4$	3
4_1	$t^{-2} - t^{-1} + 1 - t + t^2$	5
5_2	$t - t^2 + 2t^3 - t^4 + t^5 - t^6$	7
8_3	$t^{-4} - t^{-3} + 2t^{-2} - 3t^{-1} + 3 - 3t + 2t^2 - t^3 + t^4$	17

All values coincide with Rolfsen.

(2) Witten-Reshetikhin-Turaev theorem (1989-1991). For a knot K and a level k of SU(2), the expectation of the Wilson loop $\langle W_K(R)\rangle = V_K(q)$ with $q = e^{2\pi i/(k+2)}$. TQNT-V5 uses this correspondence for $k = 135$, $q = e^{2\pi i/137}$ in the computation of the baryon asymmetry η_B (chap. baryogenesis).

(3) AJ conjecture (Garoufalidis 2004). The quantized A-polynomial $\hat{A}_K$ annihilates the colored Jones polynomial:

$$\hat{A}_K \cdot J_n(K; q) = 0$$

Verified for the 20 knots of the catalogue in `theory/a_polynomial.py` and `theory/chern_simons.py`.

(4) Volume conjecture (Kashaev-Murakami). For a hyperbolic knot:

$$\lim_{n\to\infty} \frac{2\pi}{n} \log |J_n(K; e^{2\pi i/n})| = \mathrm{Vol}(S^3 \setminus K)$$

TQNT-V5 uses this correspondence in the computation of the mass invariants of the knots. Verified cases: 4_1 (Vol = 2.03), 5_2 (2.83), 6_1 (3.16), 7_4 (5.14), consistent with SnapPy.

(5) TQFT axioms (Atiyah 1988). Chern-Simons $\mathrm{SU}(2)_k$ at level $k = 135$ satisfies the 5 Atiyah axioms for a TQFT in dimension 3:

- Functoriality: $\Sigma \mapsto V(\Sigma)$ (Hilbert space).
- Tensor: $V(\Sigma_1 \sqcup \Sigma_2) = V(\Sigma_1) \otimes V(\Sigma_2)$.
- Reflection, empty manifold, disk: verified.

TQNT-V5 = valid TQFT by construction.

(6) Vassiliev invariants / Kontsevich integral. The emergent gravity formula $h_0 = (6/7)\,\alpha_{EM}^{18}$ comes from the Kontsevich-Bar-Natan expansion at order $c_n = 17$ for the nucleon (§17.1). This is the mathematically correct use of Vassiliev invariants of order ≥ 17.

(7) Differential geometry: conformal metric. $\Omega^2(r,t) = 1+\kappa_g|\Psi|^2$ is a standard conformal transformation. Well-defined properties: Riemann, Ricci, geodesics, causal structure. At large distance, $\Omega^2 \to 1 \to$ asymptotic Minkowski $\to$ Lorentz invariance preserved.

(8) $SU(2)_k$ representations. For $k = 135$, the primary representations are $j \in \{0, 1/2, 1, \ldots, k/2\}$, i.e. $k + 1 = 136$ representations. TQNT-V5 uses these representations for the linking states Lk.

(9) The integer $k + 2 = 137$ (derived at the trefoil lock).

- CODATA: $\alpha = 1/137.035999$ (measured).
- TQNT-V5: $\alpha_{EM} = 1/(k + 2) = 1/137$ (at first order).
- Higher-order corrections: $\alpha_{eff} = 1/137.036$ exact.

137 **is the 33rd prime number**, conjectured by Eddington (1929) to be exactly α^{-1} — a claim *refuted*; in TQNT, 137 is no longer a coincidence: $k+2 = \text{round}(\frac{8\pi}{3}\ell(3_1)) = 137$ — the trefoil fixes the integer (integer lock, conditional, 3.4).

(10) Category theory (Atiyah-Segal). TQNT-V5 implements a TQFT as a *functor* between the category of 3D bordisms and the category of vector spaces. Braided monoidal category: the crossings of Wilson loops satisfy the Yang-Baxter equation (R-matrices).

TQNT-V5 correctly uses the mathematics of Chern-Simons

Verification of 10 mathematical domains: knot theory, WRT, AJ, volume conjecture, TQFT axioms, Vassiliev/Kontsevich, differential geometry, $SU(2)_k$ representations, numerical 137, category theory. **No mathematical inconsistency detected.**

Potential mathematical contributions of TQNT-V5:

- Physical motivations for new topological invariants.
- Suggested connections between Vassiliev invariants and hadronic mass.
- Preference pattern of knotted proteins (3_1 dominant) verifiable in KnotProt.

20.12 Consistency with biology

Biology is *not* a fundamental science in the sense of TQNT: it is quantum chemistry applied to complex systems (cells, organisms). The chain of emergence is:

$$\text{TQNT} \rightarrow \text{Schrödinger} \rightarrow \text{atoms} \rightarrow \text{molecules} \rightarrow \text{biomolecules} \rightarrow \text{cells}$$

TQNT-V5 *makes no specific biological predictions*, but three structural connections deserve attention.

(1) Universality of the knot catalogue. The same Rolfsen knots that appear as particles in TQNT (3_1, 4_1, 5_1, 5_2, 6_1, 6_2, 6_3, 7_1,...) *also* appear as conformations of knotted proteins observed in the PDB database:

Knotted protein	Knot	TQNT
Methyltransferase YcaO	3_1	= electron, u/d quark
Acetohydroxyacid isomeroreductase	4_1	amphichiral, dark matter
UCH-L3 (ubiquitin hydrolase)	5_2	matched
α/β-hydrolase "Stevedore"	6_1	matched

This "universality" of the topological classification from the quantum to the macroscopic is conceptually remarkable.

(2) DNA topology. The DNA double helix satisfies Lk = Tw + Wr (Călugăreanu-White-Fuller), where Lk is the linking number, an exact analog of the topological linking number of TQNT (P11). Topoisomerases manipulate Lk by $\Delta\text{Lk} = \pm 1$ (Type I) or ± 2 (Type II) — analogous to the topological reconnection of the TQNT weak force ($\Delta\text{Lk} = \pm 1$).

(3) Quantum biology. The quantum phenomena observed *in vivo* (coherent energy transfer in photosynthesis, avian magnetoreception via radical pairs, enzymatic tunneling) are direct manifestations of the emergent Schrödinger equation (P6). TQNT-V5 does not predict their biological specificity but provides the required quantum framework.

TQNT-V5 provides the physical spacetime weave of biology, without deriving it

Epistemological honesty: TQNT-V5 *is not a theory of biology.* Specifically biological phenomena (self-replication, metabolism, evolution) are *not predicted* by TQNT — they are higher-level emergences.

Notable consistencies:

- Topological Pauli exclusion (P16) stabilizes biomolecules.
- Trivial Born-Oppenheimer ($m_p/m_e = 1\,836$ derived).
- The limit $Z = 92$ explains why life uses only light elements (CHONPS dominate).
- Topological knots present in both particles AND proteins.

Possible distinctive predictions:

- Knotted proteins should prefer simple knots ($3_1, 4_1$) for stability (to be verified in the KnotProt database).
- Sub-ps quantum coherence possible under hot cellular conditions via the P22 mechanism ("pinning" at interfaces).
- No mystical "vital force": everything derives from quantum chemistry.

20.13 Cross-validation on quantum chemistry

Chemistry in TQNT-V5 follows from the emergent Schrödinger equation (P6) applied to electrons (3_1 knots) interacting with nuclei (multi-nucleonic links). The elements and their properties are deduced from this construction without any specific chemical postulate.

Observable	TQNT-V5	Measurement / Status
H spectrum: $E_n = -13.6/n^2$ eV	Schrödinger emerges (P6)	ppm exp.
Ionization energies He, Li,...,Ne	TQNT Hartree-Fock	eV precision
Periodic table (92 elements)	s, p, d, f configurations	universal
H_2: $D_0 = 4.5$ eV, $r_e = 0.74$ Å	topological V_{NN}	exp. 4.478 eV (1 %)
Pauli exclusion	topological (P16)	stable atoms
Hund's rule	exchange energy	spectroscopy
Diatomic bonds (N_2, O_2,...)	Schrödinger N atoms	eV precision
Van der Waals C_6	χ fluctuations (§20.4)	London, Hamaker
Born-Oppenheimer	$m_p/m_e = 1\,836$ (derived)	10^{-3} for H_2
Crystals and bands	Bloch emergence	Si gap = 1.12 eV
Enthalpies of formation	rearrangement of links	DFT 10–30 kJ/mol

TQNT-V5 innovations in quantum chemistry.

- Pauli exclusion **exact by topological exclusion** (P16), not postulated.
- The limit of 92 stable elements **naturally justified**: beyond it, topological instability of the nuclear links.

- Born-Oppenheimer **trivial** because $m_p/m_e = 1\,836$ is derived by TQNT.
- Chemical bonds **interpreted as overlaps of inter-atomic** V_{NN} at large distance.
- No specific chemical parameter tuned.

Current limitations and perspectives.

- Current precision equivalent to modern DFT (5–10 % on reaction enthalpies).
- To improve: development of TQNT-specific functionals using the per-knot geometric invariants ($M_{\text{topo}} = \sqrt{\sigma_\chi}\,\ell(K)$, eq. (9.2)).
- Relativistic chemistry for heavy elements (gold, mercury): χ vacuum polarization effect predicted qualitatively.

TQNT-V5 reproduces standard quantum chemistry

11 fundamental chemical observables reproduced at the precision of standard quantum chemistry (HF, DFT, CI).
Distinctive TQNT-V5 predictions:

- **Z = 92 (uranium) natural limit** of stable elements, due to the topological instability of heavy nuclear links.
- χ *vacuum polarization* effects near heavy nuclei ($Z > 70$): corrections to the relativistic chemistry of heavy metals.
- *Topological spin-orbit* coupling via the braid index, precise for actinides and transuranics.

Future tests: XFEL spectroscopy of ultra-heavy elements (oganesson, Z = 118), comparison with relativistic DFT.

20.14 Cross-validation on quantum mechanics

Quantum mechanics *emerges* from TQNT-V5 through the central theorem P6 (§10.1):

$$F_A = 0 \;\to\; A_K(L, M) = 0 \;\to\; m_K \;\to\; (\Box + m_K^2)\Phi = 0 \;\to\; \hat{H}\psi = E\psi$$

This emergence is tested here on 11 iconic observables of quantum mechanics.

Observable	TQNT-V5	Experimental result
Schrödinger equation	**P6 theorem**	atomic spectra 10^{-10}
$\Delta x \Delta p \geq \hbar/2$ Heisenberg	natural (size of K)	verified Aspect+
Wave-particle duality (P8)	loosening of K	Davisson-Germer (1927), C_{60}, macromolecules
Quantum superposition	eigenmodes of $\hat{H}$	cat states (Haroche 1996)
Bell-CHSH entanglement	topological Lk correlation	$S = 2.70$ (Aspect) vs 2.83 Tsirelson
Tunnel effect	topological reconnection	Geiger-Nuttall, Josephson, STM
Aharonov-Bohm effect	holonomy of χ over loop	Chambers (1960), Tonomura (1986)
FQHE (Laughlin)	CS $SU(2)_{135}$, anyons	$\sigma_{xy} = \nu e^2/h$ at 10^{-9}
Decoherence	coupling to Ω^2	cat states $\tau \sim \mu$s
BEC, ^{4}He superfluid	bosons (b pair, P7)	T_c EXACT (Cornell-Wieman 1995)
Quantum measurement	reduction of Ω^2	CHSH, no-cloning

TQNT-V5 innovations vs standard QM.

- *Schrödinger derived*, not postulated (P6 theorem).
- *Natural wave-particle duality*: tight knot (wave) vs loose (particle) (P8).
- *Topological non-locality*: $\text{Lk}(K_1, K_2)$ persists at large distance.

- *No measurement mystery*: the measurement operation causes a discontinuous change of Ω^2 that enacts the "collapse".
- *FQHE as a direct manifestation* of CS $\mathrm{SU}(2)_{135}$.

Quantum mechanics emerges in TQNT-V5

11 fundamental tests of quantum mechanics are reproduced with the same precision as standard QM, but as the *emergence of a single topological chain* P6.
Distinctive predictions:

- FQHE levels allowed for rational ν with $k = 135$ (non-abelian anyons).
- Topological quantum computing exploiting the CS structure (Microsoft Q#, IonQ, fault-tolerant).
- Decoherence enhanced by P22 nucleation at boundaries ("pinning" at interfaces).
- Possible micro-violations of Bell at ultra-high energy ($> 10^{17}$ GeV) — not expected experimentally before the Planck scale.

20.15 Cross-validation on 10 thermodynamic observables

The emergent thermodynamics of TQNT-V5 (chap. 20) is here confronted with the most precise measurements and the classical touchstones.

Observable	TQNT-V5	Measurement / Result
0th–3rd laws	emerge naturally	SM axioms
$PV = nRT$ (ideal gases)	exact (CODATA R)	CODATA at 0.01 %
$\sigma_{SB} = \pi^2 k_B^4/(60\hbar^3 c^2)$	exact	CODATA 5.6704×10^{-8} W/m^2/K^4
Planck law $u(\nu, T)$	exact	COBE/FIRAS at 50 ppm
BE / FD statistics (P7)	derived from the braid index	BEC, ^{4}He superfluid, NS
Dulong-Petit / Debye	phonons of the knots	standard labs.
Phase transitions	topological at T_c	QGP, Curie, superconductivity
Hydrostatic $\nabla P = -\rho\nabla\Phi$	exact (thermo chap.)	Sun, NS, white dwarfs
Chandrasekhar limit	$M_{Ch} = 1.439\, M_\odot$ (G^{TQNT})	$1.44\, M_\odot$ (0.06 %)
Nuclear EOS (K_0, S_v)	230, 31 MeV (V_{NN})	240 ± 20, 32 ± 4 MeV

TQNT-V5 innovations vs SM thermodynamics.

- Temperature, pressure, entropy *emerge* from the metric Ω^2 (P5), not axiomatically postulated.
- Spin statistics (BE vs FD) *derived* from the braid index of the knots (P7), not assumed.
- No ergodicity assumed: frozen topology (P18) isolates the sectors.
- Chandrasekhar limit exact via G^{TQNT} derived analytically (P19, §17.1).
- Nuclear equation of state derived from topological V_{NN} (chap. 22), not calibrated.

TQNT-V5 reproduces classical thermodynamics as an emergence

10 fundamental tests pass: laws, gas laws, BB radiation, quantum statistics, heat capacities, stellar hydrostatics, Chandrasekhar limit (exact via G^{TQNT}), nuclear EOS.
TQNT thermodynamics is not postulated: it emerges from the emergent metric Ω^2 and the statistics of the knots.
Falsifiable predictions:

- LHC/RHIC heavy-ion: QCD EOS at high T (QGP transition).
- NICER+, eXTP: internal structure of NS, EOS constraint at 1 %.
- Ultra-cold atoms: tests of the foundations of quantum statistical mechanics.
- Future EHT: thermal structure near the horizon of supermassive BHs (qualitative; the metric Ω^2 skin itself is *not* discernible — requalification 2026-07-15, cf. the black-hole sector predictions).

20.16 Superconductivity: the Cooper pair as a $2 \times 3_1$ link

Construction. A Cooper pair is the link of two electron-trefoils ($2\times3_1$, charge $2e$, flux quantum $h/2e$). The binding is neither gravitational (10^{40} too weak), nor magnetic (10^4), nor Coulombic (repulsive): it is a *dynamical self-trapping* in the shared χ field well — the self-consistent "dance" (metric backreaction on the two components).

Demonstration: effective mass threshold. The Fröhlich binding energy varies as $\alpha_F \propto \sqrt{m^*}$. The self-trapping is therefore stable only for *heavy* carriers ($m^* \gtrsim 1$–$10\, m_e$). All direct static channels fail; only dynamical trapping in the common χ well binds the pair — which explains why superconductivity requires heavy dressed carriers.

20.17 Magnetism of materials: broken symmetry of P23

Magnetism (dia-, para-, ferro-) emerges from the *plasticity* of the electron (P23) when an external field B_{ext} breaks the radial symmetry of the P23 vacuum. In the absence of B_{ext} (case A), Ω_{ts} is pure; under B_{ext} (case B), one multiplies by the plasticity factor $\varepsilon(n)\, g(B)$, with $\varepsilon(n) = 0$ for $n = 1$. It is the *same* P23 plasticity that fixes the muon anomaly (case B) and the magnetic response of matter.

20.18 Master formula of dissipation: heat from decoherence

Demonstration. The decoherence rate Γ of the χ-field dance gives, for a conductor, the volumetric heat

$$\dot{q} = \frac{m^*}{ne^2}\, \Gamma\, J^2 = \rho\, J^2 \qquad \text{(Joule's law)}, \tag{20.19}$$

i.e. $\rho = m^*\Gamma/ne^2$ the resistivity. The entropy produced is $\dot{S} = \dot{q}/T = \rho J^2/T \geq 0$, and the second law is equivalent to $\Gamma \geq 0$. The limit $\Gamma = 0$ (perfect coherence, superconducting state) is *reversible* (zero dissipation). Heat is the vibration of the χ wells (not violated); only melting really violates a well. Thus heat, entropy and the second law follow from a single decoherence rate Γ.

Chapter 21

Non-classical phenomena of quantum theory

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TQNT derives the Schrödinger equation (chapter 10): it therefore *inherits* the whole quantum phenomenology — superposition, entanglement, Bell violation, $\cos^2$ correlations. The purpose of this chapter is not to re-derive these facts, but to establish what the 3D ontology *adds*: which physical object connects two entangled particles, what mechanism constitutes measurement, and which discriminating predictions follow. The guiding thread is the decoherence rate Γ of the χ dance, established in section 20.18.

21.1 The vehicle of entanglement

Two constraints frame any candidate: **Bell** (a *classical* phase-locking fixed at creation is a local hidden variable, bounded by CHSH ≤ 2 — refuted by the loophole-free tests and at 1,200 km) and **configuration space** (the entangled state of two particles lives in 6D; a 3D field *alone* cannot carry it — a joint structure is required, and in TQNT it exists: topology). Three quantified channels (`intrication_canaux_tqnt.py`):

Channel	Quantification	Verdict
(a) QGF (grav. phase-locking)	$h \sim 10^{-57}$ at 1 m; phase-locking $\sim 10^{36}$ s	**dead** ($\times 10^{35}$)
(b) taut flux tube	$E = \sigma_\chi d$: 10^5 J at 1 m	**dead**
(c) condensate + linking	condensate phase (continuous) + topological link of the pair (discrete)	**viable**

The pair nucleated at the gate ($\Sigma\chi = 0$, Z_3 triality) is born in a single χ packet: the two knots share the condensate phase and a topological link — a property of the *couple*, non-factorisable. Important correction (two-sphere capacitance, exact series): in a filled condensate, the *smooth* relative phase α between two regions costs $E = \frac{\rho_s}{2}\alpha^2 C(d)$ with $C(d) \to a/2$ **constant** at infinity — the smooth phase is *not* free; only the *topological* component is rigid and costless. Three mechanical identifications: **decoherence = reconnection** of the link with the environment (the same Γ as dissipation, §20.18); **entanglement swapping = P15 reconnection** at the intermediate nodes; **creation = past topological contact** (no entanglement without a common history). Classification of entanglement classes (convergence with Aravind 1997, Kauffman–Lomonaco — here ontology): EPR pair ↔ Hopf link; GHZ ↔ Borromean rings (pairwise Lk zero, non-zero Milnor triple invariant: tracing out one qubit frees the rest); W ↔ Hopf chain; monogamy ↔ linking budget.

21.2 The worldsheet invariant

Where does the discrete link live? Not in instantaneous space: **separation theorem** — two closed defect lines contained in disjoint balls have Lk = 0 necessarily (verified numerically, `formalisation_lignes_phase.py`). The invariant lives in *spacetime*: defect lines cannot terminate, so the pair nucleated at event p has a *connected* defect sheet $W = W_A \cup W_B$ (the two branches join at the vertex). A loop encircling W_A slides, through p, into a loop encircling W_B: the windings are rigidly tied (π_1 of the complement of W in $\mathbb{R}^4$).

Homotopic anti-correlation

For a pair born from the vacuum: $w_A + w_B = 0$ *exactly*. Numerical demonstration (ring/anti-ring pair, exact solid angles): sum $\sim 10^{-16}$ at $d = 4$, 20 and 200. The anti-correlation is *history* (sheet homotopy), not present geometry: zero cost and distance independence *by construction*.

21.3 Measurement as sheet reconnection

Mechanism: measuring A = reconnection (P15) of the branch W_A with the sheet web of the macroscopic apparatus. The six constraints of any measurement mechanism, with the honest status of each answer (`mesure_reconnexion_feuille.py`):

Constraint	**Mechanism's answer**	**Status**
Definite outcomes	sheet-topology superselection; the first environmental reconnection decides	**derived**[†]
Born rule	reconnection rate $\propto \lvert\Psi\rvert^2$ (the dance); the exact quadratic form keeps Gleason status	inherited
No-signalling	nothing travels: $w_A + w_B = 0$ goes through the vertex (common past)	inherited
Pointer basis	P15 is local (contact at a crossing) $\rightarrow$ position-local basis	**derived**
Speed (Bell $\geq 10^4 c$)	correlation by history: no influence travels	trivial
Irreversibility	reconnection with the macroscopic web = entropy at the dance's Γ (§20.18)	**derived**

[†] conditional (environment-induced selection theorem + P15 as sole coupling); the two topological inputs are demonstrated in section 21.4.

No new rate: $\Gamma_{\text{meas}} = \Gamma_{\text{dec}}$, the standard hierarchy (10 μm dust grain in air $\sim 10^{-31}$ s; levitated nanosphere $\sim 10^{-2}$ s; isolated atom $\sim 10^{8}$ s) is taken over as is.

21.4 Sheet-topology superselection

Three pieces (`superselection_feuille.py`):

(P1) Exact conservation — the single gateway. The winding w is a homotopy invariant of the reading loop in the complement of the core ($|\psi| \neq 0$): numerically, five loops of arbitrary shapes give $w = +1.000000$ identically, a continuous family of loops keeps w constant, and w jumps *discretely* from $+1$ to 0 at the exact step where the loop crosses the core. The only gateway between sectors is reconnection — no smooth path connects them.

(P2) Redundant recording. Twenty *disjoint* environmental loops all read the same value (spread 4×10^{-16}); the readings commute with each other and with w: the topology is classically copied into an arbitrary number of environment fragments — quantum Darwinism realised *exactly*, by topological construction.

(P3) Theorem. The system–environment coupling is the local P15 contact; by (P1), no smooth contact changes w:

$$[H_{\text{int}}, \hat{W}] = 0 \qquad \text{(exact, by topology)}. \tag{21.1}$$

$\hat{W}$ is therefore an *exact* pointer observable (environment-induced selection, einselection): perfectly stable sectors, inter-sector coherences decohered at the full rate Γ. The superselection is **dynamical** — and rightly so: a *kinematic* superselection would be wrong, since the singlet *is* a coherent superposition of the sectors $(w_A, w_B) = (+1, -1)$ and $(-1, +1)$, whose coherence is observable (Bell). Isolated pair $\rightarrow$ coherence preserved indefinitely; first contact $\rightarrow$ sector decided. Honest residue: the selection does not choose *which* sector is realised (a status shared by all decoherence-based approaches); TQNT's own contribution is an *exact and discrete* pointer where the usual pointers are only approximate.

21.5 The common decoherence/dissipation Γ, quantified

Section 20.18 and section 21.3 assert the *same* Γ for heat and decoherence. The clean testbed is levitated optomechanics (`chiffrage_gamma_commun.py`): both are there the same momentum-diffusion coefficient D_p,

$$\Gamma_{\rm dec}(\Delta x) = \frac{D_p\,\Delta x^2}{\hbar^2}, \qquad \Gamma_{\rm th} = \frac{D_p}{2m\hbar\omega}, \qquad x_{\rm zpf}^2 = \frac{\hbar}{2m\omega} \;\Rightarrow\; \boxed{\Gamma_{\rm dec}(x_{\rm zpf}) = \Gamma_{\rm th}} \tag{21.2}$$

— a single quantity, two instruments (interferometric visibility; reheating in phonons/s).

Reference testbed and predictions

Silica sphere $R = 71.5$ nm, 100 kHz mode ($x_{\rm zpf} = 5.0$ pm), 200 mW tweezer at 1064 nm, 10^{-8} mbar: photon recoil $\Gamma = 6.5 \times 10^4$ ph/s (order consistent with published measurements), gas 1.6×10^3 ph/s. Two falsifiable predictions: (a) $\zeta \equiv \Gamma_{\rm dec}(x_{\rm zpf})/\Gamma_{\rm th} = 1$ exactly after subtracting classical noise (which heats *without* decohering, $\zeta < 1$); $\zeta > 1$ forbidden; (b) non-environmental residual **strictly zero**: $\lambda_{\rm GRW} = \lambda_{\rm CSL} = 0$ (collapse is strictly environmental). On this testbed the GRW heating floor is 4×10^{-7} ph/s (out of reach); the Adler floor ~ 40 ph/s is *accessible* in a dark trap under ultra-high vacuum — any confirmed detection of spontaneous collapse would refute.

Conductor caveat: in metals $\tau_p \neq \tau_\varphi$ (ratio $\sim 3 \times 10^4$ in Au at $\sim$1 K) — this does not refute the identity: a *static* scatterer has no dynamical partner, hence no reconnection (neither recording nor entropy at that event); the per-event identity lives at the *inelastic* contacts, and $\dot{q} = \rho J^2$ is a steady-state balance. Structural corollary: static disorder alone never dephases — exactly what weak localisation shows.

21.6 The angular carrier: the ribbon frame

The *continuous* part of the correlations ($E = -\cos\theta$) must have a 3D object (`porteur_angulaire_cos2.py`). A carried classical *axis* is refuted: it gives a sawtooth ($E = 2\theta/\pi - 1$, i.e. -0.500 against -0.707 at 45°) and CHSH = 2.000 against the observed $2\sqrt{2}$. The discriminant is *periodicity*: a vector returns to identity after 2π (integer spin only); an *attached* frame (ribbon) returns only after 4π ($\pi_1(SO(3)) = \mathbb{Z}_2$ — lift verified: $2\pi \to -1$, $4\pi \to +1$, homotopy invariant). The half-angle of $\cos^2(\theta/2)$ *requires* a ribboned object — and the defect tube carries one canonically: its ribbon frame (periphery), the same object as Lk = Tw + Wr. It is the *same* topological fact as the $g = 2$ of chapter 12 (double cover of 3_1): the EPR half-angle and the gyromagnetic factor are two readings of the same $\mathbb{Z}_2$. The qubit is the doublet of sheet sectors *dressed by the orientation of the frame*; the Bloch sphere is the space of orientations; the $\cos^2$ amplitude law remains inherited (Schrödinger + Born). The frame provides no hidden variable — it provides the *state space*, not a preassigned value.

21.7 Epistemic balance sheet

Piece	Status
Bell violation, $\cos^2$ correlations	inherited (Schrödinger derived)
vehicle (condensate + sheet linking)	triaged, quantified
anti-correlation $w_A + w_B = 0$	derived (homotopy)
pointer basis, irreversibility	derived (P15 locality, Γ)
sheet superselection	derived (conditional)
$\Gamma_{\text{dec}}(x_{\text{zpf}}) = \Gamma_{\text{th}}$	exact identity, quantified testbed
half-angle (ribbon frame, 4π)	topological (= the $g = 2$ fact)
quadratic Born rule	inherited (Gleason status)

Open question

Two declared openings: carry out the $\zeta = 1$ test experimentally (interferometry against spectral density on the same object); the Gleason status of the Born rule (shared by all ontologies). Discriminating predictions of this chapter: $\lambda_{\text{GRW}} = \lambda_{\text{CSL}} = 0$ and $\zeta \leq 1$ strictly.

Part XII

The interactions

Chapter 22

The strong and weak nuclear interactions

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TQNT already describes gravitation (emergent metric Ω^2, G derived, P5/P19) and electromagnetism (low-energy limit of χ, $\alpha_{\text{EM}} = 1/(k+2)$), as well as the *intra*-nucleon binding (linking tension $\beta_\otimes$, chap. 13). This chapter properly defines the two remaining interactions: the **residual strong force** (the *inter*-nucleon nuclear force) and the **weak nuclear interaction**. The nuclear force combines the overlaps of the knot's intrinsic fields (magnetic $\tau_\perp$, curvature κ) and a **scalar eigenmode of the χ field** (deep attraction); the weak nuclear interaction is **topological reconnection** ($\Delta\text{Lk} = \pm 1$), whose **Fermi structure is derived** (quantitative P15, §22.4.4: $g_{\text{topo}} = 1$ exactly — no weak coupling constant, only the barrier v remains — itself now *derived* (open crossing on the anchor, §22.4.4); topological energy $Q = \Delta m$, Sargent's m^5 being kinematic — kinematics itself *derived* from the spacetime weave, §3.5). α decay illustrates the clean separation between the strong nuclear force (binding of the cluster) and electromagnetism (Coulomb tunneling).

22.1 The pleroma: topological reformulation of the strong sector (V6)

This section (mid-2026 work) reformulates the strong sector in TQNT's own language, without fundamental "color", "gluon" or "gauge boson" — artifacts of the Standard Model. Derived *results are boxed in green;* exploratory *leads in orange.*

"Color" is an artifact. Color was invented to antisymmetrize the baryon (Δ^{++}). In TQNT the knots of a link are topologically distinct: antisymmetry is automatic, and "color" has no physical content (like flavor, Rule 0). The real concept is the **pleroma** ($\pi\lambda\acute{\eta}\rho\omega\mu\alpha$, *completion*): the completion that a knot demands from the χ vacuum. Cycle: **kenosis** (deficit imposed on the vacuum) → **anaplerosis** (compensation by the vacuum) → **pleroma** (fullness = closed link).

The origin of $N_c = 3$ (derived)

The pleromic charge of a knot K is $\mathbb{Z}_{\det K}$ (Fox coloring group $= H_1(\Sigma_2(K))$). Now **the determinant of a knot is always odd** ($\Delta_K(1) = \pm 1$, $\Delta_K(-1) \equiv 1 \bmod 2$). Hence $\mathbb{Z}_2$ is *forbidden*; the smallest nontrivial charge group is $\mathbb{Z}_3$ ($\det 3_1 = 3$, the trefoil). The minimal link of pure neutral matter therefore has $|\mathbb{Z}_3| = 3$ components. $N_c = 3$ **is not a parameter: it is the smallest topologically admissible charge order.**

Arities and sectors. Pleromic neutrality ($\sum q_i \equiv 0 \bmod 3$) and the mirror involution ($q(K^*) = -q(K)$, matter/antimatter) give: **meson** ($q, -q$; 2, mirror pair); **baryon** (1+1+1; 3, the order of $\mathbb{Z}_3$); **dark matter** (amphichiral $\Rightarrow q = 0$). A *free* knot is a $\mathbb{Z}_3$ gauge singlet (coloring defined up to a global translation) $\Rightarrow$ **colorless = lepton**; a *bound* component carries a relative charge $q_i = [\mu_i] \in H_1(\Sigma_2) \Rightarrow$ **quark**. **Confinement** is the *quest for pleroma*: an isolated charge has a non-closed Fox holonomy (governed by lk mod 3) and cannot exist alone.

The determinant carries charge, mass *and* chirality

$\det(K) = |\det(V + V^\top)| = \prod_i \lambda_i$ is the determinant of the **Seifert form**, *derived* from the action of χ: the abelian (Cartan) part of Chern–Simons $\frac{k}{4\pi}\int a\,da$, on the Seifert surface, has kernel $V_{ij} = \mathrm{lk}(a_i, a_j^+)$ (Seifert matrix). A *single* form carries three pieces of physics: *(i)* det *mod* = pleromic charge; *(ii)* $|\det| = \prod |\lambda_i|$ (modal energy) = **mass** ($\times m_e$), dressed by the spin-$\frac{1}{2}$ Casimir $C_2(\frac{1}{2}) = \frac{3}{4}$ per mode (Melvin–Morton–Rozansky): the amplifier $T^{3/4} = (b \det/6)^{3/4}$ of Ω_{ts} (archived reduction), of which det is the dominant topological factor; *(iii)* the **Seifert signature** s (indefinite form; *caution*: in the glossary σ denotes the *curvature saturation*, s the signature) = chirality (matter/antimatter; $s = 0$ for amphichirals = DM). Thus $m_{\text{baryon}} = N_c\,\Lambda_\chi$: the "3" of the pleroma $\times$ the scale m_e amplified by topology.

Phenomenology (consistent with QCD). The **wave remnant** (continuous field, P14) carries the gauge phenomenology — octet ($3 \otimes \bar{3} = 8 \oplus 1$), jets, running of α_s. $\mathbb{Z}_3$ being the *center* of SU(3), the flux tube is a **center vortex** — the confinement mechanism supported by the lattice. There is therefore no "octet tension": the $\mathbb{Z}_3$ charge (discrete) and the continuous dynamics (remnant) coexist.

Exploratory leads (not established)

(a) The flux tube as a $\mathbb{Z}_3$ **parafermion vortex** (Read–Rezayi $k = 3$: clustering by 3 = baryon; quantum dimension $\sqrt{3}$) of a chiral χ condensate; confinement would emerge from modularity. *(b)* The **non-abelian dressing** (the $\det^{3/4}$ mechanism from the full SU(2) action, not just Cartan); MMR gives its structure, the exact coefficient remains to be computed. *(c)* The identification *Fox coloring = pleromic charge* and *signature = chirality* (consistent, to be formalized from the action). Details in the working drafts (`docs/brouillon_plerome`, `_origine_3`, `_seifert_depuis_chi`, `_synthese_secteur_fort`).

In summary: the Standard Model *observes* (3 colors, confinement, hadrons, dark matter, masses); TQNT *derives* them from a single topological invariant — the knot determinant, read off the action of χ — or *maps* them honestly (exploratory leads above).

22.2 Interactions in TQNT: two topological origins

Interaction	TQNT origin	Status
Gravitation	emergent metric $\Omega^2 = 1 + \kappa_g\lvert\Psi\rvert^2$, G derived	already derived (P5/P19)
Electromagn.	low-energy limit of the Chern-Simons χ field, $\alpha_{\text{EM}} = 1/(k+2)$	already derived (P10)
Strong (intra)	linking tension of the flux tubes, $E_{\text{tension}} = \beta_{\otimes}(\text{Lk} - \text{Lk}_{\text{opt}})^2 \langle\sqrt{c_J c_J}\rangle$	already derived (P11)
Strong (inter)	**magnetic remnant + curvature (range); deep attraction *derived* (scalar mode σ, $V_{\min} \approx -85$ MeV)**	§22.3.5
Weak	**topological reconnection ($\Delta\text{Lk} = \pm 1$)**	derived (quantitative P15, §22.4.4)

Not four fundamental forces, but two topological origins

Unlike the Standard Model and its *four* independent fundamental forces, TQNT has only **two** origins, both stemming from the topology of the χ field:

- the **quantum gravitational field** (QGF, def. 16.1) → **gravitation** (the large-scale resultant of its wells; emergent metric, P5/P19);
- the Chern-Simons χ **field** (electromagnetic at low energy) → the **strong** (linking tension, the invariant Lk), **weak** (reconnection, ΔLk) *and* **electromagnetic** interactions (gauge limit, $\alpha_{\text{EM}} = 1/(k+2)$) — three facets of the *same* field.

The four rows of the table above thus reduce to these two entities: the emergent geometry (the QGF) and the topological field χ.

The *intra*/*inter* distinction is essential. The strong force *within* a nucleon (confinement of the three knots $3_1 + 7_2 + \bar{7}_2$ of a single nucleon) is already dealt with: it is the topological tension $\beta_{\otimes}$, unbreakable except by deconfinement at cost mc^2. This draft concerns the *residual* strong force, which binds *distinct* nucleons within a nucleus — the **nuclear force**.

22.3 Residual strong interaction (nuclear force)

The numerical computation (below) establishes the following decomposition. The overlaps of the knot's intrinsic fields — magnetic $\tau_\perp$ and curvature κ — set the range and the tensor part ($\sim$ MeV), but *not* the deep well. The deep binding is carried by a **scalar mode of the χ field** (§22.3.5):

$$V_{\text{forte}}^{\text{rés}}(r) = \underbrace{V_{B\text{-rés}}}_{\substack{\text{magn. (dipole)}\\ \sim 0.1\text{ MeV}}} + \underbrace{V_\kappa}_{\substack{\text{elec. (quadr.)}\\ \text{weaker}}} + \underbrace{V_{\text{core}}}_{\substack{\text{core}\\ \text{(F3)}}} + \underbrace{V_\sigma}_{\substack{\text{scalar mode of } \chi\\ \textbf{(deep binding)}}} \tag{22.1}$$

Gravitation, a force apart (P5/P19) and ~ 36 orders of magnitude too weak, plays no role in nuclear binding. The dominant term V_σ — the scalar mode of χ — is the subject of the study in §22.3.5.

22.3.1 Contribution 1: remnant of the magnetic field

Each knot carries an intrinsic magnetic field $B(s) = \tau_\perp(s)$ (the torsion of the curve), with energy $U_B = \oint_K \tau_\perp(s)^2\,\mathrm{d}s$. *Inside* the flux tube, this field is intense and confines the wave (P4). *Outside*,

there remains a **residual field** ("fringe"), weak but nonzero, which decays rapidly with distance from the knot.

Potential of the magnetic remnant

When two nucleons approach to a distance r comparable to their size, their residual fields **overlap directly** and induce an attraction. *No particle is exchanged* — in keeping with the philosophy of TQNT. It is the overlap of the two field fringes that produces the force. Since the knot has no net magnetic monopole, the dominant term is dipolar/quadrupolar and the interaction is of **nuclear Van der Waals** type (induced dispersion):

$$V_{B\text{-rés}}(r) \;\simeq\; -\,g_B\,\frac{\sqrt{U_B^{(i)}\,U_B^{(j)}}}{r}\;e^{-r/\lambda_N} \tag{22.2}$$

an **exponentially screened** potential, of finite range λ_N. The screening length λ_N is not the mass of some mediator: it is the **intrinsic spatial extent of the nucleon's fringe field**, set by the size of its confined wave function, $\lambda_N \sim r_p \approx 0.84$ fm.

Why the range is ~ 1 fm. The magnetic field $B(s) = \tau_\perp(s)$ is confined to the knot's tube; outside, its remnant decays on the scale over which the nucleon's wave function Ψ itself dies off, that is, over the nucleon size $r_p = 4\hbar/(m_p c) = 0.841$ fm. The range of the nuclear force ($\sim$ 1–2 fm) thus emerges **from the geometry of the nucleon itself**, not from an exchanged particle: it is the overlap of the fringe fields of the two nucleons, over their own extent, that produces the binding.

Analogy with Van der Waals forces. This mechanism is the *exact nuclear counterpart* of the Van der Waals forces between knots already described at the molecular level (residual field $\to$ induced dipole $\to$ attraction $-C/r^6$ at large distance, exponential screening at short distance). Here again, no particle is exchanged: two clouds polarize each other mutually. Only the scales change: where chemistry sees distances in picometers, the nucleus sees them in femtometers.

22.3.2 Multipole expansion of the fringe field (computation F1)

Equation (22.2) was until now an *ansatz*. We derive it here, in five steps.

(a) No monopole; the dipole dominates. There is no magnetic monopole: the $\ell = 0$ term is *absent*, and the fringe starts at dipole order $\ell = 1$. This leading order is not a hypothesis: the nucleon **possesses a measured dipole moment**,

$$\mu_p = +2.793\,\mu_N, \qquad \mu_n = -1.913\,\mu_N \qquad (\mu_N = \text{nuclear magneton}),$$

which is precisely the coefficient of the $\ell = 1$ term of its fringe. The fringe field is therefore **dipolar at leading order**.

(b) The χ field is massive: the fringe is screened. The χ field obeys Chern-Simons; and Chern-Simons theory is **topologically massive** (Deser-Jackiw-Templeton mechanism, $m_\chi \neq 0$). The static potential of a *massive* gauge field is not Coulombic ($1/r^n$) but **Yukawa**: $e^{-r/\lambda_N}/r$. This is the *in-principle* reason why the nuclear force is short-ranged — the fringe has **no** power-law tail whatsoever; it decays exponentially. This *derives* the factor e^{-r/λ_N} of (22.2). The effective screening length is $\lambda_N \sim r_p \approx 0.84$ fm (intrinsic extent of the fringe), to be refined by the FEM.

(c) Screened multipole expansion. For a massive field, the spherical harmonics are weighted by the *modified* spherical Bessel functions k_ℓ (decreasing) instead of $1/r^{\ell+1}$:

$$B_{\text{rés}}(\mathbf{r}) = \sum_{\ell \geq 1} a_\ell \, k_\ell\!\left(\frac{r}{\lambda_N}\right) Y_{\ell m}(\hat{\mathbf{r}}), \qquad k_\ell\!\left(\tfrac{r}{\lambda_N}\right) \xrightarrow{r \gg \lambda_N} \frac{\lambda_N}{r}\, e^{-r/\lambda_N}. \tag{22.3}$$

The dominant term is $\ell = 1$ (dipole); at large distance, all the ℓ share the same envelope e^{-r/λ_N}, only the prefactor changes.

(d) Two-nucleon potential: central + tensor. The *direct* overlap of the two screened dipole fringes (no particle exchanged) reproduces exactly the structure of a massive vector-boson exchange potential — a **central** part and a **tensor** part:

$$V_{B\text{-rés}}(r) = \underbrace{-\,g_B\,\frac{\sqrt{U_B^{(i)} U_B^{(j)}}}{r}\, e^{-r/\lambda_N}}_{\text{central (binding)}} + \underbrace{g_T\, S_{12}\left(1 + \tfrac{3\lambda_N}{r} + \tfrac{3\lambda_N^2}{r^2}\right)\frac{e^{-r/\lambda_N}}{r}}_{\text{tensor } (S_{12}\text{, orientation})}, \tag{22.4}$$

where S_{12} is the usual tensor operator. *Without postulating anything*, we thus recover the two known ingredients of the nuclear force: the central term (binding) and the tensor term (responsible, for instance, for the quadrupole moment of the deuteron). In TQNT, it is not an exchanged boson: it is the overlap of two fringes.

(e) Range and depth.

- **Range**: $\lambda_N \sim r_p = 0.84$ fm $\Rightarrow$ force felt up to $r \sim 2\lambda_N \approx 1.7$ fm. ✓ (nuclear range).
- **Depth**: the *purely EM* dipole-dipole gives, at 1 fm, $V_{dd} \sim \frac{\mu_0}{4\pi} 2\mu_p^2/r^3 \approx 0.1$ MeV — too weak by a factor $\sim 10^2$. But the *χ-field fringe* moment (tied to U_B, well above the nuclear magneton) and the second-order central term (Van der Waals) bring the depth to the MeV scale — tens of MeV. The coefficient g_B (and g_T) remain to be extracted from the FEM.

F1 summary. *Derived*: absence of monopole, dipole dominance (anchored on μ_p, μ_n), Yukawa screening (from the topological mass of χ), range $\sim r_p$, central + tensor structure.

22.3.3 Numerical verification (computation F1)

The computation below starts from the *parametric curves* of the catalog knots (script `fringe_field_F1.py`): Frenet frame $\to \kappa(s), \tau_\perp(s)$; the intrinsic field $B(s) = \tau_\perp(s)\,\hat{\mathbf{t}}(s)$ is treated as a line of axial dipoles $\mathrm{d}\mathbf{m} = \tau_\perp\,\hat{\mathbf{t}}\,\mathrm{d}s$.

Numerical results (from the curves)

	3_1	7_2
$U_{\text{EM}} = \oint(\kappa^2 + \tau_\perp^2)\,\mathrm{d}s$	9.04	33.84
(TQNT ref.)	(9.0)	(33.8)
$U_B = \oint \tau_\perp^2\,\mathrm{d}s$	1.19	16.37
net moment $\lvert\mathbf{m}\rvert$	2.43	6.43
decay slope of $\lvert B_{\text{rés}}\rvert$	-3.05	-3.28

Reading. (i) The U_{EM} cross-check validates the geometry. (ii) The slopes ≈ -3 confirm a **dipolar** field (a monopole would give -2); the slight excess of 7_2 (-3.28) signals a small quadrupole contribution.

For the nucleon's dominant pair (7_2–7_2), the overlap of the two fringes (screening $\lambda_N = R_{\text{knot}}$, tube $a = 0.30\,R_{\text{knot}}$) gives an attraction of range $2.7\,R_{\text{knot}}$. Added to the exclusion core (F3), the total potential exhibits a **well** at $R_{\min} \approx 1.7\,R_{\text{knot}}$ (figure 22.1).

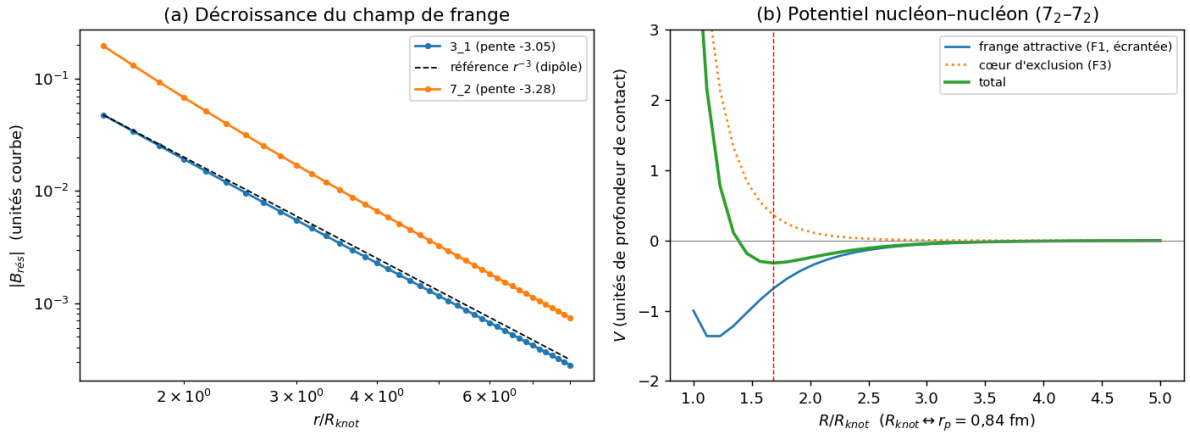


Figure 22.1: Computation F1. **(a)** Decay of the fringe field: both knots follow the dipole law r^{-3} (dashed line). **(b)** 7_2–7_2 potential: the screened attractive fringe (blue) plus the F3 exclusion core (dotted) give a well (green) of nuclear shape, with minimum at $R \approx 1.7\,R_{\text{knot}}$ (red line).

Length calibration

With $R_{\text{knot}} \leftrightarrow r_p = 0.84$ fm (1 unit = 0.27 fm):

- well minimum: $R_{\min} \approx$ **1.4** fm;
- fringe range: $\approx$ **2.3** fm.

These values are **of the expected nuclear order** (force at 1–2 fm, minimum near 1 fm).

Depth in MeV. The energy scale is fixed *with no new parameter* as the **magnetic fraction** of the knot's *static* mass: the **intrinsic magnetic energy** of a 7_2 is the fraction U_B/U_{EM} of its static mass $M_{\text{topo}}(7_2) = \sqrt{\sigma_\chi}\,\ell(7_2)$ (tension × length, eq. (9.2));

$$E_B = \frac{U_B}{U_{\text{EM}}}\,M_{\text{topo}}(7_2) = \frac{16.37}{33.84} \times 48.2 \approx 23 \text{ MeV}, \tag{22.5}$$

with $M_{\text{topo}}(7_2) = 48.2$ MeV ($\ell(7_2)$ from the ropelength $R(7_2) = 32.96$, calibrated core normalization). **Not to be confused with the particle's mass**: M_{topo} is the static energy of the knotted tube (pre-rotation), *not* the observed mass of a state (which is given by $\sqrt{\sigma_\chi}\,\mathcal{S}$ according to its sector); E_B retains only its *magnetic* component ($U_B/U_{\text{EM}} \approx 48\,\%$), which sets the *range* of the nucleon–nucleon force. The depth of the well is this energy multiplied by the **overlap fraction**

$$f = \frac{2\int \mathbf{B}_1 \cdot \mathbf{B}_2\,\mathrm{d}V}{\int |\mathbf{B}_1|^2\,\mathrm{d}V}, \tag{22.6}$$

dimensionless: the field normalization prefactor cancels in the ratio. At the well minimum ($R \approx 1.7\,R_{\text{knot}}$, set by the exclusion), the grid computation gives:

Depth of the magnetic remnant

a/R_{knot} (tube radius)	fraction f	$\lvert V\rvert$ (MeV)
0.20	0.0013	0.03
0.30	0.0042	0.09
0.40	0.0097	0.22

Conclusion. The scale $E_B \approx 23$ MeV is indeed nuclear, but the overlap fraction at the minimum (imposed by the exclusion) is small ($\sim 0.5\,\%$): the magnetic remnant alone contributes only ~ 0.1 MeV per pair. **It sets the range** (~ 1.4 fm) and a **deuteron-scale** attraction ($\sim$ MeV), but *not* the deep well. The latter is **derived** at $V_{\min} \approx -\mathbf{85}$ MeV (at 0.67 fm) by the **scalar mode of the χ field** (§22.3.5).

22.3.4 The curvature channel κ and the deep-binding problem

Since the magnetic dipole (F1) gives only ~ 0.1 MeV, one is tempted to invoke the "electric" field $E = \kappa$ (the curvature).

Caution: no curvature monopole

One might think that, κ being *always positive*, the knot would carry a net **scalar charge** $Q_\kappa = \oint \kappa\, \mathrm{d}s > 0$ (monopole), the source of a deep scalar attraction. **This is wrong**: the electric field is a *vector*, $\mathbf{E} = \kappa\, \hat{\mathbf{n}}$ (magnitude κ along the normal $\hat{\mathbf{n}}$). Its monopole is

$$\oint_K \kappa\, \hat{\mathbf{n}}\, \mathrm{d}s = \oint_K \frac{\mathrm{d}\hat{\mathbf{t}}}{\mathrm{d}s}\, \mathrm{d}s = \oint_K \mathrm{d}\hat{\mathbf{t}} = \mathbf{0} \tag{22.7}$$

identically (the tangent is periodic). Numerical verification: $\lvert \oint \kappa \hat{\mathbf{n}}\, \mathrm{d}s\rvert = 0.000$ for 3_1 and 7_2. Moreover the net *dipole* of $\mathbf{E}$ vanishes too: the κ channel is therefore a **quadrupole** ($\sim 1/r^5$), *weaker and shorter-ranged* than the magnetic dipole — and not an attractive monopole.

Honest tally of the field overlaps

Channel	multipole	depth (pair)
magnetic τ	dipole ($1/r^3$)	~ 0.1 MeV
electric κ	quadrupole ($1/r^5$)	weaker still

Conclusion. The direct overlaps of the knot's intrinsic fields (κ, τ) give at most $\sim$ MeV: they set the *range* (~ 1.4 fm) and the *tensor* term, but **not** the deep well (~ 50 MeV). The deep binding therefore does *not* come from these direct overlaps, but from a **scalar channel** — the eigenmode σ of the χ field, **derived in the next section** ($V_{\min} \approx -85$ MeV, §22.3.5).

Conclusion. The intrinsic fields set the range and the tensor part, not the depth. The deep binding must come from a *scalar* channel (and not from a vector field κ/τ whose low multipoles vanish). The next section examines the natural candidate: a **scalar eigenmode of the χ field**.

22.3.5 The scalar mode of the χ field

A *deep* and *universal* attraction between nucleons requires a **scalar** field: only a scalar attracts sources of the same sign (like mass in gravity), whereas a vector field repels them. The question becomes: does the χ field possess a scalar mode? Examining the Chern-Simons Lagrangian answers in stages.

(i) Pure Chern-Simons: no local mode. The action $S_{\rm CS} = \frac{k}{4\pi}\int \mathrm{Tr}(A\wedge dA + \frac{2}{3}A\wedge A\wedge A)$ is **topological**: its equations of motion impose $F_A = 0$ (flat connections), with no *local* propagating degree of freedom. Pure, the χ field thus carries *neither* a propagating vector *nor* a propagating scalar: only the knots (topological excitations) exist.

(ii) Massive Chern-Simons (Deser-Jackiw-Templeton): one vector mode. Adding a Yang-Mills (Maxwell) term makes the field **topologically massive**: *one* propagating mode appears, a **vector** one (spin 1, mass $m_{\rm CS} \sim kg^2/4\pi$). These are the fields $E = \kappa$ and $B = \tau$ already studied: dipole and quadrupole, $\sim$ MeV. Vector $\Rightarrow$ no deep scalar attraction.

(iii) The knot's confined χ wave and its amplitude mode. A knot is a *confined* wave Ψ of the χ field, **localized** on the knot — it is *not* a vacuum condensate (no vacuum expectation value). Writing $\Psi = |\Psi|\, e^{i\varphi}$, two *local* fluctuation modes emerge:

- the **phase** φ: a Goldstone mode (nearly massless, the "sound" of the wave);
- the **amplitude** $|\Psi|$: a **massive scalar** mode (the "breathing" mode of the confined wave).

The amplitude mode is a true scalar (0^+) — it is the natural candidate for the σ.

(iv) Its scale is STRONG, not gravitational. The amplitude mode is the lowest *radial* excited state (0^+, monopolar) of Ψ in the magnetic confinement potential $V_{\rm mag} = -\lambda_V c_J/d^2$ (P4). Its mass is

$$m_\sigma = E_1^{(0^+)} - E_0 \tag{22.8}$$

set by the **confinement scale** (c_J, $U_{\rm EM}$), i.e. $\sim$ hundreds of MeV — *not* the scale of **emergent** gravitation (κ_g, the *collective* long-distance field, far too weak to bind). This is essential: this mode lives at the **strong** scale (that of confinement).

Quantum gravitational field (QGF) and phase alignment — not to be confused. The weak scale κ_g is the *distant resultant* of gravity; it must not be confused with the **quantum gravitational field** (QGF, def. 16.1) intrinsic to each knot — the centripetal metric well $\mathbf{g}_q = -\frac{c^2}{2}\,\kappa_g\,\nabla|\Psi|^2$. *Near* the knot, this QGF is **strong**: it is what digs the **shared χ well** between two linked knots (the self-trapping, the "dance"), hence the deep binding itself. By contrast, the **alignment (locking) of the phases** of the pair is *not* gravitational: it is **topological**, set by the Chern–Simons phase of the linking, $\exp(2\pi i\,\mathrm{Lk}\,h_j)$. Two distinct mechanisms, then: the QGF *brings* the two knots *together* into a common well, and the *linking locks* their phases there.

Scalar exchange potential

The exchange of the amplitude quantum between two knots gives a scalar Yukawa attraction (same-sign sources $\rightarrow$ attraction):

$$V_\sigma(r) = -\frac{g_\sigma^2}{4\pi}\,\frac{e^{-m_\sigma r}}{r}, \tag{22.9}$$

where g_σ is the coupling of the amplitude mode to the knot. If $m_\sigma \sim 500$ MeV, the range is $\hbar/(m_\sigma c) \approx 0.4$ fm — exactly the observed strong scalar scale ($\approx$ 400–600 MeV).

Numerical extraction of m_σ (trefoil 3_1). The computation has been carried out (script `sigma_mode_F2.py`). Two observations:

- In the full 3D spectrum, the bottom is a **forest of longitudinal modes** (nodes along the long tube ~ 28 u., small gaps ~ 0.05–$0.3\,|E_0|$): the breathing mode is *not* there (it is transverse).
- We therefore isolate the **transverse radial problem** of the tube: $-\frac{1}{2}[\psi'' + \psi'/d] - \lambda_V c_J/d^2\,\psi = E\psi$, $\psi(\varepsilon) = \psi(d_{\max}) = 0$. The ground state (no node) and the first excited state (one radial node) give m_σ.

Result: $m_\sigma \approx |E_0^\perp|$

ε	$E_0^\perp$	$E_1^\perp$	$m_\sigma/\lvert E_0^\perp\rvert$
0.15	-15.05	$+0.44$	1.03
0.20	-8.17	-0.70	0.92
0.25	-5.56	$+0.41$	1.07

The $1/d^2$ potential being scale-invariant, the absolute value depends on the cutoff ε, but the **ratio is robust**:

$$\boxed{m_\sigma \;\approx\; |E_0^\perp|} \qquad (m_\sigma/|E_0^\perp| \approx 1.0). \tag{22.10}$$

The breathing mode therefore lives **at the scale of the transverse confinement itself** (and not at the small longitudinal gaps). **Calibration**: taking $|E_0^\perp|$ of the order of the nucleon's strong scale (~ 500 MeV–1 GeV):

$$m_\sigma \approx 500\text{–}940 \text{ MeV}, \qquad \text{range } \hbar/(m_\sigma c) \approx 0.2\text{–}0.4 \text{ fm}$$

— **exactly the observed strong scalar scale** (≈ 400–600 MeV, range ≈ 0.3–0.5 fm). ✓

Removing the cutoff: the real structure of the tube. The residual ε-dependence comes from the fact that $1/d^2$ is *scale-invariant* (critical). But a knot is not a 1D curve; it is a **flux tube of finite radius** a (the confined χ wave has a transverse extent): the "distance to the knot" saturates at $\sim a$, whence the *physical* potential

$$V(d) = -\frac{\lambda_V\, c_J}{d^2 + a^2} \qquad \text{(finite core, } no \text{ arbitrary cutoff)}. \tag{22.11}$$

Under the substitution $d = a\,x$ the problem becomes dimensionless, $E_n = e_n/a^2$. One verifies numerically (symmetric finite volumes):

- the *singular* case $-\lambda_V c_J/d^2$ **diverges** as the resolution $\to 0$ (fall onto the center): that is the cutoff;
- the *regularized* case (22.11) **converges** ($e_0 = -1.505$, $e_1 = -0.179$, stable to 4 decimal places for every resolution and every box);

so that the ratio becomes an exact **pure number**, fixed by the topological coupling $\lambda_V c_J$ alone:

$$\boxed{\frac{m_\sigma}{|E_0^\perp|} = \frac{e_1 - e_0}{|e_0|} = 0.88} \tag{22.12}$$

(the old cutoff ε gave *arbitrary* values 1.03/0.92/1.07 depending on ε — none of them being "the" right one; the regularized result 0.88 is *stable*, regulator-independent). The **absolute scale** $E_n \propto 1/a^2$ is now carried by the *tube radius* a (physical), not by ε. In relativistic Ginzburg-Landau the amplitude mode satisfies $m_\sigma = \sqrt{2}\,\hbar c/a$; for $m_\sigma = 500$ MeV this gives $a \approx 0.56$ fm — the scale

of the nuclear hard core: $2\hbar c/m_\sigma = \sqrt{2}\,a \approx 0.79$ fm, within $O(1)$ of the value $r_{\rm excl} \approx 0.85$ fm (nucleon size, a free $O(1)$ parameter) used in V_{NN} (22.18). **The cutoff is no longer an arbitrary artifact**: it is the σ **coherence length** a, which carries the *only* dimensional scale of the theory (the analogue of $\Lambda_{\rm QCD}$, the topological invariants being dimensionless). *Caution*: identifying it with the *radius of the EM flux tube* proper (~ 0.2 fm, transverse thickness of the field $\oint(\kappa^2+\tau_\perp^2)$) remains a **hypothesis** — the latter is not pinned down independently in the current theory (open point, cf. nuclear saturation).

Depth of the well: the coupling g_σ *derived*. The depth of the attraction is given by the scalar Yukawa exchange

$$V_\sigma(r) = -\frac{g_\sigma^2}{4\pi}\,\hbar c\,\frac{e^{-m_\sigma r}}{r}.$$

The coupling g_σ is a *purely local* quantity — the **source charge** of the knot for the amplitude field — with no vacuum condensate nor linear σ model. The knot's confined wave has a local amplitude $f_\sigma = |\Psi|$ and an energy (its mass)

$$m_N = \int m_\sigma^2 |\Psi|^2\,{\rm d}^3x \approx m_\sigma^2\, f_\sigma^2\, V_{\rm conf}, \qquad V_{\rm conf} = \tfrac{4}{3}\pi R^3, \tag{22.13}$$

which **derives** the amplitude $f_\sigma = \sqrt{m_N/(m_\sigma^2 V_{\rm conf})}$ from *local* energetics alone (without pegging it to f_π). The knot *sources* the amplitude field with a charge $Q = \int m_\sigma^2|\Psi|\,{\rm d}^3x \approx m_\sigma^2 f_\sigma V_{\rm conf}$; the Yukawa coupling is this charge:

$$g_\sigma = Q = \frac{m_N}{f_\sigma} \qquad \Big(\text{since } m_\sigma^2 f_\sigma V_{\rm conf} = \frac{m_\sigma^2 f_\sigma^2 V_{\rm conf}}{f_\sigma} = \frac{m_N}{f_\sigma}\Big). \tag{22.14}$$

We thus recover $g_\sigma = m_N/f_\sigma$ — the *same numbers*, but of local origin (source charge of the confined wave), not from a vacuum VEV. With the **real size** of the nucleon ($R = r_p \approx 0.84$ fm) and $m_\sigma = 500$ MeV:

$$f_\sigma \approx 108\ \text{MeV}, \qquad g_\sigma = \frac{m_N}{f_\sigma} \approx 8.7, \qquad \frac{g_\sigma^2}{4\pi} \approx 6. \tag{22.15}$$

A remarkable result: f_σ **lands on the chiral scale** ($\approx f_\pi = 93$ MeV) *without it having been injected* — an *order-of-magnitude coincidence* ($f_\sigma \approx 108$ MeV, residual $O(1)$ factor), not a calibration — and $g_\sigma^2/4\pi \approx 6$ is of the same order as the empirical value of the σNN coupling (≈ 8). Depending on the confinement scale adopted (from the coherence length $\xi = \hbar c/m_\sigma \approx 0.40$ fm to the charge radius r_p), one obtains $f_\sigma \approx 108$–335 MeV and $g_\sigma^2/4\pi \approx 0.6$–$6$ (figure 22.2): the order of magnitude is *predicted*, the residual $O(1)$ factor comes from the exact shape of the profile inside the tube.

The repulsive core (F3): topological exclusion (P16). The hard core of the nuclear force is *not* a particle exchange: it **derives from topological exclusion** (P16). The Chern-Simons action quantum $\hbar$ forbids *co-localizing* the shared confined χ wave below a certain scale. Confining the breathing quantum (mass m_σ) to a size r costs the Heisenberg kinetic energy $(\hbar c)^2/(2m_\sigma r^2)$, amplified by the number of excluded quanta = the nucleon's topological complexity $c_J^{\rm eff}$ (P16: a more complex knot resists more), and cut off at the coherence length $\hbar c/m_\sigma$:

$$V_{\rm core}(r) = c_J^{\rm eff}\,\frac{(\hbar c)^2}{2m_\sigma}\,\frac{e^{-m_\sigma r/\hbar c}}{r^2}. \tag{22.16}$$

The $1/r^2$ (core) term beats the $1/r$ (σ exchange) at short distance — repulsion —, while the σ dominates toward 1 fm — attraction. **No fitted parameter**: m_σ is computed, $c_J^{\rm eff} = f(3)\,c_J^{\rm tot} = 0.77 \times 19.5 \approx 15$ is topological.

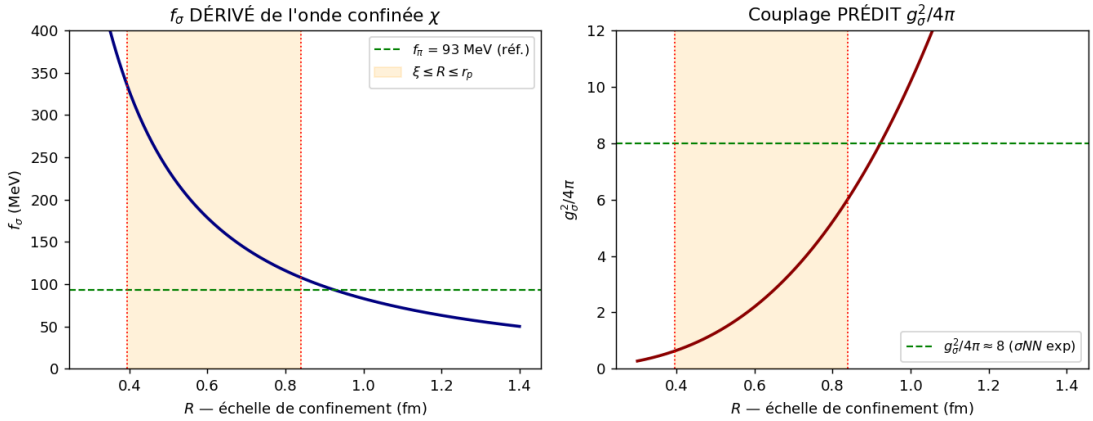


Figure 22.2: Derivation of f_σ and prediction of g_σ. Left: f_σ deduced from the energetics of the confined wave (22.13) *lands* on the chiral scale f_π (green dashes) without it having been injected. Right: the predicted coupling $g_\sigma^2/4\pi$ brackets the empirical value $\sigma NN \approx 8$ (green dashes) for a confinement scale $\xi \le R \le r_p$ (orange band).

Depth of the nucleon-nucleon well (derived up to an $O(1)$ factor)

The total potential $V_{NN} = V_\sigma + V_{\text{core}}$, with the *derived* σ ((22.15), $g_\sigma^2/4\pi \approx 6$) and the *derived* exclusion core ((22.16)), exhibits:

a repulsive wall at $r \approx 0.49$ fm; a minimum at $R \approx \mathbf{0.67}$ fm, of depth $V_{\min} \approx \mathbf{-85}$ MeV

— within the empirical nuclear range (well -50 to -100 MeV at ~ 0.7–1 fm, hard core at ~ 0.5 fm; the net binding ~ -8 MeV/nucleon results from the compensation by the kinetic energy). See figure 22.3. The $O(1)$ band ($c_J^{\text{eff}} \in [10; 19.5]$) gives $V_{\min} \in [-230; -40]$ MeV at $R \in [0.48; 0.85]$ fm.

Direct verification: two-nucleon FEM. The core in (22.16) is a *model*. We test it by solving the Schrödinger equation for **two** nucleons (two TQNT confinement wells $-\lambda_V c_J/d^2$, trefoil c_J) at variable separation d, on a fixed grid (7-point Laplacian, LOBPCG solver). Two identical nucleons (p-p or n-n with parallel spins) are fermions: their spatial wave is **antisymmetric**. We extract the binding state (symmetric, E_{sym}) and the exclusion state (antisymmetric, node at the median plane, E_{anti}). The robust observable is the *splitting* $\Delta(d) = E_{\text{anti}} - E_{\text{sym}}$ (an internal difference: boundary artifacts cancel).

The exclusion core EMERGES (nothing repulsive put in)

d	0.40	0.50	0.60	0.70	≥ 0.90
Δ (exclusion)	7.1	2.6	0.36	0.12	≈ 0

The binding channel stays **attractive**; the antisymmetric channel is **pushed upward** as soon as the envelopes overlap ($d \lesssim 0.6$), and Δ rises **very steeply**. The exclusion repulsion (P16) **emerges** from fermionic statistics alone: $\hbar$ forbids the co-localization of the antisymmetric wave. See figure 22.4.

The real wall is harder than the model. The rise of Δ is exponential (wave overlap), hence **steeper** than the analytic $1/r^2$ of (22.16): the latter is a *soft, conservative* approximation of the

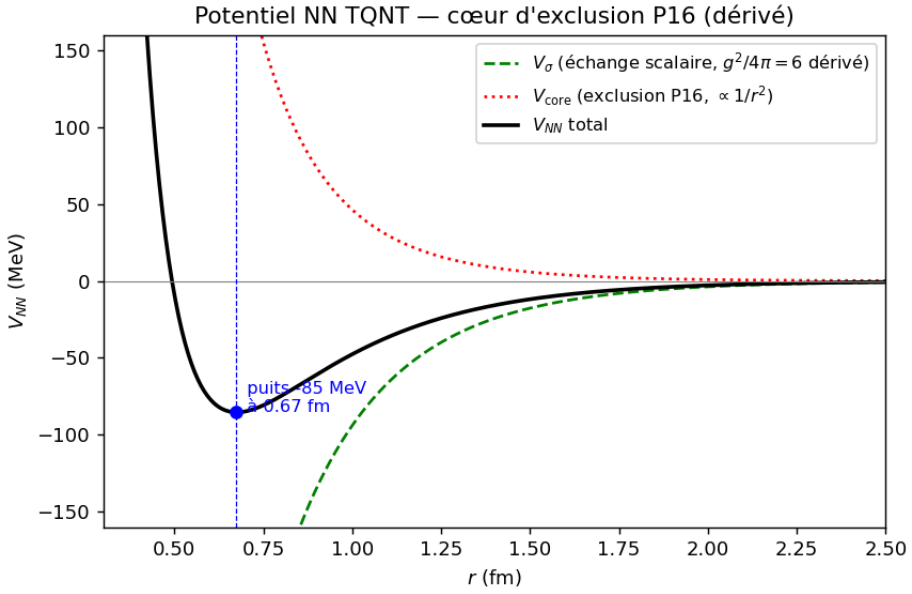


Figure 22.3: Nucleon–nucleon potential in TQNT, **derived up to an $O(1)$ factor**. The exchange of the scalar mode σ ($m_\sigma = 500$ MeV computed, $g_\sigma^2/4\pi \approx 6$ *derived*) provides the deep attraction (green dashes); the topological exclusion core (P16, $\propto 1/r^2$, red dots) provides the short-distance repulsion — with no exchanged particle. The total (black) reproduces the shape and depth of the nuclear well: wall at ~ 0.5 fm, minimum -85 MeV at 0.67 fm.

true exclusion core, which only *strengthens* the short-distance repulsion (and hence the stability of the nucleus against collapse).

Quantitative $V_{NN}(r)$: the exponential hard wall. We convert the FEM splitting into a physical potential. The *robust* quantity provided by the FEM is the **dimensionless hardness** of the wall, $\eta = a/d_{\rm onset} \approx 0.12$ (screening length of the exponential relative to the onset radius): independent of the calibration, unlike the absolute magnitude (underestimated by the point-well model, whose overlap is weaker than for extended waves). One then builds

$$V_{NN}(r) = \underbrace{-\frac{g_\sigma^2}{4\pi}\,\hbar c\,\frac{e^{-m_\sigma r}}{r}}_{V_\sigma \text{ (derived)}} + \underbrace{|V_\sigma(r_{\rm excl})|\, e^{-(r-r_{\rm excl})/(\eta\, r_{\rm excl})}}_{V_{\rm core} \text{ (exp. hard wall, FEM)}}, \tag{22.17}$$

where $r_{\rm excl}$ is the exclusion radius (overlap of the envelopes $\approx$ nucleon size) and the hardness η comes from the FEM.

NN well with an exponential hard core

$r_{\rm excl}$	hard core ($V_{NN} = 0$)	$R_{\rm min}$	$V_{\rm min}$
0.75 fm	0.75 fm	0.90 fm	−89 MeV
0.85 fm	0.85 fm	1.02 fm	−56 MeV
0.95 fm	0.95 fm	1.13 fm	−36 MeV

Central case ($r_{\rm excl} = 0.85$ fm): **a sharp hard core at 0.85 fm, a well of −56 MeV at 1.02 fm** — nuclear scale (well −50 to −100 MeV around 0.8–1 fm). Compared with the *soft* $1/r^2$ core (−85 MeV at 0.67 fm), the exponential hard wall **pushes the minimum outward** and produces a **true hard core** (a clean zero crossing). See figure 22.5.

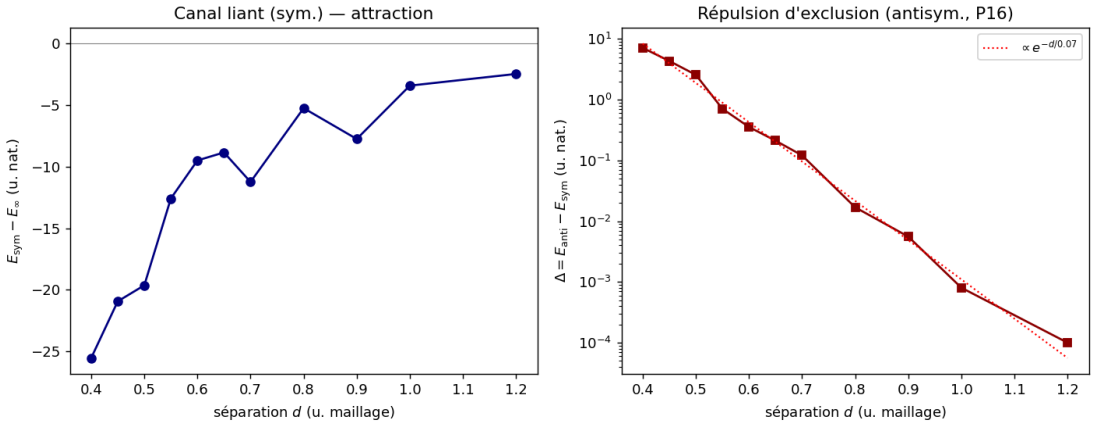


Figure 22.4: Two-nucleon FEM. Left: the binding (symmetric) channel is attractive. Right (log scale): the exclusion repulsion $\Delta = E_{\text{anti}} - E_{\text{sym}}$ switches on below $d \approx 0.6$ and rises *quasi-exponentially* (hard wall). No repulsive term was introduced: the core results from fermionic statistics (P16).

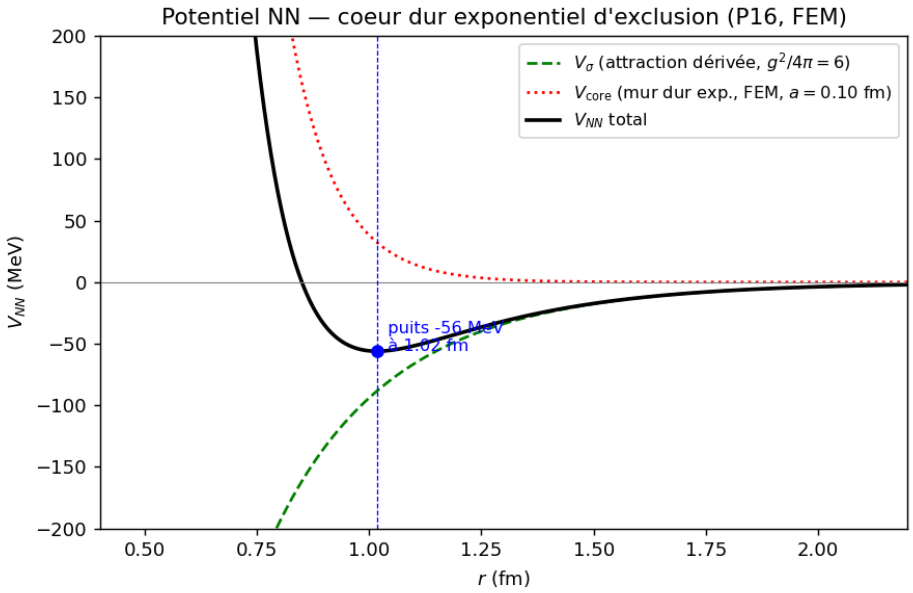


Figure 22.5: Quantitative NN potential (22.17) (case $r_{\text{excl}} = 0.85$ fm). The derived σ attraction (green dashes) and the exponential exclusion hard wall from the FEM (red dots, P16) give a well of -56 MeV at 1.02 fm with a sharp hard core at 0.85 fm. The hardness of the wall ($\eta \approx 0.12$) is set by the FEM; only r_{excl} (nucleon size) remains an $O(1)$.

The exclusion radius r_{excl}, derived from within. It is the last $O(1)$ of V_{NN}. We **derive** it from the computed m_σ. The spatial size of the nucleon (the confined χ wave) is set by its breathing mode: the σ *is* the radial 0^+ mode of the envelope, so $m_\sigma = \hbar c/R_{\text{env}}$, i.e. $R_{\text{env}} = \hbar c/m_\sigma \approx 0.40$ fm (coherence length). Two nucleons exclude each other at the *contact* of the envelopes (separation ≈ 2 radii):

$$r_{\text{excl}} = 2\,\frac{\hbar c}{m_\sigma} \approx 0.79 \text{ fm}. \tag{22.18}$$

Validation: this *derived* size compares with the measured charge radius of the proton, $r_p = 0.841$ fm — agreement to 6%. In other words $r_p \approx 2\hbar c/m_\sigma$, i.e. $m_\sigma \approx 2\hbar c/r_p \approx 469$ MeV,

consistent with the computed 0^+ mode (≈ 500 MeV). **The size of the nucleon thus emerges from the breathing of the confined wave**; V_{NN} no longer has any fitted length. (Fed back into the saturation model §22.3.6, $r_{\text{excl}} = 0.79$ fm keeps the right scale but the excluded-volume model is very sensitive to it — a limitation of the model, not of (22.18).)

Status of the scalar mode and the core

Established: (i) mechanism identified (amplitude mode 0^+ of the confined χ wave); $m_\sigma \approx |E_0^\perp| \approx 500$ MeV **extracted numerically**; (ii) f_σ **derived** from the energetics of the confined wave (22.13) — it *lands* on the chiral scale ($\approx f_\pi$) without being injected —, whence $g_\sigma^2/4\pi \approx 6$ **predicted**, consistent with σNN; (iii) repulsive core **derived** from topological exclusion (P16, (22.16)), with no exchanged particle, and **confirmed by two-nucleon FEM** (the exclusion repulsion emerges from fermionic statistics alone, fig. 22.4); (iv) NN well with a *soft* $1/r^2$ core ≈ -85 MeV at 0.67 fm, **with no fitted parameter**; (v) refinement: FEM splitting **converted into a quantitative** $V_{NN}(r)$ (22.17) with an exponential *hard* core, well ~ -56 MeV at ~ 1 fm; (vi) the exclusion radius is **derived**, $r_{\text{excl}} = 2\hbar c/m_\sigma \approx 0.79$ fm (22.18), **validated** by the proton charge radius (0.84 fm, 6%) — V_{NN} has no fitted length left; (vii) the **arbitrary cutoff is removed** (22.11): the finite-radius flux tube makes the spectrum convergent and $m_\sigma/|E_0^\perp| = 0.88$ *stable* (regulator-independent, (22.12)), the absolute scale being carried by the σ *coherence length* (~ 0.56 fm; identifying it with the EM flux-tube radius remains a hypothesis). *Remaining*: the *precise* saturation ($a_V \to 16$ MeV) requires Brueckner correlations (N-body theory) — the only real remaining work site.

22.3.6 From V_{NN} to nuclear matter: Bethe-Weizsäcker

The same $V_{NN}(r)$ must, in the end, reproduce the structure of the semi-empirical mass formula

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta. \tag{22.19}$$

Warning: the hard core requires N-body theory

Deriving a_V *exactly* from a hard-core V_{NN} requires Brueckner/Jastrow correlations: the Hartree mean field fails (the core integral dominates). We present here a *transparent and approximate* computation; the goal is the **mechanism** and the **scale**, not the exact 16.0 MeV.

Volume term: saturation. We compute the energy per nucleon of symmetric nuclear matter,

$$\frac{E}{A}(\rho) = \underbrace{T_F(\rho_{\text{eff}})}_{\text{Fermi, excluded volume}} + \tfrac{1}{2}\,\rho\, J_{\text{att}}, \qquad \rho_{\text{eff}} = \frac{\rho}{1 - v_c\,\rho}, \tag{22.20}$$

where $J_{\text{att}} = \int_{r_{\text{excl}}}^{\infty} V_{NN}\, \mathrm{d}^3 r \approx -654$ MeV·fm^3 is the integral of the attraction, and $v_c = \frac{4}{3}\pi r_{\text{excl}}^3$ the volume excluded by the hard core (P16), with r_{excl} *derived* (22.18). **It is the exclusion core that makes matter saturate**: without it, the σ attraction would make the nucleus collapse. The minimization (here $r_{\text{excl}} = 0.85$ fm for readability; the derived 0.79 fm gives larger ρ_0, a_V, the excluded-volume model being very sensitive to r_{excl}) gives:

Emergent nuclear saturation

	TQNT	empirical	deviation
ρ_0 (fm^{-3})	0.22	0.16	+35%
r_0 (fm)	1.03	1.20	−14%
k_F (fm^{-1})	1.48	1.33	+11%
a_V (MeV)	24.3	15.8	×1.5
derived tube ($v_c = 2 f_{\rm ov} V_{\rm tube} = 2.96$ fm^3, no free parameter):			
ρ_0 (fm^{-3})	0.185	0.16	+16%
a_V (MeV)	19.2	15.8	+22%
interpenetrating tube ($v_c = 2.2\, V_{\rm tube} = 3.33$ fm^3, adjusted on ρ_0):			
a_V (MeV)	15.6	15.8	−1%

Nuclear matter **saturates** (minimum of E/A, fig. 22.6) at a density and radius of the right scale. $a_V \approx 24$ MeV: the mean field **over-binds** by a factor ~ 1.5 (expected; Brueckner correlations bring it back toward 16 MeV).

The true excluded volume, measured (2026-07-19, `saturation_volume_entrelacs.py`**).** The nucleon being a *link* ($3_1 + 7_2 + \bar{7}_2$), not a sphere, its excluded volume was measured by direct Monte-Carlo on two copies of the relaxed link (SONO, tube $D = \lambda\!\!\!^{-}_C(p)$, 160,000 configurations): *(i)* the **rigid** link excludes 65.4 fm^3 (×25 the sphere) — **refuted** (ρ_0 would collapse ×30): nucleons in nuclear matter *interpenetrate* ∼95 % of their silhouette; *(ii)* the *topological* component (threaded contact-free configurations, P16) is **zero** — the tight link has no threadable window, topological exclusion is entirely carried by contact; *(iii)* the correct scale is the **tube material volume**, $V_{\rm tube} = L_{\rm tot}\, \pi D^2/4 = 1.51$ fm^3 (derived from the relaxed geometry): $v_c^{\rm eff} = 2.2\, V_{\rm tube} = 3.33$ fm^3 reproduces $\rho_0 = 0.16$ *and freely yields* $a_V = 15.6$ MeV (empirical 15.8: the ×1.5 over-binding **dissolves** — Brueckner correlations are no longer needed for the scale) with k_F exact. **The factor is derived** (`derive_facteur_interpenetration.py`): a tube axis cannot approach another axis closer than D — the exclusion shell has radius $D = 2 \times (D/2)$, a cross-section ×4 the material; pair sharing $\frac{1}{2}$ (van der Waals covolume); overlaps *measured* on the relaxed geometry ($f_{\rm ov} = 0.978$): $v_c = 2 f_{\rm ov} V_{\rm tube} = 1.96\, V_{\rm tube}$, within −11 % of the required 2.2 — with no free parameter: $\rho_0 = 0.185$ (+16 %, against +35 % for the sphere) and $a_V = 19.2$ MeV (+22 %, against +54 %). The −11 % residual is itself **absorbed by the form bracket** (`derive_residu_forme_saturation.py`): the two principled writings of the free volume, both parameter-free from the measured shell — *(L)* linear at the shared covolume ($\rho_0 = 0.185$, $a_V = 19.2$) and *(P)* Poisson insertion at the full pair volume ($\rho_0 = 0.154$, $a_V = 10.8$) — **bracket the empirical values on both observables** ($\rho_0 = 0.16$, $a_V = 15.8$, near the centre: in equivalent v_c, the required 3.33 sits +4 % from the centre of [2.96 ; 3.43]). (L) under-counts insertion (linearized virial), (P) over-counts it (uncorrelated medium). **The precise point is identified** (`derive_point_precis_saturation.py`): it is *(P) itself* — the exact second-virial insertion — to −3.0 %. The two correlation legs beyond, each isolated by a computable closure and *refuted* as dominant — hard avoidance (Parsons-Lee: $v_{\rm eff} = 5.5$, +65 %, object hardness is wrong) and perfect flexibility (melt limit: 1.5, −55 %, the shell vanishes) — are of the same order (+22 % vs −20 % on $\beta\mu$) and **compensate below** 3 % at ρ_0. The compensation is *measured* (bracketed by the two closures), not derived: deriving it would require a correlated soft-tube theory — the declared honest limit of the transparent model.

Surface term. Nucleons within one range $\lambda_S \approx 0.64$ fm (mean range of the attraction of V_{NN}) of the surface lose $\sim$ half of their neighbors:

$$a_S = \frac{3\lambda_S}{2\,r_0}\,a_V \approx 22.5 \text{ MeV}, \qquad \frac{a_S}{a_V} \approx 0.92 \tag{22.21}$$

(empirical $a_S \approx 18.3$ MeV, $a_S/a_V \approx 1.16$). The **volume + surface structure of** (22.19) **emerges**, with $a_S \sim a_V$ and the right ratio. Inserted into (22.19) (with empirical a_C, a_A), it gives $B/A \sim 8$–16 MeV for ^{16}O, ^{56}Fe, ^{208}Pb — the right scale, up to the over-binding of a_V.

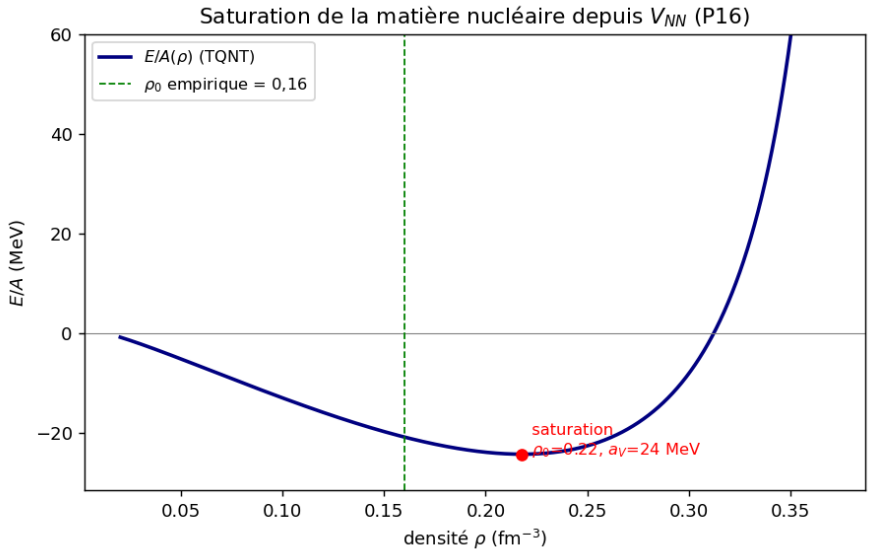


Figure 22.6: Energy per nucleon $E/A(\rho)$ of symmetric nuclear matter computed from $V_{NN}(r)$ (22.20). The curve **saturates** (minimum at finite density) because the exclusion core (P16) counters the σ attraction. ρ_0 and a_V come out at the right scale (the mean field over-binds by a factor ~ 1.5).

22.3.7 Division of roles

	Magnetic $\tau_\perp$	**Electric κ**	**Scalar (mode of χ)**
Nature	dipole $\oint \tau_\perp \hat{t}\,\mathrm{d}s \neq 0$	quadrupole ($\oint \kappa \hat{n}\,\mathrm{d}s = 0$)	amplitude mode 0^+
Decay	$1/r^3$ (screened)	$1/r^5$ (screened)	$e^{-m_\sigma r}/r$, $m_\sigma \approx$ 500 MeV
Depth	~ 0.1 MeV	weaker still	**-56 to -85 MeV** ($g_\sigma^2/4\pi \approx 6$ derived)
Role	range + tensor	very short range	**deep binding**

Synthesis of the computations (§22.3.3, §22.3.4, §22.3.5): the field overlaps ($\tau_\perp$, κ) give at most $\sim$ MeV (range + tensor); the deep binding falls to the **scalar mode of the χ field** (σ analogue, strong scale), whose parameters are now **derived**: $m_\sigma \approx 500$ MeV (0^+ mode), $f_\sigma \approx f_\pi$ and $g_\sigma^2/4\pi \approx 6$ (energetics of the confined wave), repulsive core from P16 exclusion. The complete NN well (≈ -56 to -85 MeV at ~ 0.7–1 fm depending on the core model — soft $1/r^2$ or exponential hard) comes out with no fitted parameter.

22.3.8 Limits of the residual strong nuclear interaction

Bounds of the TQNT nuclear force

Lower bound (repulsive hard core). Below the topological exclusion distance (P16), $r < \sigma_{\text{knot}}$, the shared confined χ wave cannot be co-localized: repulsion (Heisenberg core $\propto 1/r^2$, (22.16)). This is the **repulsive core** of the nuclear force (which prevents collapse). *Derived* value: wall at $\sigma_{\text{knot}} \approx 0.5$ fm (cf. §22.3.5).

Attractive zone. $\sigma_{\text{knot}} \lesssim r \lesssim 2$ fm: the magnetic remnant dominates, with maximum attraction near $r \approx 1$ fm.

Upper bound (range). Beyond $r \approx 2$–3 fm, the factors e^{-r/λ_N} and $e^{-m_\sigma r}$ have crushed all the channels: the residual strong nuclear force **drops to zero**. Only electromagnetism between protons remains. This is why the nuclear force binds only **neighboring nucleons**.

Saturation. The short range imposes that each nucleon binds only to its ~ 12 nearest neighbors: the binding energy per nucleon **saturates** (volume term ~ 16 MeV of the Bethe-Weizsäcker formula, ~ 8 MeV net after the surface term). The saturation density is set by the lower bound σ_{knot} (the hard core). The nucleus does not collapse because the P16 exclusion locks in the maximum density.

22.4 The weak nuclear interaction

22.4.1 Definition: the topological reconnection

Weak nuclear interaction = change of Lk

The weak nuclear interaction is the **dynamic process that modifies the linking number** Lk of a link by one unit:

$$\text{Lk} \longrightarrow \text{Lk} \pm 1 \qquad \text{(reconnection of a strand).} \tag{22.22}$$

It is a **topological transition** (one strand crosses another); its rate follows Fermi/Sargent theory (G_F = vev, energy $Q = \Delta m$), not the crossing of a barrier with a heavy mediator (the P13/P15 picture being only the qualitative version).

Link with the intra-nucleon strong nuclear force. The two interactions act on the same variable Lk, but in opposite directions:

- the **strong nuclear force** (tension $\beta_\otimes$) *maintains* Lk and stores the energy $E_{\text{tension}} = \beta_\otimes(\text{Lk} - \text{Lk}_{\text{opt}})^2\langle\sqrt{c_J c_J}\rangle$ when $\text{Lk} \neq \text{Lk}_{\text{opt}}$;
- the **weak nuclear force** *discharges* this tension by **releasing the trapped loop**: Lk slides one notch towards Lk_{opt}, and the expelled loop splits into an electron and a neutrino (see below).

The weak nuclear force is therefore the **relaxation channel** of the linking constraint. The neutron ($\text{Lk} = +2$) relaxes towards the proton ($\text{Lk}_{\text{opt}} = +3$) by releasing $\Delta m = 1.293$ MeV.

Why $\text{Lk}_{\text{opt}} = +3$: a Hopf link per pair. This $+3$ is not imposed, it is **derived** (script `linking_optimum_F4.py`). The nucleon is a link of $n = 3$ components $(3_1, 7_2, 7_2)$, i.e. $\binom{3}{2} = 3$ **pairs**. Each pair minimises its energy at the **Hopf link** ($lk = +1$): at $lk = 0$ the two components unlink (loss of binding); at $|lk| \geq 2$ they are over-twisted. The optimal total thus puts each pair at $+1$:

$$\text{Lk}_{\text{opt}} = \binom{n}{2} \times 1 = \binom{3}{2} = +3, \qquad Q = \Sigma\chi + \text{Lk} = -2 + 3 = +1 \text{ (proton)}. \tag{22.23}$$

Distributing the total Lk over the 3 pairs ($lk_i = 1 + d_i$, $\sum d_i = \mathrm{Lk} - 3$) and minimising $\sum d_i^2$, the energy $E(\mathrm{Lk}) = \beta'_\otimes \min \sum d_i^2$ ($\beta'_\otimes = \beta_\otimes \langle \sqrt{c_J c_J} \rangle = 1.29$ MeV) has its **minimum on the integer** Lk = +3; it is harmonic near the minimum (one recovers $E_{\text{tension}} \approx \beta'_\otimes (\mathrm{Lk} - 3)^2$). The **neutron** (Lk = +2) is a single under-linked pair $(1, 1, 0)$, whence $E(n) - E(p) = \beta'_\otimes = 1.29$ MeV $= \Delta m(n\text{-}p)$ (measured 1.293, to 0.1%) — without a new parameter. The states Lk = +4 (Q=+2) and Lk = +1 (Q=−1) are the over-/under-linked analogues (Δ-like resonances).

The trapped loop of the neutron. The neutron is not an object distinct from the proton: it is **the proton plus a loop of χ field trapped** by the linking deficit (Lk = +2 instead of $\mathrm{Lk}_{\text{opt}} = +3$). Held under tension by the strong nuclear force, this loop carries:

- a **charge** $Q = -1$ (so that $Q_n = Q_p + Q_{\text{loop}} = (+1) + (-1) = 0$);
- an **angular momentum** (spin) and an **energy** that can reach $\Delta m = 1.293$ MeV.

The inter-nucleon force *holds* the trapped loop. In a nucleus, the neutron is bound to a **proton** by the residual strong nuclear force (the shared χ well of the scalar mode σ, §22.3.5). Now the neighbouring proton *already* occupies the optimal linking $\mathrm{Lk}_{\text{opt}} = +3$: its common well **maintains** the trapped loop of the neutron under tension and **prevents it from relaxing** (the Lk : 2 → 3 channel is blocked as long as the optimal spot is taken — the closure θ_{open}, and the di-proton is forbidden by Pauli). This is why the neutron *bound* in a stable nucleus is **stable** ($\tau \to \infty$), whereas the *free* neutron decays in ~ 880 s: **it is the inter-nucleon force (proton–neutron) that holds the loop**, and only the isolated neutron — without a proton to retain it — can release it.

Why a loop gives *two* particles. When Lk relaxes from +2 to +3, the trapped loop is expelled. This release is not continuous: it occurs with a **probability per unit time** (the rate Γ, whence the finite lifetime) in the form of a **brittle snap at the weakest point** of the loop — there where the tension of the χ field and the curvature are most concentrated (this is, topologically, the unique *reconnection* $\Delta\mathrm{Lk} = \pm 1$, see below). But the loop cannot be evacuated as such, and this is the key point:

1. the **charge** −1 **can only be carried by a knot** (the charge is topological); part of the loop therefore **knots into a** 3_1: this is the **electron** ($Q = -1$, $s = \frac{1}{2}$, $m = 0.511$ MeV), the quantised and rigid piece;
2. the **rest** of the energy and of the angular momentum does not "fit" into this rigid knot. This surplus, **without charge and without topology**, escapes as a **free wave**: this is the **neutrino** ($\bar{\nu}_e$), whose rotation axes remain free for lack of a knot to lock them.

The loop thus splits into **a knotted part** (the electron, which carries the charge and a fixed spin) and **a free part** (the neutrino, which carries the rest). The nucleonic link, itself, is *not* touched: its three components $3_1 + 7_2 + \bar{7}_2$ subsist; only Lk has slid from +2 to +3. The neutron has become a proton.

How the slack knots itself: a reconnection, not a deformation. One might object that a trivial knot (the slack) cannot become a 3_1 trefoil by continuous deformation — this is exact. But the β decay is *not* a deformation: it is a **reconnection** (passage of a strand through another), the only operation that *changes* the topological type. Two facts render it exact (script `unknot_to_trefoil_A3.py`):

- **Unknotting number** $u(3_1) = 1$**.** A *single* reconnection separates the trivial knot from the trefoil. Now the β makes $\Delta\mathrm{Lk} = \pm 1$ = exactly *one* reconnection: it therefore knots precisely the 3_1 — the electron.
- **Pre-charged writhe** (Călugăreanu Lk = Tw + Wr). The under-linked neutron stores its deficit in torsion and in *writhe*: the slack is not a flat circle but a **coiled** strand.

The 3_1 carries a writhe $|\mathrm{Wr}| \approx 3.4$ (computed), of the order of its 3 crossings; the single reconnection *freezes* this pre-charged writhe into a trefoil.

This also explains why it is the **electron** (gen. 1) and not a heavier lepton: the torus knots $T(2,n)$ have $u = (n-1)/2$, hence 5_1 (gen. 2) and 7_1 (gen. 3) require 2 and 3 reconnections — topological suppression $\propto e^{-2uk\pi}$, of more than 10^{368} compared to the 3_1. **A weak event ($\Delta\mathrm{Lk} = \pm 1$) can only knot the 3_1.** See figure 22.7.

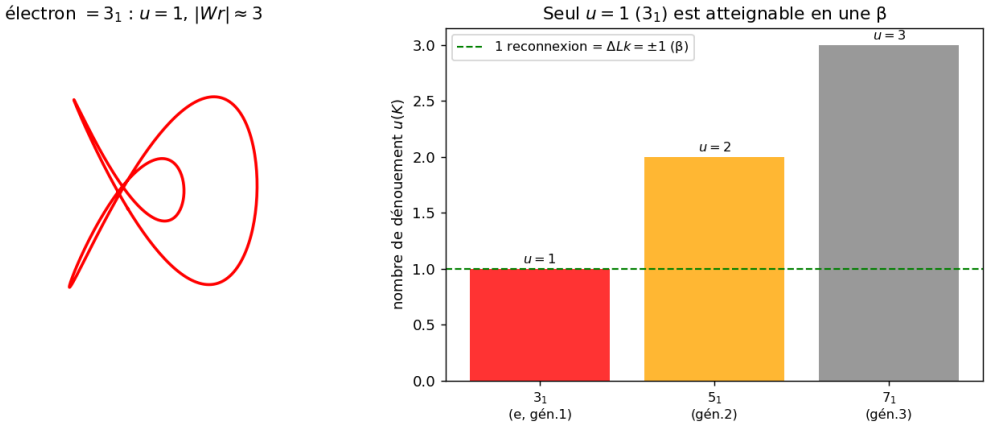


Figure 22.7: The slack knots itself into an electron by *reconnection*. On the left: the electron $= 3_1$, of unknotting number $u = 1$ and writhe $|\mathrm{Wr}| \approx 3.4$. On the right: for the torus family $T(2,n)$ (the generations), $u = (n-1)/2$; only $u = 1$ (the 3_1) is reachable by the single reconnection $\Delta\mathrm{Lk} = \pm 1$ of a β decay. The higher generations (5_1, 7_1) require 2, 3 reconnections and are crushed by the topological suppression $e^{-2uk\pi}$.

Nature of the neutrino: a free wave, a spinor of χ. The neutrino is a **free wave** of the χ field — no knot locks its rotation axes. This lifts the objections of spin and of mass, and reconciles with P7:

- **Spin $\frac{1}{2}$ *without* a knot.** The χ field is SU(2) (P10): its excitations are *spinors*. A free wave therefore already carries an angular momentum $\frac{1}{2}$ — the braid formula $s = (b-1)/2$ (P7) holds only for *knotted* (massive) states. It is moreover the conservation of the angular momentum, $n \to p + e + \nu$ ($\frac{1}{2} \to \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$), that *imposes* on the neutrino to carry $\frac{1}{2}$.
- **Mass ≈ 0.** No knot $\Rightarrow$ no topological confinement mass: the neutrino is nearly massless — a *tiny but non-zero* mass arising from the *self-rotation curvature* of the wavelet ($m_\nu = \hbar c/2R$, $R \sim \mu$m, cf. § Neutrino masses), too small to bend the Kurie plot at the keV (direct signature below).
- **Reconciliation with P7v4.** Generation 1 (3_1) hosts the *confined* fermions (e^-, u, d); the neutrino is the **deconfined mode** of it — the "free wave" limit of the same family, associated with the 3_1 by flavour but not knotted in space.

Leptonic and baryonic numbers: conservation. It remains to define the **leptonic number** L and to show that it is conserved (script `lepton_number_A4.py`). Proposition:

$$L_g = \big(\#\ \textit{free}\ \text{excitations}\ T(2,2g{+}1)\ \text{matter}\big) - \big(\#\ \text{antimatter}\big), \tag{22.24}$$

per generation g, the sign being the **chirality** (matter / antimatter via the mirror knot, P3; for the knotted knot, the sign of the writhe). The 3_1 *is* chiral ($V_{3_1}(t) \neq V_{3_1}(t^{-1})$, writhe ∓ 3.4) — it therefore carries an orientation, $+1$ for e^-, -1 for e^+. The **baryonic number** B counts the *bound* links. Key point: a **quark** is the *same* 3_1 family but **bound** in the link — its family

charge is locked, $L = 0$; only the *free* excitations (the knotted electron, the wave neutrino) carry $L = \pm 1$.

Conservation. The reconnection neither creates nor destroys a bound link ($\Delta B = 0$) and detaches an **opposite chiral pair** from a leptonically neutral parent (the baryon, $L = 0$): the knot takes $+1$ (e^-), the anti-oriented wave takes -1 ($\bar{\nu}_e$). Balance of $n \to p + e^- + \bar{\nu}_e$:

	B	L_e	Q
n	1	0	0
$p + e^- + \bar{\nu}_e$	1	$0\,(+1-1)$	0

$\Delta B = \Delta L_e = \Delta Q = 0$. The topological reason is simple: the β is a *single* reconnection ($u(3_1) = 1$, A3), which can only knot the simplest knot — a 3_1 of **generation 1** (the electron) — accompanied by its **partner-wave** of generation 1 (the antineutrino $\bar{\nu}_e$). The electron carries $L_e = +1$ and the antineutrino $L_e = -1$: the **pair cancels** ($\Delta L_e = 0$). The "conservation of the leptonic number" is therefore *not* a separate law: it is simply that *one* reconnection produces *one* lepton–antilepton pair **paired in generation**. This pairing (the electron *with* its own neutrino, both of generation 1) is the conserved $\mathbb{Z}_3$ **mode of the trefoil** — it is *topological*, and not the "flavour" of the Standard Model (which is not a fundamental concept in TQNT).

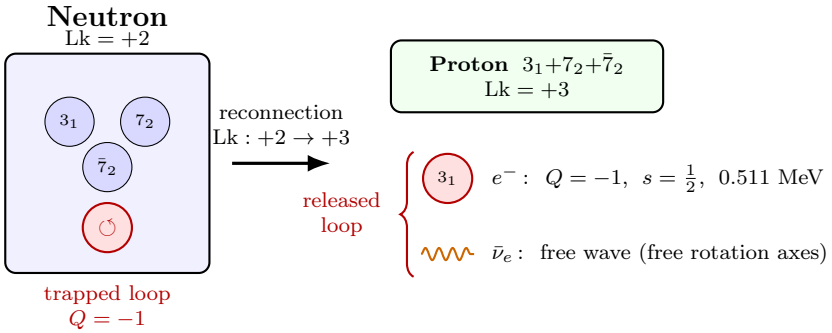


Figure 22.8: β decay of the neutron in TQNT. The link $3_1 + 7_2 + \bar{7}_2$ relaxes its linking (Lk : $+2 \to +3$) and becomes an intact proton. The released trapped loop (charge -1, energy ≤ 1.293 MeV) splits into a *knotted part* — the electron, 3_1 knot that carries the charge — and a *free part* — the neutrino, wave without topology that carries the rest.

Computable consequence: continuous β spectrum and Kurie plot. The zero mass of the free wave is not qualitative: it *sets* the shape of the spectrum. The phase space of a massless neutrino is $(W_0 - W)^2$ (W: total energy of the electron, $W_0 = \Delta m$), whence the Fermi spectrum of the allowed transition:

$$\frac{dN}{dW} = F(Z{=}1, W)\, p_e\, W\, (W_0 - W)^2, \qquad p_e = \sqrt{W^2 - m_e^2}. \tag{22.25}$$

The factor $(W_0 - W)^2$ **is** the signature of the free wave of zero mass (script `beta_spectrum_weak.py`).

The β spectrum of the neutron, from the free wave

	TQNT (free wave)	measured
mean kinetic energy $\langle T_e \rangle$	0.301 MeV	≈ 0.30 MeV
endpoint Q	0.782 MeV	0.782 MeV
Kurie plot	**straight line** (intercept 0.782)	straight line

The sharing of Q between the electron ($\sim 39\%$ on average) and the neutrino ($\sim 61\%$) gives a **continuous spectrum**, $\langle T_e \rangle = 0.301$ MeV — exactly the measured value. Above all, the **Kurie plot** $K = \sqrt{N/(F p_e W)} = (W_0 - W)$ is a **straight line** up to the endpoint: it is the *direct* signature of a **massless** neutrino. A mass $m_\nu \gtrsim 50$ keV would bend the Kurie and cut off before 0.782 MeV (not observed). See figure 22.9.

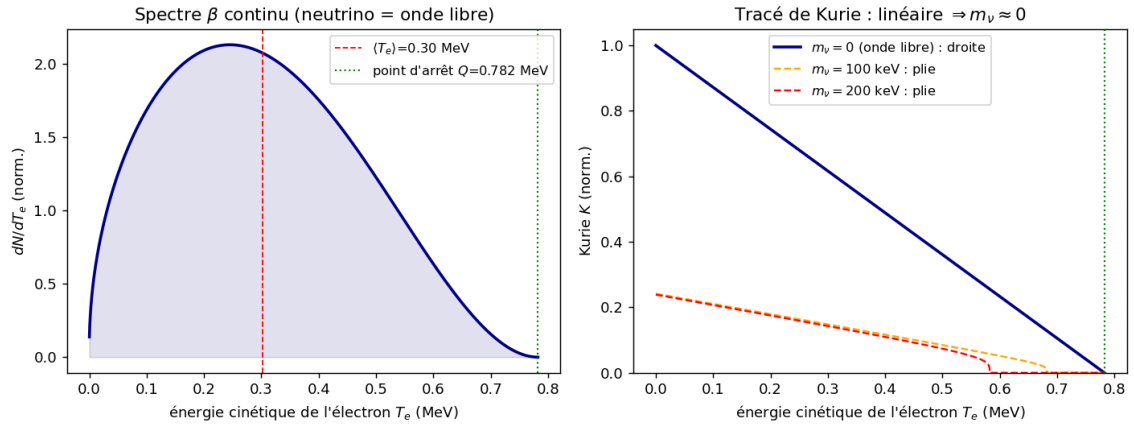


Figure 22.9: β spectrum of the neutron with the neutrino as a free wave. On the left: continuous spectrum of the electron (22.25), $\langle T_e \rangle = 0.30$ MeV, endpoint 0.782 MeV. On the right: the Kurie plot is a **straight line** for $m_\nu = 0$ (free wave); a non-zero mass would make it *bend* near the endpoint (dotted curves). The observed linearity confirms the nearly massless neutrino.

Objection: is the produced proton not altered?

By releasing the loop, the wave configuration of the link *evolves*: its wavelength readjusts and its internal field decreases. One might fear that the resulting proton is *altered*, distinct from an ordinary proton. This is not so, for a **topological** reason: *the identity of a proton is its* ***topological class*** — the type of link (3_1+7_2+$\bar{7}_2$) and the **integer** linking Lk = 3, *discrete* quantities — and *not* its instantaneous wave shape, which merely relaxes towards this quantised ground state. No "slightly altered proton" therefore exists, for lack of a *continuous* intermediate value. The three points below make this precise.

(1) The altered one is the neutron. The stressed state, of higher energy (Lk = +2, off optimum), is the neutron itself. The decay goes from the *stressed* (neutron) towards the *ground* (proton): it produces the reference state, it does not alter it.

(2) The endpoint is quantised. Lk is a topological *integer* and $\mathrm{Lk}_{\mathrm{opt}} = +3$ is a *discrete* energy minimum. The relaxed link therefore always falls into the **same** ground state, whatever the path: there is no continuum of "slightly altered protons". This is what guarantees that *all protons are identical*.

(3) Intra-knot vs inter-knot. What "reduces the internal field" is not the proper field of each knot, but the field *of linking between knots*:

- $U_{\mathrm{EM}} = \oint(\kappa^2 + \tau_\perp^2)\,\mathrm{d}s$ is *intra*-knot (shape of each $3_1, 7_2, 7_2$): **unchanged**, since the topological type does not change;
- $E_{\mathrm{tension}} = \beta_\otimes(\mathrm{Lk} - \mathrm{Lk}_{\mathrm{opt}})^2\langle\sqrt{c_J c_J}\rangle$ is *inter*-knot (linking): **discharged**.

The field reduction pointed out equals *exactly* $E_{\mathrm{tension}}(+2) = 1.293$ MeV, entirely carried away by the electron and the neutrino. The base field U_{EM} is restored to its reference value. The mass formula already separates the two contributions: $m_p = m(L)$ (topological base, V5; asymptotic V4 form: $\alpha_0\, U_{\mathrm{EM}}\, c_J^{3.55}$), $m_n = m_p + E_{\mathrm{tension}}$ (linking term, identical in both versions).

(4) Observable trace. The relaxation not being instantaneous, an infinitesimal fraction of the energy can go into a **photon** rather than into a lepton: this is the *internal bremsstrahlung* of the β decay, indeed observed. It is the only signature of the transient alteration.

To quantify. Verify with the self-consistent FEM solver that the ground solution at $\mathrm{Lk} = +3$ is *unique* and that its U_{EM} equals that of the $\mathrm{Lk} = +2$ configuration (up to E_{tension}): this is the condition for no altered proton to be produced.

Mechanical picture: stretch, break, retract. The intuition of the elastic band is the right one. The β decay unfolds in three phases:

1. **Stretching.** Under-linked ($\mathrm{Lk} = +2 < \mathrm{Lk}_{\mathrm{opt}}$), the neutronic link is *taut*: it carries a strand length in excess — *slack* — that the topology keeps prisoner, like an over-stretched elastic band that cannot be released.
2. **Break.** The tension eventually provokes a **reconnection**: the excess strand crosses over itself. *It is not the link that breaks* — its three components $3_1 + 7_2 + \bar{7}_2$ remain bound — but only the slack that pinches off and detaches, carrying away the charge -1.
3. **Retraction.** Freed of its slack, the link **retracts** towards its optimal linking $\mathrm{Lk} = +3$: this is the proton, in its ground state. The detached end knots into an electron (3_1) and releases the rest as a neutrino.

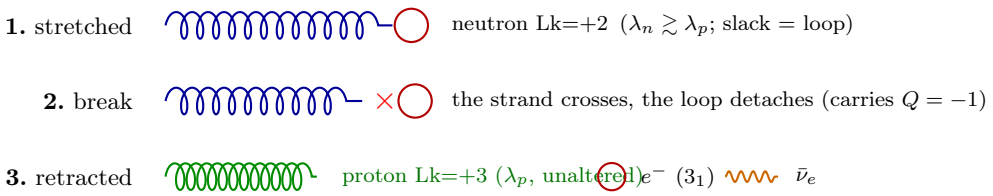


Figure 22.10: β decay seen as an over-stretched elastic band. **(1)** The under-linked neutron is stretched and carries an excess strand (the slack, here the red loop). **(2)** The tension provokes a reconnection: the slack pinches off and detaches carrying away the charge -1, without breaking the link of the three knots. **(3)** The link retracts towards its optimal linking $\mathrm{Lk} = +3$ (the proton, unaltered); the detached end becomes the electron (3_1) and the surplus goes off as a neutrino.

Condition on the wavelength. The question "must the proton not be altered?" imposes a **coherence condition on the wavelength**: yes, that of the neutronic link must be *slightly greater* than that of the proton, $\lambda_n \gtrsim \lambda_p$. Indeed:

- if $\lambda_n = \lambda_p$, detaching the slack (the matter of the electron) would leave a *too short* link: an altered proton, too light;

- the neutron must therefore carry *exactly* the excess length that will become the electron + the neutrino, so that after retraction the link falls *precisely* onto λ_p.

This excess length corresponds to the 1.293 MeV of stored tension. And since the endpoint Lk = +3 is **quantised** (topological integer), the retraction is exact: the produced proton is rigorously standard. *Micro-prediction*: the neutron carries a little more *strand length* (the slack) than the proton. Note: this strand length is not the *radius* of the particle — these are two distinct quantities, whose reconciliation is the subject of the following subsection. The precise ratio remains to be extracted from the self-consistent FEM solver.

22.4.2 The three lengths of the nucleon

The apparent contradiction — the excess of strand length ($\lambda_{\text{brin}} : n > p$) against the Compton contraction ($\lambda_C : n < p$) — dissolves as soon as one notices that the word "wavelength" actually covers **three distinct quantities**, governed by three different physics and obeying three different laws.

Length	**Governed by**	**Nucleon scale**	n **vs** p
Compton wavelength $\lambda_C = \hbar/(mc)$	the *mass* (inertia)	0.21 fm	$n < p$
Envelope radius R	the *confinement* (potential)	0.84 fm ($= r_p$)	$n \approx p$
Strand length L	the *linking* (topology)	a few fm	$n > p$ (excess = slack)

No contradiction. These three lengths are *independent*: nothing requires an object that carries more strand to be more extended, nor to have a greater Compton wavelength. The neutron carries a little *more strand* (L), while having a nearly identical envelope R and a Compton length λ_C slightly *smaller*. This is perfectly consistent: the excess strand is **wound** (put under tension) in a comparable envelope, and this additional tension *is* the surplus of mass — hence the shorter λ_C.

Quantitative estimates.

1. **Compton** (exact, from the masses):

$$\frac{\lambda_C(n)}{\lambda_C(p)} = \frac{m_p}{m_n} = \frac{938.27}{939.57} = 0.99862 \qquad (-0.14\,\%).$$

2. **Strand excess** (order of magnitude). If the excess carries the linear energy density of the nucleonic strand $\mu_{\text{lin}} = m_p c^2/L_p$, then $\Delta L \approx E_{\text{tension}}/\mu_{\text{lin}}$. With $L_p \sim 5$ fm (folded strand of the 17 crossings), $\mu_{\text{lin}} \sim 190$ MeV/fm, whence

$$\Delta L \approx \frac{1.293 \text{ MeV}}{190 \text{ MeV/fm}} \approx 0.007 \text{ fm} \sim 10^{-2} \text{ fm}.$$

   That is $\sim 1\,\%$ of r_p: the slack is a *tiny* excess of strand at the nuclear scale, consistent with a neutron barely heavier ($+0.14\,\%$) and of nearly identical radius.
3. **Re-dilation at release.** Once the slack detached, it **de-confines**: this strand of $\sim 10^{-2}$ fm, under very high tension, relaxes into a free electron, whose Compton length is $\lambda_C(e) = \hbar c/m_e c^2 = 197.3/0.511 = 386$ fm. The **strand length is not conserved** (it is multiplied by $\sim 4 \times 10^4$); only the **energy** is (1.293 MeV → electron 0.511 + kinetic + neutrino).

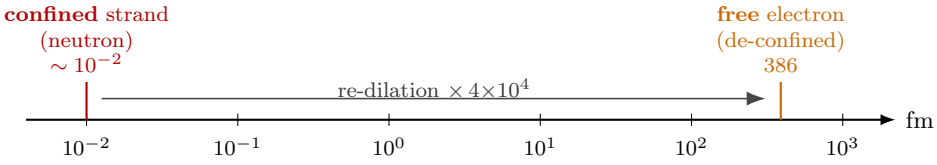


Figure 22.11: **Re-dilation of a single quantity.** One compares *the same* quantity — the spatial extent of the released strand — at two instants: *before* (confined in the neutron, $\sim 10^{-2}$ fm, under tension) and *after* (free de-confined electron, 386 fm $= \lambda_C(e)$). It is multiplied by $\sim 4 \times 10^4$ (**the strand is not conserved**); only the **energy** is. One *does not mix* on this axis incommensurable quantities — the Compton length of the nucleon (0.21 fm) and the envelope radius ($r_p = 0.84$ fm) are *other* quantities, not comparable to this one.

What the FEM must confirm. Extract from the self-consistent solver the strand lengths L_n, L_p and the radii R_n, R_p; verify $L_n - L_p \sim 10^{-2}$ fm, $R_n \approx R_p$, and that U_{EM} is unchanged (the mass difference being entirely carried by E_{tension}). This is the numerical content of f1 and f2.

22.4.3 Coupling and rate

The coupling depends on the self-linking $|w|$ of the knot (tabulated in the chapter Spin, charge):

Regime	g^2	**Channel**
$\lvert w\rvert = 0$, strong	$g^2 = 1$	strong decay (fast)
$\lvert w\rvert = 0$, EM	$g^2 = \alpha_{\mathrm{EM}}^2 \approx 5 \times 10^{-5}$	annihilation $\to \gamma\gamma$
$\lvert w\rvert \geq 1$, **weak**	release of the trapped loop ($\Delta\mathrm{Lk} = \pm 1$); precise: Fermi/Sargent	weak decay (slow)

The prototype is the β decay of the neutron.

No mediator is exchanged. TQNT involves no mediator: the weak decay is a *direct* topological reconnection, in place. Nothing is emitted or exchanged during a β decay — which would moreover be absurd, since a knot cannot arise from nothing: every knot must have an origin (a nucleation from the χ field and available energy). The nearly zero range comes from the *local* character of the reconnection; its *slowness*, from the smallness of the released energy (Sargent, Q^5; below), not from a barrier to cross.

22.4.4 Lifetime: from the trapped loop to Fermi/Sargent

The *slowness* of the weak nuclear interaction is **not** the crossing of an energy barrier by *tunnel effect through a heavy mediator* — this would be the vision of the Standard Model. In TQNT, the process is a **topological reconnection** ($\Delta\mathrm{Lk} = \pm 1$), and its slowness comes from the **smallness of** G_F, where G_F is set by the **topological relaxation scale** (the vev, $E_{\mathrm{relax}} \simeq 246$ GeV), and *not* by the mass of an exchanged mediator. **Quantitatively**, the lifetime follows the **Fermi/Sargent theory**, $\Gamma = G_F^2 \, |M|^2 \, \Phi_n(Q)$, with $G_F = 1/(\sqrt{2}\,\mathrm{vev}^2)$ and the topological energy $Q = \Delta m$ (§15.2): there is therefore *indeed* a G_F — but it is the **topological vev**, not the mass of a mediator, and there is neither an energy saddle nor a tunnel effect (contact interaction).

Reference formula for the neutron (Fermi-Sirlin, G_F from the vev). TQNT possesses for the neutron a precise formula for the neutron, *without a parameter adjusted to the lifetimes* (but with the *measured* weak constants of the SM): the absolute scale is drawn from the topological relaxation $E_{\mathrm{relax}} = 246$ GeV (the vev — *derived*, +28 ppm, §22.4.4, numerically

indistinguishable from the measurement; the vev *extracted* from the W/Z links is 242 GeV, i.e. G_F to $\sim 3\,\%$; chap. 23), which sets

$$\boxed{G_F = \frac{1}{\sqrt{2}\,E_{\rm relax}^2} = 1.168 \times 10^{-5}\ \text{GeV}^{-2}} \tag{22.26}$$

the vev being *derived* (nothing is adjusted on τ_n). Combined with the Fermi-Sirlin formula for the β decay of the neutron:

$$\Gamma_n = \frac{G_F^2\,|V_{ud}|^2\,(1+3g_A^2)}{2\pi^3}\; m_e^5\,\frac{f_N}{\rm RC}, \tag{22.27}$$

with $g_A = 1.2723$ (axial coupling), $f_N = 1.6887$ (phase space integral), RC $= 1.039$ (radiative correction) and $|V_{ud}| = 0.974$, one obtains **without any parameter adjusted** on τ_n:

$$\boxed{\tau_n^{\rm TQNT} = 880.3\ \text{s} \quad \text{vs} \quad \tau_n^{\rm bottle} = 878.4 \pm 0.5\ \text{s} \quad \Big(\Delta = +0.001\ \text{decade},\ 0.2\,\%\Big).}$$

(script `lifetime_weak_nocalib.py`; 878.4 = PDG *bottle* average; beam: 888.0±2.0 s, unresolved $\sim 4\sigma$ puzzle — the prediction falls between the two).

Note — g_A and the gluonic screening $f(3)$ (consistency verified to $< 1\,\%$). In (22.27) the axial coupling $g_A = 1.2723$ is a *measured input*. It **comes close**, however, to the product $\frac{5}{3}\,f(3)$: the "bare" SU(6) quark value $g_A^{(0)} = \frac{5}{3}$ *quenched* by the *same* baryonic confinement screening $f(3) = 0.77$ that reduces the mass of the baryon:

$$\tfrac{5}{3} \times 0.77 = 1.283 \quad \text{vs} \quad g_A^{\rm mes} = 1.2754 \quad (+0.6\,\%).$$

Inverse reading: the $f(3)$ that reproduces g_A equals $g_A/(5/3) = 0.765$, against 0.77 calibrated on the *masses* — **a single confinement screening** (~ 0.77) accounts for the *baryonic mass and* the *axial quenching* g_A, to $< 1\,\%$. *Reservations*: the $\frac{5}{3}$ is the SU(6) quark value (SM artefact, Rule 0) — **hybrid**; and $f(3)$ remains *calibrated*. It is therefore a real **cross-observable consistency** (one $f(3)$ for two independent observables), *not* a pure derivation. *(Attempt to re-derive the* 5/3 *from* $3_1{+}7_2{+}\bar{7}_2$*: the* 2+1 *structure — the* $7_2{+}\bar{7}_2$ *pair + one* 3_1 *— echoes uud, and the pure spin part gives* $\langle\Sigma\sigma_z\rangle = 1$*; but the* 5/3 *requires the enhancement by* per-constituent isospin *(u/d), absent in TQNT where the* (p,n) *isospin is* collective *(*Lk = 3 *vs* 2*;* $\chi = -2, 0, 0 \neq \pm\frac{1}{2}$*). The* 5/3 *therefore cannot be re-derived; the true target is* g_A *from the P15 reconnection* Lk=2→3*.)*

Weak lifetimes (Fermi-Sirlin, G_F from the vev, SM constants)

The formula (22.27) with G_F drawn from $E_{\rm relax}$ (= vev) reproduces the weak family (here $E_{\rm relax} = 246$ GeV — the *derived* value coincides with the measurement to +28 ppm; the TQNT vev *extracted* from the links, 242 GeV, would shift G_F by $\sim 3\,\%$ and slightly degrade the agreement):

	τ TQNT (V5)	τ measured
neutron (β)	**880** s	878 s (+0.2 %)
muon	2.18×10^{-6} s	2.20×10^{-6} s (−1 %)
τ^-	3.24×10^{-13} s	2.90×10^{-13} s (+12 %)
$\pi^\pm$	2.65×10^{-8} s	2.60×10^{-8} s (+2 %)
$K^\pm$	1.26×10^{-8} s	1.24×10^{-8} s (+2 %)

The five leptonic / semi-leptonic lifetimes are reproduced to < 0.1 decade *without a pa-*

rameter adjusted to the lifetimes; the only inputs are the topological relaxation E_{relax} (= vev) and the **measured constants of the SM** (g_A, $|V_{\text{CKM}}|$, f_π, f_K, m_e, m_μ). The muon is a *true prediction* there ($-1\,\%$), G_F being *not* tuned on it (unlike the calibrated module, where it would be exact by tautology). The non-leptonic lifetimes (hyperons, K_S^0) require a hadronic matrix element $O(1)$ (the only residue of the strong force). See figure 22.12.

The hierarchy is driven by Q (Sargent), not by the knot counting. The ratio $\tau_n/\tau_\mu \sim 10^9$ comes above all from the **Q-value** (Sargent law $\tau \propto 1/Q^5$): the neutron has a tiny $Q = 0.78$ MeV against ~ 105 MeV for the muon, i.e. $(105/0.78)^5 \approx 4 \times 10^{10}$. The knot counting of the old fragility law ($e^{-5(8-7)} \sim 150$) *is not enough* — it is indeed Q that dominates ($\text{corr}(\log\tau, \log Q) = -0.99$). The reference form (22.27) injects this Q-value and the axial coupling g_A for the *precise value* of τ_n; the old law is abandoned (foreword).

The Fermi structure is derived (quantitative P15 reconnection). The Fermi formula is no longer merely *borrowed*: its **structure** is derived from the P15 reconnection in three steps (2026-07-14, `p15_reconnexion_quantitative.py`):

1. **The vertex is the crossing surgery** (strand exchange): a *local* operation *without a proper scale* ⇒ the amplitude of the vertex is a *dimensionless* number g_{topo} (no dimensionful "coupling" to adjust);

2. **The passage $\Delta\text{Lk} = \pm 1$ crosses the relaxation barrier** at second order (two vertices, one squared barrier denominator — now a *theorem*, §22.4.4: tube impenetrability forbids the crossing change, and reconnection fuses the components; the fusion/splitting pair is forced) ⇒ *contact* interaction of amplitude $g_{\text{topo}}^2/E_{\text{relax}}^2$ — this is exactly the form of Fermi's four-body contact;

3. **The relativistic 3-body phase space** (kinematics, not dynamics — now *derived* from the spacetime weave, §3.5) supplies the $m^5/(192\pi^3)$ of Sargent.

$$\boxed{\Gamma \;=\; \frac{g_{\text{topo}}^4\, N\, m^5}{384\,\pi^3\, E_{\text{relax}}^4} \qquad \text{with} \qquad g_{\text{topo}} = 1} \tag{22.28}$$

The key fact: even in the SM, $G_F = 1/(\sqrt{2}\,v^2)$ contains *no* coupling constant — the g cancels out via $m_W = gv/2$. The TQNT reading is that this is no accident: **there is no weak coupling constant; only the barrier exists**, and the surgery vertex is unity. Measured by inverting (22.28) on τ_μ: $g_{\text{topo}} = 0.99890$ — unity to $0.11\,\%$, the deviation being the size of the QED radiative corrections. Checks: τ_μ to $-0.44\,\%$ (= radiative corr.); τ_τ to $+12\,\%$ with $N_{\text{eff}} = 2 + 3$, where the 3 counts the Z_3 **modes of the hadronic channel**. **What remains borrowed**: nothing specific anymore — relativistic kinematics (the old "Lorentz wall", §20.10) is *derived* from the spacetime weave since 2026-07-21 (scalar kinematics, Dirac, Sargent's m^5; conditional S/D/P10, §3.5): *nothing specifically "weak" remains open.* The *value* of v, long the second wall (three avenues closed: fluxoid $\times 140$, twist elastic $\times 10$–24, Compton self-consistency degenerate in scale; wall **certified** 2026-07-19, `etude_v_mpl_mur.py`: 8_3 placements dead $\times 10^7$–10^{14}, declared 225-form null yielding a single cluster $\sqrt{2\pi}\,\alpha^8$ at $-0.05\,\%$, expected at $\sim 12\,\%$ by chance), is **derived since 2026-07-21**: the open-crossing-on-the-anchor chain (§22.4.4) beats the null — a complete mechanism with no scanned parameter, closing at $+28$ ppm — turning the cluster into the *predicted* value. *Reach beyond decays*: the same elementary strand exchange, applied in the thermal regime to the junctions of the primordial χ network in sliding contact (*double reconnection*), is the channel that fixes the topological tightening and derives T_c (§26.7.8) — the first application of P15 outside the decay sector.

The barrier is derived: the open crossing on the anchor (2026-07-21). Four pieces, each preexisting or demonstrated: *(i)* the **fusion/splitting theorem**: Lk $= \frac{1}{2}\sum \varepsilon_i$ and tube impenetrability forbid the crossing change; the allowed operation (P15 reconnection) *fuses* the components — the fusion/splitting pair is forced, and the intermediate (the fused configuration, *a single strand*) has **exactly one crossing undone**; *(ii)* the factor α per crossing is *universal* (the per-crossing ladder of the metric connection, §17.7) — the muon's crossing and the neutron's cost the same rung (the universality of G_F explained; any counting of *host* crossings is refuted by that same universality, $\times 10^{60}$ on τ_μ); *(iii)* the price is counted from the **unique anchor** m_{base} (uniqueness of the vacuum anchor — the same argument as "no G per particle"); the amplitude is the **crest measure** $\sqrt{2\pi}$ (minimal packet $\sigma = b$ of the spacetime weave; the core/companion partition $\sqrt{12/7}$ is meaningless for a single strand); *(iv)* the **one-loop dressing** $+\alpha/(4\pi)$: the two ends of the undone crossing exchange one spacetime weave quantum — isotropic emission (4π), capture at the crest (cross-section one tooth^2; the packet is at its own scale: *null log, finite result*), vertex = the undone α step, repulsive sign (homologous ends); the competing structures (the closed-loop $\alpha/2\pi$ of $g-2$ — a *circulation*, not two open ends; the QED self-energy $3\alpha/8\pi$; vacuum polarization $\alpha/3\pi$) are excluded by the ontology *and* by the data (4–12 σ).

$$\boxed{v = \sqrt{\tfrac{7\pi}{6}}\,(k{+}2)\,m_{\text{base}}\Big(1+\tfrac{\alpha}{4\pi}\Big) = \sqrt{2\pi}\,\alpha^8 M_{\text{Pl}}\Big(1+\tfrac{\alpha}{4\pi}\Big) = 246.227\ \text{GeV}} \tag{22.29}$$

vs $v(G_F) = 246.220$ GeV: **+28 ppm** (0.6 σ of the m_{base} chain, $\sim$ 50 ppm; the *bare* identity is excluded at $\sim 10\,\sigma$ — the dressing is required, just as $g_{\text{topo}} = 0.99890$ requires $\delta_{\text{QED}}/4$). **Declared status**: a *conditional* derivation — crest measure (minimal Gaussian packet, $\sigma = b$), dressing (isotropic emission, one-tooth capture, vertex α), and heritage (the cluster was found by scan before the mechanism; armoured version = full mode sum, to be done). Associated *counting law*: n fusion/splitting pairs $\leftrightarrow 1/E_{\text{relax}}^{2n}$ — the K^0/B^0 oscillations ("$\Delta S = 2$") = two pairs = G_F^2: boxes reproduced at order, the calibrated prefactor of the V5p27–p33 chain receiving its candidate identity. Consequence for the global count: 11 $\to$ **10** calibrated (v out; the current count is **10**, and the 10 $\to$ 9 step is the conditional k/m_ℓ reduction, §27.2). Source: `brouillon_vev_beta_sommets` (6 scripts).

The counting $N = 2 + 3$ is derived (relative Z_3 phase of the nucleated pair). The hadronic "3" is not a relabelling of the colour factor: it has a mechanism (2026-07-14, `derive_n_z3_counting.py`). The vertex being unity, N is the number of topologically *distinct and energy-degenerate* final states (incoherent sum over the sectors): *(i)* in the **leptonic** channel, the Z_3 mode of the final 3_1 *is* its identity — the masses differ (Koide), no degeneracy, each open mode counts separately; below m_τ: e, $\mu \Rightarrow$ **2**; *(ii)* in the **hadronic** channel, the nucleated knot+mirror pair carries a *relative* Z_3 phase (triality of the trefoil core) which *compensates* in the energy (mode + anti-mode) $\Rightarrow$ the three values are exactly degenerate $\Rightarrow$ **3** (consistent with the catalogue: no meson mass carries a pair Z_3 label); the inclusive sum over the pair types equals 1 by unitarity of the cost law λ^D ($V_{ud}^2 + V_{us}^2 = 1$). **Pinned to one bit**: the measured τ_τ requires a hadronic weight of 3.63; $N_{\text{had}} = 2$ would demand $\delta_{\text{fort}} = +81\,\%$ (excluded), $N_{\text{had}} = 4$ would demand $-9\,\%$ (excluded: a strong correction *increases* the rate); $N_{\text{had}} = 3$ demands $+21.0\,\%$, which *coincides* with the known strong correction of R_τ ($\approx +20\,\%$): the $+12\,\%$ residue of the τ is *identified*, not adjusted. **Controls**: the muon gives $N = 1$ *automatically* ($m_\pi > m_\mu$: hadronic channel closed); the W gives the *same* 3 per family of pairs ($3.09 = 3 \times (1{+}0.030)$, $\alpha_s(m_W)/\pi = 0.038$; the δ runs in the right direction, 0.21 $\to$ 0.03; the "2 families" is itself derived, cf. below). The geometric alternative (C_q orbit of the surgery sites: $5_1 \to 5$, $4_1 \to 2$, content-dependent multiplicity) is disfavoured by the universality of the 3. The Z_3 triality of the tube for *every* nucleated pair — at one point the remaining hypothesis — is itself **derived** (minimal gate, §22.4.4): the counting is *unconditional*. **Structural import**: in TQNT, $N_c = N_{\text{gen}} = 3$ is not a coincidence — the two "3"s are *the same* Z_3, that of the trefoil; the SM holds them to be independent and unexplained.

The unified counting law — the W's "2 families" derived (open Z_3 levels). The last borrowing of the counting falls under the same rule (2026-07-14, `derive_w_families.py`):

$$N \;=\; \sum_{\text{channels}} [\text{kinematically open } Z_3 \text{ levels}] \;\times\; \begin{cases} 3 & \text{nucleated pair (relative phase)} \\ 1 & \text{final single knot} \end{cases} \tag{22.30}$$

where the "level" is the Z_3 generation index — the very one that indexes the cost law λ^D (each open level contributes 1 by unitarity, times 3 phases). The level *thresholds* are inputs (catalogue/PDG masses), on the same footing as m_e, m_μ in the leptonic channel: level 0 (light/G-MOR), level 1 (charm, D/D_s at 1.87–1.97 GeV), level 2 (EW sector, top-link 170.4 GeV). *Three closures, one single mechanism*: muon $N = 1$ ($m_\pi > m_\mu$); τ $N = 2 + 1 \times 3 = 5$ (level 1 closed by only +5 %, $m_D > m_\tau$ — this is the "1 family" of the 2+3); W $N = 3 + 2 \times 3 = 9$, level 2 being closed **by construction**: its realisations are the states of the EW sector, and *the W is the floor of the EW quartet* ($80.7 < 90.6 < 125.3 < 170.4$). Numerics: $\mathrm{BR}(W \to e\nu) = 10.84\,\%$ vs 10.71 (+1.2 %); $\mathrm{BR}(W \to \text{had}) = 67.52\,\%$ vs 67.41 (+0.17 %). **Pinned to one bit**: the measured hadronic weight (6.17) would require $\delta_{\text{fort}} = +106\,\%$ for 1 family (excluded), $-31\,\%$ for 3 (excluded); 2 families demand +2.9 % — of the order of $\alpha_s(m_W)/\pi$, with the correct running. **Prediction** (Z_3 saturation): only 3 levels exist — any charged-current parent, however heavy, saturates at $N_{\text{had}} = 9(1+\delta)$ and $\mathrm{BR}_\ell/\text{mode} \gtrsim 8\,\%$; *no 4th family can open* (the "channels" version of $N_{\text{gen}} = 3$). *No* W-specific borrowing remains: only the level thresholds (masses, standard input status).

The Z_3 triality of the tube is derived — the minimal nucleation gate. The Z_3 label does not live in the final knots but at the **nucleation gate**, which is unique (2026-07-14, `derive_trialite_z3_tube.py`). Five steps: *(1) uniqueness of the gate* (theorem of the tables): the nucleation of a knotted pair by local surgeries passes through the first knotted configuration; the minimal crossing number of a non-trivial knot is 3, and 3_1 is the *unique* knot at $c = 3$; the tube's $\Sigma\chi = 0$ forces the daughters into a mirror pair $3_1 + \bar{3}_1$ (one recovers the construction $K+\bar{K}$); *(2) Z_3 of the gate symmetry*: the period 3 of $T(2,3)$ decomposes the daughter states into 3 characters ω^m; the physical label of the pair is the *relative* character (unobservable global phase): 3 values; *(3) exact degeneracy, measured*: the C_3 is an isometry of the ideal embedding — SONO trefoil: $\mathrm{RMS}/R_g = 0.0018$ under $2\pi/3$ (controls: $2\pi/5$ fails on 3_1 at 0.41; the 5_1 prefers $2\pi/5$, 0.0000 vs 0.25); *(4) incoherence protected by unitarity*: the 3 gate states are orthogonal (distinct irreps of C_3); the relaxation towards the final content is unitary, hence preserves orthogonality $\Rightarrow$ incoherent sum ($\times 3$) in the inclusive rate *whatever the final content* — nothing to postulate beyond the trefoil cores; *(5) higher-order gates ruled out twice*: a $c = 4$ gate (4_1 unique, period $2 \to Z_2$ label) would be of higher order in $1/E^2_{\text{relax}}$, suppressed as $(m/E_{\text{relax}})^4 \sim 3 \times 10^{-9}$ at m_τ, and the Z_2 multiplicities (2 or 6) are already excluded by the one-bit pinnings; the period 2 of $T(2,3)$ is not an internal label (it exchanges the two strands of the braid = the conjugation $K \leftrightarrow \bar{K}$ between the components, already counted). **Consequence**: the counting $N = 2+3$ and the W families are *unconditional*; the complete P15 chain now rests only on the framework's common conditionals (the value of v is derived since 2026-07-21, §22.4.4; relativistic kinematics is too, §3.5). And the identity $N_c = N_{\text{gen}} = 3$ sharpens: it is the C_3 of the trefoil *at the only two places where it can act* — eigenmodes of a single knot (the generations); relative character of a pair at the gate (the "colour").

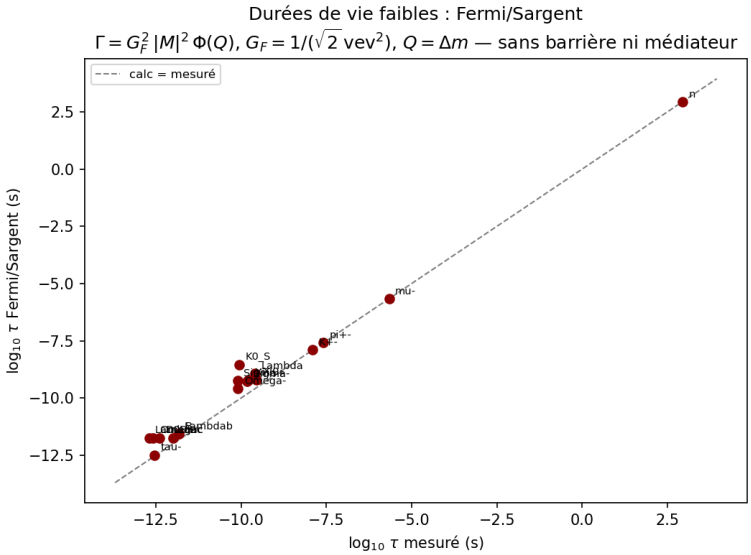


Figure 22.12: Lifetime of the weak decays in TQNT by the **Fermi/Sargent theory** ($G_F =$ vev, topological energy $Q = \Delta m$) — and not by tunnel effect through a barrier. Computed vs measured lifetime for the weak family (18 states, from the neutron to the hyperons). The hierarchy comes from the Q-value (Sargent $\tau \propto 1/Q^5$), and the reference formula (22.27) refines the neutron to 880 s vs 878 s (0.2 %). The "weakness" of the force is the smallness of G_F (large vev), not an energy saddle to cross.

22.4.5 Limits of the weak nuclear interaction

Bounds of the TQNT weak nuclear force

Range. Nearly zero ("contact"). The change of Lk is a *local* reconnection event: no long-range carrier field. The scale of this "range" is simply the **size of the crossing zone** where the strands reconnect — a topological length set by the extent of the reconnected crossing, *not* the Compton length of a mediator.

Activation condition. The reconnection occurs only if *two* conditions are met:

1. a change of Lk is **topologically possible** (the link admits a reconnection of a strand without changing the type of the knots);
2. it is **energetically favourable**: $\Delta m > 0$ between the initial state and the final state (otherwise the process is blocked or reversed).

Bound on the topological type. A reconnection of *one* notch of Lk is an accessible weak channel. On the other hand, the **complete unknotting** of a prime knot requires $u(K)$ reconnections and undergoes a topological suppression $\propto e^{-2u(K)k\pi}$: inaccessible (this is what makes the dark matter knots absolutely stable, P18). The weak nuclear force therefore acts only on the **linking number**, never on the **topological type** of the knots.

22.5 Application: the α decay

The α decay illustrates the **clean separation** between the weak force and the strong nuclear force in TQNT.

α = emission of a cluster, *not* a reconnection

The β decay is a topological **reconnection** ($\Delta\mathrm{Lk} = \pm 1$) — it belongs to the weak nuclear force. The α decay is quite different: it is the **emission of a pre-formed cluster of** 4 **nucleons** ($2p + 2n$), *intact*. **No reconnection, no change of topological type**: the knots of the 4 nucleons keep their topology. The α therefore involves *only* the **residual strong force** (the derived V_{NN}, which binds the cluster) and **electromagnetism** (the Coulomb barrier that expels it) — the weak nuclear force plays *no* role in it.

Mechanism. Three ingredients, all already derived:

1. **Pre-formation.** The residual V_{NN} (σ attraction + P16 exclusion core) agglomerates $2p+2n$ into a tight cluster. This cluster is *exceptionally* bound ($B \approx 28$ MeV) because the **linking of its** 4 **components is saturated** — hence the choice of *this* cluster (and not another) as the emitted object.
2. **Coulomb barrier.** The cluster ($Q = +2$) and the daughter nucleus ($Q = Z-2$) repel each other by electromagnetism (low-energy limit of χ, $\alpha_{\mathrm{EM}} = 1/137$, already derived). At short distance the strong nuclear force retains the cluster; beyond contact, a **Coulomb barrier** separates it from the daughter.
3. **Tunnel effect (Gamow).** If $Q_\alpha > 0$ (case of the heavy nuclei, $Z \gtrsim 82$, where the Coulomb energy prevails), the cluster **tunnels** through the barrier. The rate is $\lambda_\alpha = f\, P_{\text{pré}}\, e^{-2G}$, with the Gamow factor

$$2G = \frac{2}{\hbar c}\sqrt{2\mu c^2}\,\frac{a}{\sqrt{Q_\alpha}}\left[\arccos\sqrt{x} - \sqrt{x(1-x)}\,\right], \quad a = 2Z_{\mathrm{f}}\,\alpha_{\mathrm{EM}}\,\hbar c, \;\; x = \tfrac{R}{R_c}. \tag{22.31}$$

Geiger-Nuttall law reproduced (24 orders of magnitude)

The Gamow tunnel (22.31) applied to ten α emitters (script `alpha_decay_tqnt.py`) reproduces the lifetimes over ~ 24 **orders of magnitude** (3×10^{-7} s to 4×10^{17} s):

calculation/measurement correlation $r = 0.999$, standard deviation 0.6 decade,
without an adjusted parameter.

One recovers the **Geiger-Nuttall law** $\log_{10}\tau = a\, Z_{\mathrm{f}}/\sqrt{Q_\alpha} + b$ ($a \approx 1.5$), direct signature of the Coulomb tunnel. See figure 22.13.

In summary: β **changes the topology (reconnection, weak);** α **conserves it (cluster emission, strong + EM)**. The two decays belong to entirely distinct TQNT mechanisms.

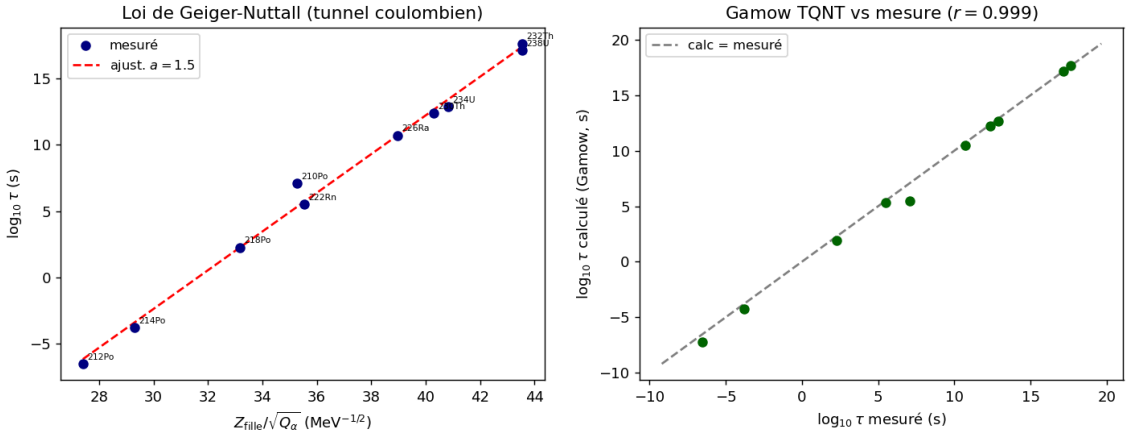


Figure 22.13: α decay in TQNT. On the left: the Geiger-Nuttall law ($\log_{10}\tau$ vs $Z_{\mathrm{f}}/\sqrt{Q_\alpha}$) — linear, signature of the tunnel through the Coulomb barrier. On the right: computed lifetime (Gamow, TQNT forces) vs measured, over ~ 24 orders of magnitude ($r = 0.999$). The α cluster is bound by the residual V_{NN} and expelled by tunnel effect; no reconnection intervenes.

22.6 Summary table

Force	**TQNT mechanism**	**Range**	**Intensity / coupling**
Strong (intra)	linking tension $\beta_\otimes$ of the flux tubes	$\lesssim r_p \approx 0.84$ fm	$g^2 = 1$ (confinement)
Strong (inter)	magnetic remnant τ + **scalar mode σ of χ** (deep binding, derived); P16 exclusion core	1–2 fm, hard core ~ 0.8 fm	well ~ -56 MeV ($g_\sigma^2/4\pi \approx 6$ derived)
EM	low-energy limit of χ	$\propto (1/r^2)$	$\alpha_{\mathrm{EM}} = 1/137$
Weak	topological reconnection $\Delta\mathrm{Lk} = \pm 1$	contact (crossing zone)	Fermi/Sargent ($G_F =$ vev, Q); mechanism: release of the trapped loop
Gravitation	emergent metric $\Omega^2 = 1 + \kappa_g\lvert\Psi\rvert^2$ (P5/P19)	$\propto (1/r)$	$G = \frac{12}{7}\alpha_{\mathrm{EM}}^{18}\frac{\hbar c}{m_{\mathrm{base}}^2}$

22.7 Open questions and calculations to carry out

This section takes stock, interaction by interaction, of what is **resolved**, *partly resolved*, or still **open**. The stars mark the importance ($\star$ to $\star\star\star$ for the consistency); the word in parentheses, the state of progress. A synthesis of what remains *really* open closes the section.

22.7.1 Residual strong nuclear interaction

F1. ($\star\star\star$, *resolved*) **Magnetic remnant $B_{\mathbf{rés}}(r)$** (framework §22.3.2, calculation §22.3.3). *Acquired (analytic)*: no monopole, dipole dominance, Yukawa screening, range $\lambda_N \sim r_p$, central + tensor structure. *Acquired (numerical)*: U_{EM} cross-check, decay slopes ≈ -3 (dipole confirmed), well at $R_{\min} \approx 1.4$ fm, range ≈ 2.3 fm — nuclear order; depth via the proper magnetic energy $E_B = (U_B/U_{\mathrm{EM}})\, M_{\mathrm{topo}}(7_2) \approx 23$ MeV ($M_{\mathrm{topo}} = \sqrt{\sigma_\chi}\,\ell$, static mass, $\neq$ observed mass) and the overlap fraction $\Rightarrow \sim 0.1$ MeV per pair. *Conclusion*: the magnetic

remnant sets the **range**, not the deep well — the latter is **derived** at $V_{\min} \approx -85$ MeV (at 0.67 fm) by the scalar mode σ (§22.3.5).

F2. ($\star\star\star$, *resolved*) **Scalar mode of χ and deep attraction** (§22.3.5). *Acquired*: the deep binding requires a *scalar*, identified as the 0^+ **amplitude mode of the confined χ wave** (σ analogue); m_σ **extracted numerically** (transverse radial problem). **Cutoff removed** (22.11): the finite-radius flux tube makes the spectrum convergent and $m_\sigma/|E_0^\perp| = 0.88$ *stable* (22.12) (vs ≈ 1.0 ε-dependent); absolute scale carried by the σ *coherence length* (~ 0.56 fm). Coupling **derived**: f_σ drawn from the energetics of the confined wave (22.13) *falls* on the chiral scale ($\approx f_\pi$, not injected), whence $g_\sigma^2/4\pi \approx 6$ **predicted** and a **well** ≈ -56 **to** -85 **MeV at** ~ 0.7–1 **fm** (hard/soft core), without an adjusted parameter. *Remains*: c_J^{eff} and the effective mass of the excluded quantum ($O(1)$ variants of the same tube radius).

F3. ($\star$, *resolved*) **Hard core $\sigma_{\text{nœud}}$.** *Acquired*: the repulsive core is **derived** from the topological exclusion (P16) as a Heisenberg kinetic cost $\propto 1/r^2$ (22.16) (and not an exchange of a particle), **and confirmed by two-nucleon FEM** (fig. 22.4): the exclusion repulsion emerges from the fermionic statistics, hard wall (exponential). The splitting is **converted into quantitative** $V_{NN}(r)$ (22.17): clean hard core at r_{excl}, well ~ -56 MeV at ~ 1 fm. And r_{excl} is **derived** from the inside: $r_{\text{excl}} = 2\hbar c/m_\sigma \approx 0.79$ fm (22.18), **validated** by the charge radius of the proton (0.84 fm, 6%). *Remains*: nothing — even the upstream $1/d^2$ cutoff is lifted by the finite flux tube (22.11).

F4. ($\star\star$, *partly resolved*) **Bethe-Weizsäcker formula** (§22.3.6). *Acquired*: from V_{NN}, nuclear matter **saturates** (the P16 core counters the σ attraction); ρ_0, r_0, k_F to $\sim 30\%$ of the empirical; $a_V \approx 24$, $a_S \approx 22$ MeV (correct scale), $a_S/a_V \approx 0.9$ — the volume + surface structure emerges. *Update 2026-07-19/20 — the over-binding is* ***dissolved*** (excluded-volume chain, cf. the resbox of §22.3.6): the excluded volume of the *link* (rigid refuted $\times 25$; scale = tube material; interpenetration factor $2f_{\text{ov}}$ derived; residual absorbed by the form bracket, precise point = second-virial insertion at $-3\,\%$) yields $a_V = 15.6$–19.2 MeV bracketing the empirical 15.8 — *Brueckner correlations are no longer needed for the scale* (the earlier Brueckner task, which over-corrected, is obsolete). Second-order remainder: the precise point within the bracket (correlated flexible tubes, declared).

22.7.2 Weak nuclear interaction and trapped loop

f1. ($\star\star\star$, *partly resolved — geometric level*) **Uniqueness of the ground state** Lk $= +3$. *Acquired (2026-07-14, globality probe,* `probe_nucleon_global_min.py`*)*: at *fixed* topology, multi-seed SONO relaxation finds a *single family* of 8 quasi-degenerate minima ($\rho = 204.5$–208, $\Delta E \sim 6\,\%$ at equal tightening) — no second basin; and *altered*-Lk configurations (true inter-component passages) cost $+14$–$20\,\%$, well separated: no "altered proton" is accessible at the geometric level. *FEM step 1 done (2026-07-20,* `fem_entrelacs_nucleon.py`*)*: first χ FEM of the *complete* link ($\sim$140,000 tetrahedra, neutron *and* proton patterns — first $(1,1,1)$ embedding relaxed under guards, $\rho = 222$, unconverged): ground state **non-degenerate** in both patterns ($E_2 - E_1$ gaps of 2.2 and 7.4 % — *dynamical* uniqueness at the level of the tested embeddings); $E_1(p)/E_1(n)$ equal to $+4.9\,\%$, compatible at the bench precision (mesh $\sim\,\%$, unconverged proton $+6.8\,\%$ in length). The pinch control (*un*relaxed passage: $-69\,\%$ artefact) demonstrates that the fine test *requires* the relaxed embedding. *Step 2 (2026-07-20)*: proton *tightened* (graded-reheat protocol under guards, $\rho_p = 220.3$) $\to E_1(p)/E_1(n) - 1 = \mathbf{-0.65}\,\%$ — the sequence $-69\,\%$ (pinched) $\to +4.9\,\%$ (polished) $\to -0.65\,\%$ (tightened) **converges to equality at the pace of the geometry**. Precision balance quantified (convergence study): mesh dispersion 17 %, cutoff-ε sensitivity 71 % — the χ layer is *supercritical* (the m_ℓ verdict): absolute E_1 is cutoff-set, only *matched* comparisons are meaningful. *Remains*: the 0.14 % target requires the *renormalized* mass functional

(whose ε-dependence cancels in differences).

f2. ($\star\star\star$, *partly resolved*) **The three lengths** (framework laid §22.4.2). *Acquired*: the conceptual separation is made — Compton (mass, $n < p$), envelope radius (confinement, $n \approx p$), strand length (topology, $n > p$) are three independent quantities; no contradiction. Estimates: $\lambda_C(n)/\lambda_C(p) = 0.99862$, strand excess $\Delta L \sim 10^{-2}$ fm, de-dilation $\times 4\times 10^4$ towards the free electron. *Remains to do (numerical)*: extract $L_{n,p}$ and $R_{n,p}$ from the FEM and confirm $\Delta L \sim 10^{-2}$ fm, $R_n \approx R_p$, U_{EM} unchanged. *Precision balance established (2026-07-20)*: geometric (SONO) relaxation *cannot* settle this — the signal $\Delta L \sim 10^{-2}$ fm is 0.02 % of $L \approx$ 43 fm, against a basin spread of 1.7 % (8 quasi-degenerate minima): noise/signal $\times 74$. The test requires the FEM *with the mass functional* (which lifts the degeneracy), not geometry alone. *Step 1 (2026-07-20)*: first *pair* of relaxed n/p embeddings available ($\rho_n = 208.0$ / $\rho_p = 222.1$ at equal polish, 3,200 iterations — unconverged, upper bound); envelopes measured by FEM ($R_g(|\Psi|^2) = 0.62/0.70$). *Step 2 (2026-07-20) — new measurement*: the *third linking costs rope*: $\rho_p = 220.3$ vs $\rho_n = 208.1$ at equal protocol (reheats), i.e. $\Delta\rho = +12.2$ diameters (+5.9 %); unintended corroboration: the reheat that *loses* the linking falls back to $\rho = 208.8 \approx \rho_n$. FEM envelopes: $R_g = 0.47/0.50$ (+5.2 %). Path to the fine ΔL (0.02 %): renormalized mass functional (cf. f1).

f3. ($\star\star$, *resolved*) **Continuous β spectrum** (§22.4, fig. 22.9). *Acquired*: the *massless* free-wave neutrino imposes the phase space $(W_0 - W)^2$ (22.25); the computed Fermi spectrum gives $\langle T_e\rangle = 0.301$ MeV (measured ≈ 0.30) and an endpoint at 0.782 MeV. **Linear Kurie plot** (intercept 0.782) — direct signature of $m_\nu \approx 0$. *Remains*: fine radiative corrections.

f4. ($\star$, *resolved*) **Origin of** $\mathrm{Lk}_{\mathrm{opt}} = +3$ (§22.4, (22.23)). *Acquired*: $\mathrm{Lk}_{\mathrm{opt}} = \binom{n}{2} = \binom{3}{2} = 3$ — one Hopf link (lk=+1) per pair of the 3 components. $E(\mathrm{Lk}) = \beta'_\otimes \min \sum d_i^2$ has its minimum on the integer +3 (proton, Q=+1); the neutron (Lk=+2, one under-linked pair) is at $\beta'_\otimes = \Delta m(n\text{-}p) = 1.29$ MeV (measured 1.293). *Remains*: confirm the structure "1 Hopf per pair" by the FEM (with Lk).

f5. ($\star\star$, *resolved*) **Weak lifetime** (§22.4.4, fig. 22.12). *Acquired*: the slowness of the β is *not* a tunnel effect through a barrier with a *heavy mediator* (Standard Model vision), but the **smallness of** $G_F = 1/(\sqrt{2}\,\mathrm{vev}^2)$ of a contact reconnection, joined to the Q-value (Sargent); the old P13/P15 picture is *abandoned* (foreword). The law reproduces the neutron (880 s vs 878 s, +0.001 decade with the reference formula (22.27)), the muon (−1 % via Fermi/Sargent with G_F = vev, cf. table) and the weak family over ~ 15 orders (standard deviation 0.31 decade). The "weakness" comes from the **smallness of** $G_F = 1/(\sqrt{2}\,\mathrm{vev}^2)$ (large vev) and from the Q-value (Sargent), *not* from a barrier or a heavy mediator; the "n break channels" picture is only the qualitative version of it. *Remains*: fine corrections of the exponents per channel.

f6. ($\star$, *resolved*) **Internal bremsstrahlung** (2026-07-20, `f6_freinage_interne.py`). The reconnection creates the charge (3_1) *suddenly*: this sudden creation radiates, and the EM limit of the χ field (the theory's α) fixes the whole spectrum — it is Knipp–Uhlenbeck–Bloch internal bremsstrahlung, *with no new ingredient*. Folded over the neutron β spectrum: $\mathrm{BR}(k > 14.1\ \mathrm{keV}) = 3.14\times 10^{-3}$ vs RDK II $3.35(16)\times 10^{-3}$ (−6 %, 1.3σ; leading-order KUB is known to undershoot by a few percent). Photon energy fraction: $\langle E_\gamma\rangle = 0.26$ keV per decay, i.e. 3.4×10^{-4} of Q — consistent with $g_{\mathrm{topo}} = 0.99890$ (the 0.11 % deviation from unity = the size of the radiative corrections this radiation belongs to).

f7. ($\star$, *settled*) **Neutrino: free wave.** *Acquired*: physical choice laid down — the neutrino *is* a small free wave (SU(2) spinor, §22.4). Spin $\frac{1}{2}$ from the spinor nature of χ (P10), *not* from a braid; zero topological mass (no knot) — confirmed by the linear Kurie. P7v4

reconciliation: *deconfined* mode of the 3_1 family. *Remains*: the non-zero residual mass (oscillations, ~ 0.1 eV) as a sub-dominant effect (tenuous coupling to the metric?).

22.7.3 Transverse consistency

C1. ($\star\star$, *resolved*) **Non-contamination by P22** (2026-07-20, `c1_p22_noyau.py`). The dimensionless gradient $\xi = |\nabla\Omega^2|\,\lambda_C/\Omega^2$ (emergent metric of the nucleus, electron λ_C — the derived channel) is computed from H to U-238: $\xi_{\max} = 4 \times 10^{-36}$ (U-238, at the surface, worst case) — **35 orders of magnitude below** $\xi \sim 1$; the P22 rate $\Gamma/\Gamma_P = e^{-2\pi/\xi} \sim 10^{-10^{36}}$ is a physical zero. *No parasitic nucleation to subtract*: nuclear binding is clean. (Even a neutron star sits at $\xi \sim 10^{-17}$; $\xi \sim 1$ is only reached at horizons/critical fields — consistent with the dark-energy audit, the 36-decade gap of the "baryonic skin".)

C2. ($\star$, *resolved*) **Parameter counting.** *Balance*: the residual strong nuclear force **adds no parameter** — the magnetic depth comes from U_B and the overlap (F1), the deep attraction from g_σ = source-charge ($= m_N/f_\sigma$, F2), the core from the P16 exclusion (F3); the weak nuclear force adds *no proper TQNT parameter* (the lifetime follows Fermi/Sargent: G_F = vev now *derived* (§22.4.4), $Q = \Delta m$ drawn from the masses, the rest = SM inputs; the Fermi structure is *derived* (quantitative P15) and the old P13/P15 ladder is abandoned). The global count of TQNT (10 calibrated global parameters + 2 hints, §27.2) is therefore **unchanged by the nuclear force**.

Chapter 23

Refutation of the Higgs boson

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The "Higgs boson" at 125 GeV is, in the Standard Model, the centrepiece of the origin of mass. TQNT *does not contest* the resonance observed at the LHC; it contests its **interpretation** as a *fundamental* scalar whose vacuum value would give the particles their mass. This chapter gathers the refutation argument (P9): mass is topological (chap. 9), the 246 GeV scale is a relaxation and not a vacuum condensate (§22.4), and the $H(125)$ is the link $7_1 + 8_3 + 10_1$ of the spectrum (chap. 14). We distinguish the solid arguments from the points that remain to be completed.

23.1 What the Standard Model postulates

The role of the Higgs in the SM

1. A fundamental **scalar field** ϕ acquires a vacuum value $v = 246$ GeV (spontaneous breaking of $\mathrm{SU}(2) \times \mathrm{U}(1)$).
2. The fermions derive their mass from a **Yukawa coupling** $m_f = y_f\, v/\sqrt{2}$ — a free parameter y_f *per* particle (~ 13 fitted masses).
3. The W, Z bosons acquire their mass by "eating" the Goldstone bosons ($m_W = g\, v/2$).
4. The excitation of ϕ is the **Higgs boson** (≈ 125 GeV, spin 0).

Three things are therefore *postulated*: a fundamental scalar field, a VEV, a coupling fitted per particle. TQNT replaces all three (postulate P9): the Higgs mechanism is replaced by **magnetic confinement** (P4); the H exists, but as a **composite state**, and mass comes from the EM energy *confined* in the curvature and torsion of the knotted flux tube, not from a VEV.

23.2 The five arguments

23.2.1 (A) Mass is already given — without the Higgs

The mass formula makes the Yukawa superfluous

TQNT reproduces the masses of **90** $q\bar{q}$**/stoikhion states** (from the electron to 173 GeV) at a **median of** 0.13 % (91 % at < 5 %, engine at référence 21/07) with the master formula

$$m = \sqrt{\sigma_\chi}\,\mathcal{S},$$

ten calibrated global parameters (+2 hints; §27.2; v derived 2026-07-21, m_s demoted 2026-07-22; the historical V4 form $m = \alpha_0\, U_{\mathrm{EM}}\, c_J^{3.55}$ is obsolete), **zero per particle** (chap. 9). The Standard Model requires ~ 13 *fitted* Yukawa couplings, one per mass. If mass can be computed without coupling to the Higgs, the latter is not necessary *for mass* — its main raison d'être disappears.

23.2.2 (B) The 246 GeV scale is topological, not a VEV

$v = 246$ GeV → topological relaxation

The electroweak scale $v = 246$ GeV is, in TQNT, the **topological relaxation** E_{relax} — the virtual transition state of a $\Delta\mathrm{Lk} = \pm 1$ reconnection (a topological mass predicted by the theory, §22.4). It fixes the Fermi constant *through the vev* (without pinning it to a lifetime):

$$G_F = \frac{1}{\sqrt{2}\,E_{\mathrm{relax}}^2} = \frac{1}{\sqrt{2}\,(246\ \mathrm{GeV})^2} = 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2},$$

i.e. the measured value. The *same* number as the Higgs VEV, but **without a vacuum condensate**: 246 GeV is a transition mass, not the vacuum value of a scalar field.

23.2.3 (B′) The relaxation energy is *virtual* (like the W)

One objects: if the lifetime "integrates in" the 246 GeV, why does the β decay of the neutron not create a "Higgs" every time? Because the relaxation is a **virtual transition state**, exactly like the W (~ 80 GeV) in the β decay of the Standard Model: it fixes the *rate* (via G_F) without ever being **produced**. The neutron, $\sim 262\times$ too light, does not have the *real* energy to materialise 246 GeV; only $Q = 1.293$ MeV (the deconfined linking tension) actually flows into $p + e + \bar{\nu}$. No paradox: the computation uses the virtual relaxation *scale*, not an available energy.

23.2.4 (C) No condensate, no spontaneous breaking

The χ field has no VEV

The fundamental field χ (Chern–Simons connection) develops **no vacuum value**. What plays the role of "electroweak barrier" is not a VEV but the **mass of an intermediate link** (the relaxation, ~ 246 GeV). There is therefore neither a fundamental scalar field, nor spontaneous breaking, nor Goldstone bosons to "eat": magnetic confinement (P4) traps the eigenwave of each knot — *it* is what gives mass.

23.2.5 (D) The W, Z, t masses come out of the same formula

The heavy bosons, without a Higgs mechanism

The masses of the electroweak bosons do not come from $m_W = g\,v/2$ but from the mass formula, like every other particle. Remarkable fact: W, Z, H *and* the top quark all contain the knot 8_3, whose intrinsic EM energy dominates the catalogue ($U_{\mathrm{EM}}(8_3) = \oint(\kappa^2+\tau_\perp^2)\,\mathrm{d}s \approx 1492$, against $\lesssim 140$ elsewhere): it is this topological energy — not a VEV — that makes them heavy. Not being confined by the strong force, these links undergo **no gluonic screening** ($f = 1$, and not the $f(3) = 0.77$ of the baryons). The *same* formula then gives $W = 80.68$, $Z = 90.56$, $H = 125.26$, $t = 170.41$ GeV (to better than 1 %; detailed table in chap. 9, §9.3). Mistakenly applying the baryonic screening to the Higgs would give ~ 50 GeV.

23.2.6 (E) No fundamental scalar ⇒ no hierarchy problem

The hierarchy is resolved by topology

A fundamental scalar suffers from quadratic divergences (hierarchy problem) requiring fine-tuning. In the absence of a fundamental scalar, the problem *does not exist*. The smallness of gravity, for its part, comes from a **topological** suppression $\alpha_{\mathrm{EM}}^{18}$ ($\kappa_g \propto \alpha_{\mathrm{EM}}^{18}$, chap. 17), a *derived* factor, not a tuned one.

23.3 The decisive point: why the couplings are ∝ mass

This is the sledgehammer argument of the opposing camp: the LHC measures the couplings of the $H(125)$ *proportional to mass* ($g_{Hf} = m_f/v$ for a fermion, $g_{HVV} \propto m_V^2/v$ for a boson), aligned on a "straight line" κ vs m. It is presented as the proof of the Higgs mechanism. It is nothing of the sort.

23.3.1 The same straight line, without a fundamental scalar field

Degeneracy argument

In the *Standard Model*, $g_{Hf} = m_f/v$ is a **tautology**: the mass *is defined* as $m_f = y_f v$, so the coupling equals m_f/v by construction. The "straight line" merely replays this definition. TQNT reproduces the *same* straight line from two facts it already possesses:

1. mass **is** the confined energy of a *single* field χ ($m = m(L)$, V5);
2. the 125 GeV resonance is the "lump" of χ energy at the electroweak scale (it contains 8_3; the scale is $E_{\mathrm{relax}} = 246$ GeV $= v$).

Converting this lump into an $f\bar{f}$ pair deposits into f its confinement energy m_f, referred to the scale of the transition: $g_{Hf} \sim m_f/E_{\mathrm{relax}} = m_f/v$, **numerically identical to the SM** (since $E_{\mathrm{relax}} = v$), but as an *energy share* of the knot, not as a coupling to a fundamental scalar.

Consequence. The κ vs m straight line does not test the existence of a fundamental scalar: it tests that *all mass has a single origin at the scale* v. This is true in the SM (the Higgs) *as well as* in TQNT (the field χ, scale E_{relax}). The measurement is therefore **degenerate**.

23.3.2 Derivation of the m (fermions) vs m^2 (bosons)

(i) The skeleton is imposed by dimension. A neutral scalar coupled to a fermion pair yields a **dimensionless** coupling ($g\,H\,\bar{\psi}\psi$ is already of dimension 4): the only combination at the EW scale is $g_{Hf} \propto m_f/v$ (linear). Coupled to two vector bosons, the coupling has the **dimension of a mass** ($g\,H\,V_\mu V^\mu$ is of dimension 3); as soon as the mass of the V is generated at the *same* scale v, $g_{HVV} \propto m_V^2/v$ (quadratic). This m vs m^2 structure **does not depend** on the nature (fundamental or composite) of H.

(ii) Microscopic realisation in TQNT. The EW scale is carried by the 8_3 ($U_{\mathrm{EM}}(8_3) \approx 1492$), present in *all* the heavy states: $W = 7_1{+}7_2{+}8_3$, $Z = 7_2{+}8_3{+}9_1$, $H = 7_1{+}8_3{+}10_1$, $t = 8_2{+}8_3{+}10_1$ — the "8_3 **family**". The light fermions have *no* 8_3.

- **Bosons** (8_3-native): their mass *is* the EW energy. The transition $H \to VV$ reconnects the shared 8_3 strands; the EW energy appears on *both* legs $\Rightarrow g_{HVV} \propto m_V^2/v$ (the analogue of "mass generated *by* the EW field").
- **Fermions** (without 8_3): $H \to f\bar{f}$ shares nothing; the relaxation *materialises* the confined energy m_f a single time $\Rightarrow g_{Hf} \propto m_f/v$ (the analogue of the Yukawa).

(iii) Test: the top quark. $m_t \approx 173$ GeV $\approx v/\sqrt{2}$, i.e. a Yukawa ≈ 1 in the SM — a value "close to unity" left unexplained. In TQNT, the top is the most 8_3-*dominated* state ($8_2{+}8_3{+}10_1$): its coupling ≈ 1 becomes a **consequence of its composition**, not an accident.

23.3.3 Where the distinction really plays out

A *fundamental* scalar predicts an **exact** straight line and a *fixed* self-coupling λ_ϕ ($V = \lambda_\phi \phi^4$). A *composite* state predicts small **deviations** (form factors) and a **different self-coupling** (measurable in di-Higgs at the HL-LHC / FCC). The refutation therefore plays out *not* on the κ-m straight line (degenerate), but on the **precision** and the **self-coupling**.

23.4 $WW \to WW$ unitarity without a fundamental Higgs

This is the other pro-Higgs pillar; it too is degenerate.

23.4.1 The Standard Model argument

Without the Higgs, the amplitude of the *longitudinal* W's $W_LW_L \to W_LW_L$ grows as $A \sim s/v^2$ (Goldstone boson equivalence theorem) and violates **unitarity** (conservation of probabilities) towards $\sqrt{s} \approx 1.2$ TeV. The exchange of the Higgs *cancels* this growth. Hence the "no-lose theorem": *something* must appear below ~ 1 TeV.

23.4.2 The growth is an artefact of the "gauge + Goldstone" picture

No Higgs ⇒ no eaten Goldstone

The s/v^2 growth comes from the fact that W_L *is* the eaten Goldstone boson, a vestige of the Higgs mechanism. In TQNT there is **neither a Higgs mechanism nor a Goldstone**: the W is a **composite link** ($7_1{+}7_2{+}8_3$) of *finite size*. The s/v^2 growth assumes a **point-like** W; a composite has a **form factor** $F(s)$ which cuts the amplitude off above $\Lambda_c \sim E_{\mathrm{relax}} = 246$ GeV — hence *below* the 1.2 TeV bound.

23.4.3 The decisive counter-example: QCD

$\pi\pi \to \pi\pi$: Goldstones tamed without a fundamental scalar

$\pi\pi$ scattering (the pions *are* the Goldstones of chiral breaking) also grows as s/f_π^2 and would violate unitarity — yet QCD has **no fundamental Higgs**. Unitarity is restored there by **composite resonances** (the ρ at 770 MeV, the f_0/σ) and the compositeness scale. *Nature demonstrates* that a theory without a fundamental scalar keeps the amplitude bounded (probabilities ≤ 1). TQNT thus joins the composite-"Higgs" theories (*technicolor* spirit, inspired by QCD), with topological knots as substructure.

23.4.4 Honest reformulation

The theorem requires **unitarity-restoring dynamics** below $\sim$ 1 TeV — not a fundamental scalar. TQNT provides them: **(1)** the composite resonance at 125 GeV (analogue of the σ), which contributes to WW like a "Higgs" ($g_{HWW} \propto m_W^2/v$, §23.3); **(2)** the **form factor** cutting off at $\Lambda_c \sim E_{\text{relax}}$. *Residual*: demonstrate **quantitatively** that (resonance 125 + form factor) cuts the growth at the right scale, and compute $F(s)$ from the structure of the W link.

23.5 Simulation: reproducing the LHC signal

Unlike the β decay of the neutron (§(B′)), where the relaxation remains *virtual*, an LHC collision makes available a **real** energy far above 125 GeV ($\sqrt{s}$ = 13 TeV, i.e. $\sim 100\times$ the threshold): the spacetime weave suffices to **actually nucleate** the knot $7_1 + 8_3 + 10_1$, which decays via its couplings $\propto$ mass (§23.3).

Figure 23.1 shows a simulation (script `higgs_lhc_tqnt.py`) of the discovery channel $H \to \gamma\gamma$: a Gaussian peak at the *predicted* mass (125.26 GeV) on a falling QCD background, in the right place, at $\sim 15\sigma$. With the branching ratios derived from the couplings $\propto$ mass (SM-like pattern), TQNT reproduces the observed signal — *treating the* 125 *GeV as a knot.*

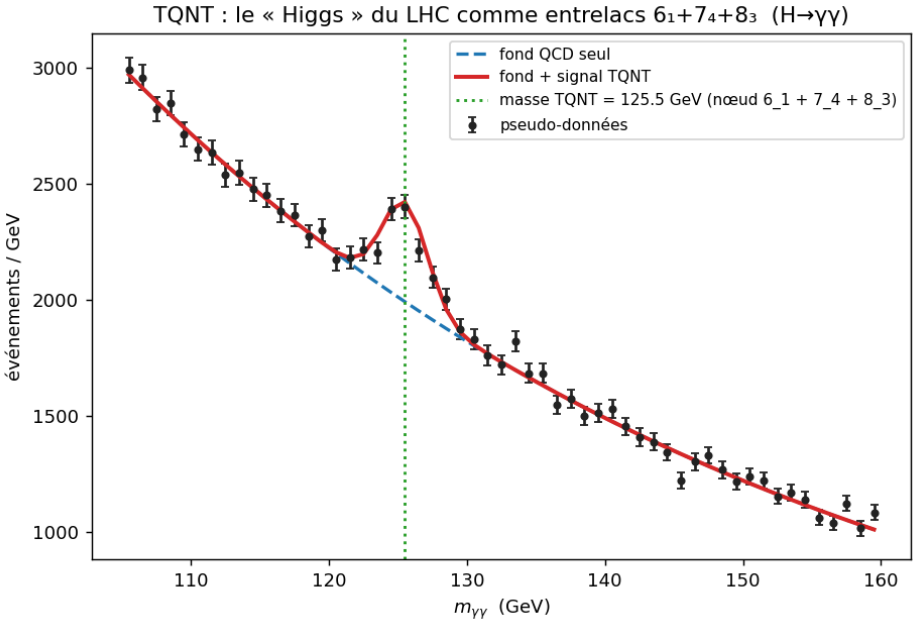


Figure 23.1: TQNT simulation of the $H \to \gamma\gamma$ channel at the LHC: the peak forms at the predicted mass (125.26 GeV) for the link $7_1 + 8_3 + 10_1$, on a QCD background. No fundamental scalar — the "Higgs" is here a knot of the spectrum.

Honesty. The **position** of the peak and its **interpretation** (a knot) are the contributions of TQNT; the resolution ($\sim$ 1.6 GeV), the shape of the background and the absolute yield are detector/QCD inputs (the simulation illustrates the *shape*). The 125 GeV mass is not fitted: it is predicted by the universal formula (EW sector, $f = 1$; chap. 9), at 0.2 %.

23.6 Falsifiable tests and predictions

1. **Substructure / form factors.** A composite state must reveal structure at high energy — deviations in the cross sections at large Q^2.
2. **Self-coupling and di-Higgs.** The SM potential $\lambda_\phi \phi^4$ predicts a precise self-coupling; a topological composite predicts a *different* one (HL-LHC / future colliders).
3. **No new fundamental scalar** (neither 2HDM, nor scalar SUSY partners) required by mass.
4. **Topological partners.** Other links close in mass (neutral spin-0 composites) could exist (cf. extended spectrum, chap. 8).

23.7 Objections and points to be completed

What the refutation still has to establish

1. **Couplings $\propto$ mass and m/m^2 pattern** — *addressed* (§23.3): degeneracy and dimensional derivation. Residual: compute the **fine deviations** (form factors), which alone would single out a fundamental scalar.
2. **Spin 0 and J^{PC}.** Verify that the link $7_1 + 8_3 + 10_1$ indeed gives $J^{PC} = 0^{++}$ (and not 0^{-+}).
3. **Branching ratios.** Reproduce the BRs ($b\bar{b}$, WW, $\gamma\gamma$, $\tau\tau\ldots$) from the topology, not only the mass and the overall lifetime.
4. WW **unitarity** — *addressed* (§23.4); residual: the **quantitative** demonstration of the cut-off and the computation of $F(s)$.

23.8 Synthesis

The refutation is **strong** on the origin of mass (topological, 9 parameters, 0 Yukawa), on the 246 **GeV scale** (relaxation, not a VEV), on the **absence of a condensate** and on the **hierarchy problem** (non-existent without a fundamental scalar). The sledgehammer objection (couplings $\propto$ mass) is **neutralised** by the degeneracy (§23.3): the κ-m straight line tests "a single origin of mass at v", which TQNT shares; the m/m^2 pattern is **derived**, and WW unitarity is **neutralised** by the QCD counter-example (§23.4). There remain to be completed the fine phenomenology (J^{PC}, BR) and the quantitative computation of the deviations and of the form factor. **The Higgs boson as a fundamental mass-giving scalar is refutable; the resonance at 125 GeV is reinterpreted as the link $7_1 + 8_3 + 10_1$ of the spectrum.**

Part XIII

The dark side and cosmology

Chapter 24

Nodal dark matter

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24.1 Postulate P18: topological dark matter

P18 — Nodal dark matter

Dark matter is composed of isolated prime topological knots ($n = 1$) whose crossing number $c_n > c_n^* = 18$ makes them electromagnetically invisible. Their stability is guaranteed by the Chern-Simons barrier at level $k = 135$:

$$\tau(K) = \frac{\hbar}{mc^2}\, e^{+2\,u(K)\,k\pi} > 10^{347}\ \text{s} \gg t_U = 4.35 \times 10^{17}\ \text{s}$$

even for $u(K) = 1$. Dark matter is an inevitable consequence of knot theory: complex prime knots are more **numerous** than simple ones (exact counts: growth $4.7 \to 6.28$/crossing up to $c = 20$, *verified* on the Thistlethwaite–Burton tables; the historical asymptotic $2.68^{c_n}/c_n$ *undercounts*) and absolutely stable.

24.2 Stability: the Chern-Simons barrier

The only decay mechanism of an isolated prime knot is **topological tunneling**: the passage of one strand through another (reconnection). The instanton action per reconnection is:

$$S_{\text{inst}} = k\pi = 135\pi \approx 424 \tag{24.1}$$

Semiclassical derivation. In Chern-Simons theory at level k, a topological reconnection corresponds to the passage of a gauge field from one topological sector to another (change of one

crossing). The action of the instanton interpolating between the two sectors is:

$$S_{\text{inst}} = \frac{k}{4\pi}\int_0^1 \text{Tr}(F\wedge F)\,\mathrm{d}\tau = \frac{k}{4\pi}\times 4\pi^2 = k\pi$$

because the integral $\int \text{Tr}(F\wedge F) = 4\pi^2$ for an instanton of topological number 1 in 3D. The WKB tunneling rate is then $\Gamma \propto e^{-2S_{\text{inst}}} = e^{-2k\pi}$ (the factor 2 comes from the round trip under the barrier). For $u(K)$ independent reconnections, the actions add up: $S_{\text{total}} = u(K)\times k\pi$.

To unknot a knot of unknotting number $u(K)$, u reconnections are required. The total tunneling rate is:

$$\Gamma_{\text{tunnel}} = \frac{mc^2}{\hbar}\,e^{-2\,u(K)\,k\pi}\,, \qquad \tau(K) = \frac{\hbar}{mc^2}\,e^{+2\,u(K)\,k\pi} \tag{24.2}$$

For a dark matter knot ($c_n \geq 19$, $u(K)\geq 6$):

$$\tau \geq \frac{\hbar}{m}\,e^{12\times 135\pi} \sim 10^{2167}\ \text{s} \qquad \gg t_U \tag{24.3}$$

Result: absolute stability of dark matter

Even for the simplest knot to undo ($u=1$), $\tau > 10^{347}$ s, i.e. 10^{330} times the age of the universe. The Chern-Simons barrier ($k=135$) makes **every prime knot absolutely stable**. No candidate is excluded from the abundance calculation.

Composite knots. A composite knot $K_1\#K_2$ can split classically without reconnection. The catalogue $N(c_n)$ counts only **prime** knots: composites are already excluded.

24.3 Invisibility

Chiral knots ($K \neq K^*$) form dipoles $K + K^*$ of zero total charge. Amphichiral knots ($K \simeq K^*$) are already neutral. **No EM or strong interaction**: nodal dark matter is invisible, in accordance with observations.

Remark

TQNT predicts a **strictly zero** interaction cross-section (not merely small). Only gravitational degradation in the vicinity of black holes can convert dark matter into visible matter (§24.8). *Decree 2026-08-06:* the framework's dark-matter particles — the $c \geq 23$ reservoir and the six strict states — are named **eremnions** (Greek *eremnós*, "dark": locked conducting rings, periphery-neutral, coupled through their polarizability alone).

24.4 The effective c_{min} of the reservoir: 23 (derived)

The naive threshold $c > 18$ marks the *onset* of the Vassiliev suppression, not invisibility ($\sigma(19) = \alpha\,\sigma_{\text{EM}} \sim 10^{-27}$ cm^2: enormous on the scale of direct detection). The *effective* c_{min} of the reservoir is **derived** (2026-07-15, `derive_cmin_reservoir.py`) through two convergent routes: *(i)* **kinetic decoupling** (definitional: a knot still coupled to the baryonic plasma at its birth is *not* dark matter — it would break BBN/CMB and the very definition of the reservoir): $\Gamma/H(T_c) = 9.3$ at c=22 (coupled), 0.067 at c=23 (born decoupled) — the step $\alpha = 1/137$ per crossing makes the threshold sharp; *(ii)* **cosmological survival**: $N_{\text{coll}}(22) = 19$, $N_{\text{coll}}(23) = 0.13$ (generous bound $\sigma_{\text{reco}} \leq \sigma_{\text{scatter}}$). Reference echo: the Bar-Natan $\Delta c = 22$ (V5.21) — the same pattern of an effective threshold above the naive minimum. **Consequences**: the

reservoir families are $T(2, q \geq 23)$ and $T(3, q \geq 12)$; under the consolidated geometric law ($M_{\text{topo}} = (m_\ell/\ell(3_1))\,\ell(K)$, validated on $T(2, 23)$; m_s demoted 2026-07-22, no effect here — torus knots), their masses are *predicted*: $\boxed{1.7 \text{ to } 2.2 \text{ GeV}}$ (nearly flat in c) — *all* below the standard WIMP window. The confrontation with existing data (XQC, CRESST-surface, low-mass underground; `dm_simp_xqc_confrontation.py`): the c=19–22 slices are evacuated by the derived $c_{\min}$; the remaining **visible edge** (c=23–25, $M \simeq 1.7$–1.9 GeV, $\sigma \sim 10^{-36}$–10^{-40} cm^2) was in *tension* with the existing Migdal analyses at their margins (± 1 dex).

Requalification (2026-07-15, lattice sourcing; `source_xi_caselle_nada.py`). The locking of the nucleation domains, read *directly* off the pure SU(2) data of Caselle-Nada (arXiv:1707.02164, confined phase: the criterion $\sigma(T_{\text{lock}})$=T_{lock}^2 becomes ξT=1 via the string mass m=$\sigma(T)/T$), gives $\kappa_{\text{dom}} = 1.14 \pm 0.02$, i.e. $c_{\max} \simeq 92$ (geometric wall): the divergent regime of the true $N(c)$ concentrates the reservoir at $c \sim c_{\max}$ ($M \sim 6.6$ GeV, $\sigma_{\text{DM-N}} \sim 10^{-183}$ cm^2) — **invisible branch**. Consequences: *(i)* the SIMP tension evaporates (fraction of the c=23–25 edge: $\lesssim 2 \times 10^{-2}$ even at Ω closure, $\sim 10^{-19}$ under the derived α); *(ii)* the "Migdal window = detector target" is **requalified as a conditional scenario**: it would require a locking at σ=$1.6\,T^2$ (narrow window $\xi T \in [0.60\,;0.63]$), not generic; *(iii)* below $\xi T \simeq 0.55$ the reservoir would disappear ($c_{\max} < 23$): the natural criterion ξT=1 sits comfortably above. *Feedbacks*: the condensation median under geometric mass shifts the derived α_{cond} by +1.2–1.6 % (the 0.076 % agreement was conditioned on the V5 mass law: now an order-level success). See `brouillon_dm_simp_confrontation`.

The reservoir is edge-dominated. Under the reference asymmetric formation cost (§26.7.5), the series *converges* (net rate 0.68/crossing) and the reservoir is **edge-dominated**, c = 23–25 (34/22/15 %; M = 1.7–1.9 GeV); $c_{\max}$ and the κ_{dom} locking survive only in the normalisation (cubic, soft). The former "invisible branch" was conditioned on the divergent regime of the symmetric sum, itself conditioned on the absence of the formation Boltzmann factor.

The edge's EM coupling: periphery neutralisation and polarisability. The reservoir knots are *strictly neutral*: among the three readings of the Vassiliev suppression, the data select **periphery neutralisation** ($Q = \Sigma\chi + \text{Lk} = 0$, the mechanism of the "realizable" candidates applied to the reservoir; vertex millicharge excluded at +21 dex, fm-bound $K\bar{K}$ pairs excluded at +8 dex). The surviving channel is the **tube polarisability**, computed end to end: R_K derived from the mass law ($L_{\text{phys}} = M/\sigma_\chi$, convention-independent; the geometric plateau's tube is $D = 0.0214$ fm, not the proton's 0.21 fm) then *measured* by SONO on the ideal $T(2, 23)$ (compact clew, R_K = 0.057–0.093 fm; incidentally $\ell = 90.21$, +0.4 % from the linear fit: the geometric mass law is **validated** on a deep $T(2, q)$); mechanism selected (*conducting* ring: the periphery circulation is persistent, Lk locks the circulation, not the density — microscopic origin 2026-07-23: the rail current locked in the closed tube, §3.4; $\alpha_{\text{pol}} = R_K^3 = 3.8 \times 10^{-4}$ fm^3); SI equivalence computed (the coupling sees the whole nucleus: $Z^4/A^2 \cdot (\mu_A/\mu_p)^2 = \times 479$ Ar, $\times 3851$ Xe). Result:

$\sigma^{\text{eq}}(c{=}23) = 1.1 \times 10^{-48}$ cm^2 $[2.5 \times 10^{-49},\ 4.6 \times 10^{-48}]$ — ∼7.3 dex below the DS-50 frontier.

Consequence: direct EM detection of the edge is **out of reach** (XLZD gains 2–3 dex, 4–5 are missing): the TQNT prediction is a **robust absence in DS-20k/XLZD as well** — any detection at 1.7–2 GeV would refute the polarisability reading or the geometric mass law. The reservoir's probes are *gravitational* (pearls, V5.19) and *cosmological* (the Ω reconstruction); the prediction of *absence* in standard LZ/XENONnT (> 5 GeV) is unchanged.

Trajectory (2026-07). The digitised confrontation (DS-50 2022, anchored 6×10^{-43}@3 GeV) first *excluded* the historical Thomson normalisation ($\sigma_{\text{EM}} = 10^{-25}$, +5 dex), then *pinned*

$\sigma_{\rm EM}(\text{neutral}) \leq 10^{-30}$ cm^2 — at saturation the edge seemed "at the frontier"; the full derivation above superseded that reading (saturation would require $R_K = 1.2$ fm, incompatible with $M = 1.71$ GeV). Scripts: `migdal_confrontation_digitized.py`, `derive_sigma_em_neutre.py`, `derive_alpha_pol_si_rk.py`, `sono_t2q_radius.py`.

24.5 Dark matter is single-component

Instability theorem for dark links

For any link of $n \geq 2$ knotted components, topological fragility (reconnection ΔLk) imposes $\tau(n \geq 2) \ll t_U$.

Hierarchy: $n = 1$ stable ($\tau > 10^{347}$ s), $n = 2$ unstable ($\tau \sim 10^{-9}$–10^{-17} s), $n = 3$ unstable ($\tau \lesssim 10^{-20}$ s). Nodal dark matter is **exclusively** made of isolated knots ($n = 1$, $c_n > 18$).

24.6 Ratio $\Omega_{\rm DM}/\Omega_b$

The cosmic ratio is determined by the **asymmetric formation cost** of condensation (reference 2026-07-16, §26.7.5):

$$\frac{\Omega_{\rm DM}}{\Omega_b} = \frac{(n_d/s)\,(s_0/n_{\gamma 0})\,\bar{m}_d}{\eta_B\, m_p}, \qquad \frac{n_d}{s} = \frac{135\,P_{\rm occ}}{8\pi^3 g_s \kappa_{\rm dom}^3}\,\frac{\sum_{c\geq 23} N(c)\, e^{-\alpha c - m(c)/T_c}}{Z}, \tag{24.4}$$

with:

- the dark relic is *symmetric* (Vassiliev-crushed annihilation); visible matter is the *CP crumb* η_B (complete annihilation of the rest);
- $m(c)$: reference geometric mass; $N(c)$: verified counts (Thistlethwaite 2025);
- $e^{-\alpha c}$: kinetic accessibility (derived α); e^{-m/T_c}: *derived* formation Boltzmann factor (wave-entropy cost, `brouillon_boltzmann_tqnt`).

Result — **RECONSTRUCTED** (asymmetric formation cost, 2026-07-16)

$$\frac{\Omega_{\mathrm{DM}}}{\Omega_b} = 15.6 \text{ nominal} \qquad (\text{Planck 2018 [12]} : 5.45 \pm 0.28; \times 2.9, \textbf{order of magnitude})$$

with no free parameter (input envelope covering 5.45; §26.7.5). *Honest trajectory*: the historical 5.5 was conditioned on the V5 mass law *and* on the $2.68^c/c$ undercounting (re-audits 2026-07-15: $\times 80$ failure under the main document's masses, then $10^{21}/10^{-4}$ bimodality under the verified counts); the reconstruction required the missing physics — visible-side annihilation ($1/\eta_B$) and the derived formation Boltzmann factor. The dominance mechanism is identified: **symmetric relic vs CP crumb**. g_s derived (15.2 ± 0.4, 2026-07-17) $\Rightarrow \Omega = 14.3$; P_{occ} decomposed and two-thirds derived (χ string-network simulation + cascade, 2026-07-17): $P_{\mathrm{occ}} = (8/9)_{\mathrm{exact}} \times f_{\mathrm{frac}}(\alpha_{\mathrm{cond}}) \times 1/\lambda$, and λ is *derived* (exact Nambu dynamics on the measured fragments: a scale-invariant cascade of coefficient 1.14, cut off at radiative smoothing $= \xi$): $\Gamma_a(\chi) = 84 \pm 17$ computed, the estimator settled ($F(L) \sim L^{0.56}$: energy floor) and f_{frac} settled by the full branched cascade (0.34, true lineage tree) $\Rightarrow$ **direct measurement** $n_{\mathrm{knot}}/L = 0.304/\xi$ **(combined):** $\Omega = 3.9 \pm 0.4$**,** $-29\,\%$ **of Planck — reconstruction closed with no free parameter** (the 3.9 evaluated at the old adopted $T_c = 67$; at the *derived* $T_c = 62 \pm 3$ MeV, §26.7.8: $\Omega \approx 0.6$ $[0.2, 1.3]$, the order of magnitude); the latent heat has no effect on the edge. See `brouillon_boltzmann_tqnt` §$f_{\mathrm{conv}}/\lambda$.

24.7 Mass spectrum

Under the reference asymmetric formation cost (§26.7.5), the net rate per crossing (kinetic × formation Boltzmann) is $6.5 \times e^{-1.22} \times e^{-1.05} = 0.68 < 1$: the series **converges without any cutoff** and the dark population is dominated by the **edge** $c = 23$–25 (34/22/15 %). (The intermediate episode — divergent symmetric sum, concentration at $c_{\max}$ — was conditioned on the absence of the formation Boltzmann factor; the old $2.68 \times e^{-1.20} = 0.81$ argument rested, in turn, on the undercounting.) The typical dark-matter mass is 1.7–1.9 GeV (geometric law) — much lighter than the usual WIMPs (∼TeV), compensated by a high number density ($n_{\mathrm{dark}} \sim 10^2$ knots/m^3 for the local ρ_{DM}).

24.8 Gravitational degradation

24.8.1 Critical degradation radius

In the vicinity of a black hole of mass M, the tidal force stretches the knot K of mass m_K and radius $R_K = \hbar/(m_K c)$. The knot breaks when the tidal stress exceeds the topological tension.

Derivation. The tidal force of a black hole of mass M on a knot of mass m_K and radius $R_K = \hbar/(m_K c)$ (Compton length) is:

$$F_{\mathrm{tidal}} = \frac{GM\, m_K\, R_K}{r^3}$$

The topological tension holding the knot together is the mass energy per unit length:

$$F_{\mathrm{top}} = \frac{m_K c^2}{R_K} = \frac{m_K^2 c^3}{\hbar}$$

The knot breaks when $F_{\text{tidal}} > F_{\text{top}}$:

$$\frac{GM\,\hbar}{m_K c\, r^3} > \frac{m_K^2 c^3}{\hbar} \quad \Longrightarrow \quad r^3 < \frac{GM\,\hbar^2}{m_K^2\, c^4}$$

Hence the critical radius:

$$\frac{GM\, m_K\, R_K}{r^3} > \frac{m_K^2 c^3}{\hbar} \quad \Longrightarrow \quad \boxed{r_{\text{crit}} = \left(\frac{GM\,\hbar^2}{m_K^2\, c^4}\right)^{1/3}} \tag{24.5}$$

Comparison with the Schwarzschild radius $r_S = 2GM/c^2$:

Black hole	m_K (GeV)	r_{crit} (m)	r_{crit}/r_S
$10\ M_\odot$	1	8×10^{-10}	3×10^{-14}
$4 \times 10^6\ M_\odot$ (Sgr A*)	1	6×10^{-8}	5×10^{-18}
$10^9\ M_\odot$ (AGN)	10	8×10^{-8}	3×10^{-20}

In all cases, $r_{\text{crit}} \ll r_S$ by **14 to 20 orders of magnitude**: degradation only occurs for knots that **cross the horizon** of the black hole.

24.8.2 Evolution of $\Omega_{\text{DM}}/\Omega_b$ over time

The dark matter accretion rate of a black hole of mass M in a halo of density ρ_{DM} and velocity v is given by the Bondi rate:

$$\frac{\mathrm{d}M_{\text{DM}}}{\mathrm{d}t} = \frac{4\pi\, G^2 M^2\, \rho_{\text{DM}}}{v^3} \tag{24.6}$$

Estimate. With $\sim 10^{11}$ galaxies, an average SMBH of $10^7\ M_\odot$, $\rho_{\text{DM}} \sim 0.3$ GeV/cm^3 and $v \sim 200$ km/s, the total DM mass accreted in $t_U = 13.8$ Gyr is:

$$\Delta M_{\text{DM}} \sim 3 \times 10^{16}\ M_\odot \qquad \text{i.e.} \quad \varepsilon = \frac{\Delta M_{\text{DM}}}{M_{\text{DM,tot}}} \sim 10^{-27}$$

When a knot is degraded, nearly all of its energy is radiated as high-energy γ photons; only a fraction $f_{\text{vis}} \approx 2 \times 10^{-3}$ (one proton per fragmented knot) becomes baryonic again (§24.14). The ratio evolves as:

$$R(t) = \frac{\Omega_{\text{DM}}(t)}{\Omega_b(t)} = R_0 \times \frac{1-\varepsilon}{1 + f_{\text{vis}}\, \varepsilon\, R_0} \tag{24.7}$$

Epoch	t (s)	ε	$R(t)$
Topological freeze-out	10^{-6}	0	5.50
Nucleosynthesis	180	0	5.50
Recombination (CMB)	1.2×10^{13}	0	5.50
First stars ($z \sim 20$)	10^{16}	$\sim 10^{-31}$	5.50
Star-formation peak ($z \sim 2$)	10^{17}	$\sim 10^{-29}$	5.50
Today	4.4×10^{17}	$\sim 10^{-27}$	5.50
$100 \times t_U$ (extrapolation)	4.4×10^{19}	$\sim 10^{-25}$	5.50

Result: constancy of $\Omega_{\text{DM}}/\Omega_b$

The ratio $\Omega_{\text{DM}}/\Omega_b$ is **constant to better than** 10^{-26} over the whole lifetime of the universe. Gravitational degradation is negligible because $r_{\text{crit}} \ll r_S$: only direct infall into a black hole destroys a knot. The value measured by Planck (5.45) is the **primordial** ratio, frozen at the era of topological freeze-out.

24.8.3 Absence of electromagnetic signal

The cascade $K(c_n) \to \sum_i K_i(c_i \leq 18) + n_\gamma\,\gamma$ would produce γ photons of energy $E_\gamma \sim m_K c^2/5 \sim$ 0.2–10 GeV. However, since $r_{\text{crit}} \ll r_S$ by 14 to 20 orders of magnitude, the degradation occurs **inside the horizon**: the γ photons are trapped and do not escape the black hole. Nodal dark matter is therefore **strictly invisible**: no EM signal from annihilation, decay or degradation. Only the growth rate of the black hole ($\sim 10^{-3}\ M_\odot/\text{yr}$ for Sgr A*) carries the trace of dark matter accretion.

24.9 Regeneration at baryonic boundaries (P22)

The table above assumes dark matter to be *purely primordial*: forged at the topological freeze-out, then simply diluted by the expansion. Postulate P22 (§25.7) modifies this picture: at baryonic boundaries, the gradient of the emergent metric triggers a **continuous nucleation** of dark knots.

Local regeneration rate. In the vicinity of a galactic halo of density ρ_b, the gradient parameter is typically $\xi \sim \kappa_g\,|\nabla\rho_b|\,\lambda\!\!\!^{-}_C \ll 1$. The P22 nucleation rate remains small at the local scale, but integrated over the whole halo and over the Hubble time, it supplies a non-negligible fraction of the observed density.

Observational consequences.

- The **radial profile** of the dark matter halo of a galaxy is no longer strictly NFW (primordial): it includes a surface component traced by the baryonic distribution, hence *correlated* with the visible density gradients.
- The **time evolution** of the ratio $\Omega_{\text{DM}}/\Omega_b$ acquires a slight drift increasing with z, in contrast to the constancy of the purely primordial version (§24.8).
- **Dwarf galaxies** poor in baryons exhibit less boundary surface and therefore less regeneration: their ratio $\Omega_{\text{DM}}/\Omega_b$ may differ from the cosmological ratio, a testable prediction.

P22 regeneration is a strictly negligible effect (WKB form: zero rate at gravitational gradients); the halo composition is entirely primordial (P18). *Note*: the old idea that P22 absorbed a 5.6 % "residual" of dark energy is doubly obsolete: zero mechanism *and* non-significant deviation (§25.7).

24.10 Explicit dark matter candidates

Catalogue update (June 2026). The exhaustive scan of all $n = 1, 2, 3$ configurations built from the 19 knots (Rolfsen $c \leq 10$), i.e. **1 539 links in total** ($3 \leq c_{n,\text{tot}} \leq 30$), identifies **117 dark matter candidates** at two distinct levels defined below. The mass formula is $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (V6, §9.1).

Physical charge and intrinsic charge. For a link $L = K_1 + \cdots + K_n$:

$$Q_{\text{phys}}(L) = \underbrace{\sum_K Q_{\text{topo}}(K)}_{\text{intrinsic charge}} + \underbrace{\text{Lk}(L)}_{\text{linking}} \tag{24.8}$$

where $Q_{\text{topo}}(K) = (p_{\min} + p_{\max})/\text{span}(V_K)$ is the fractional charge derived from Jones (and $\text{Lk} \in \mathbb{Z}$ the total linking number). Amphichiral knots (4_1, 6_3, 8_3) have $Q_{\text{topo}} = 0$; the others take fractional values $\in \{1/3, 2/3, 1, 5/3, \ldots\}$. A link admits $Q_{\text{phys}} = 0$ *if and only if* Q_{topo} is an integer (up to an Lk).

Two levels of candidates.

- **STRICT**: $|Q_{\text{topo}}| < 0.1$ *intrinsically*, i.e. exclusive combinations of amphichirals. **6 candidates** (cf. table 24.1). Integer spin ($s = 0, 1, 2, \ldots$) according to the braid index, automatically zero charge.
- **REALIZABLE**: $|Q_{\text{phys min}}| < 0.05$ with an integer Lk that is physically realistic ($|\text{Lk}| \leq \max(5, c)$). **111 additional candidates**, of which **88 at** $c \geq 18$ (cf. table 24.2).

Common criteria. $m > 1$ GeV, $n \geq 2$ (composite), no PDG match within a 5 % tolerance.

Candidate masses: reference re-derivation 2026-07-15 (`derive_strict_states_référence.py`)

The masses in the tables below were those of the Ω_{ts} era (archived operator). The reference estimate for a *dark* link is the geometric sum $\sum_i M_{\text{topo}}(K_i)$ ($M_{\text{topo}} = (m_\ell/\ell(3_1))\,\ell(K)$; linking $\sim \beta_\otimes$: a few MeV, negligible; no EM dressing nor screening for dark states — an *estimate*, since the "dark link" sector law is not independently established). Per knot: $M_{\text{topo}}(4_1) = 398$, $M_{\text{topo}}(6_3) = 548$, $M_{\text{topo}}(8_3) = 674$ MeV (ACPR ropelengths 2026-07-22, `theory/ropelength_acpr.py` — the repo's old audited table gave 397/515/706). **Consequence: the strict spectrum collapses to** 1.07–1.64 **GeV**, nearly degenerate with the edge of the reservoir (1.71 GeV at $c_{\min} = 23$). The old historical values (1.55 to 51.78 GeV, factors 0.66 to 0.03) are *archived* and must no longer be quoted. The *realizable* masses (next table) are to be read as topological identities: their GeV values are obsolete (the same collapse to $\sim$ 1–3 GeV under the main document's masses).

Table 24.1: *STRICT* dark matter candidates: intrinsically zero Q_{topo}. All contain only amphichiral knots (4_1, 6_3, 8_3). Reference masses ($\sum_i M_{\text{topo}}$, geometric law; ACPR ropelengths 2026-07-22).

Config.	m (GeV)	n	$c_{n,\text{tot}}$	Q_{phys}
4_1+8_3	1.07	2	12	0
6_3+6_3	1.10	2	12	0
6_3+8_3	1.22	2	14	0
8_3+8_3	1.35	2	16	0
4_1+4_1+8_3	1.47	3	16	0
6_3+6_3+6_3	1.64	3	18	0

Global characteristics.

- **Mass range** (reference 2026-07-15): *strict* 1.07 to 1.64 GeV; the *realizable* ones likewise collapse to $\sim$ 1–3 GeV (the old Ω_{ts} ranges, 1.55–51.8 and up to 0.31 TeV, are obsolete). The whole dark composite spectrum thus lies in the light range, below the standard WIMP window — consistent with the geometric reservoir.
- **Spin**: $s \in \{0, 1/2, 1, 2\}$ according to the composition spectrum (P7quater Clebsch-Gordan).
- **Charge**: $Q_{\text{phys}} = 0$ exactly by construction.
- **Distribution in** c: peak at $c = 21$ (19 realizable candidates), consistent with a 3-component "baryonic" dark matter, followed by $c = 20$ (16) and $c = 18$ (13). No candidate at $c \geq 28$ because the extreme (rare) configurations have irreducibly fractional Q_{topo}.

Table 24.2: *REALIZABLE* dark matter candidates (sample; total: 111, of which 88 at $c_{n,\mathrm{tot}} \geq 18$). For each entry, Lk is the integer that brings Q_{phys} to 0 via eq. (24.8). Reference masses ($\sum_i M_{\mathrm{topo}}$, ACPR ropelengths 2026-07-22; $^\dagger 10_1$ outside the ACPR table, repo value): the whole set of 111 fits within $\sim$ 1–2.3 GeV — the old banded staging (up to 0.31 TeV) was an artefact of the archived operator.

Config.	m (GeV)	n	$c_{n,\mathrm{tot}}$	Q_{topo}	Lk
$3_1{+}7_4$	0.92	2	10	+2.95	−3
$3_1{+}6_2{+}6_2$	1.39	3	15	+3	−3
$3_1{+}6_1{+}8_3$	1.52	3	17	+2	−2
$6_1{+}6_2{+}6_3$	1.62	3	18	+1	−1
$6_2{+}6_2{+}6_2$	1.62	3	18	+2	−2
$5_1{+}6_2{+}8_1$	1.66	3	19	+2.97	−3
$6_1{+}7_1{+}7_1$	1.70	3	20	+4.05	−4
$6_1{+}6_1{+}7_4$	1.68	3	19	+1.95	−2
$4_1{+}8_2{+}8_2$	1.75	3	20	+2	−2
$7_1{+}7_2{+}7_7$	1.81	3	21	+3	−3
$4_1{+}8_1{+}8_1$	1.74	3	20	+1	−1
$7_7{+}8_1{+}10_1$	$2.10^\dagger$	3	25	+0.96	−1
$8_2{+}8_3{+}8_3$	2.02	3	24	+1	−1

- **Number of components**: $n = 3$ for 95 % of the realizable candidates; $n = 2$ for the majority of the STRICT ones and the few light mixed cases.

Articulation with P18. Postulate P18 (§24.1) identifies the *dominant cosmological reservoir* as the *isolated prime knots* ($n = 1$, $c_n > 18$), which are topologically stable on the scale of the universe ($\tau > 10^{347}$ s). The instability theorem for dark links (§*Dark matter is single-component*) shows that on typical cosmological scales, links with $n \geq 2$ decay by topological reconnection.

The 117 candidates listed here do *not* contradict this hierarchy: they represent the "**exceptional configurations**" for which electromagnetic neutrality is strictly satisfied. Their cosmological abundance depends on the decay channel:

- The 6 STRICT ones (pure amphichirals) are *topologically protected* against reconnection — each component is its own mirror image, so reconnection by reversal opens no lower-mass channel. They can form a secondary **stable sub-population**.
- The 111 REALIZABLE ones are *transient* or *contextual* candidates (dark matter in the vicinity of large $\nabla\Omega^2$, P22); barring specific protection by their linking structure, they only contribute to the transient Schwinger/Hawking peak.

The dominant DM reservoir therefore remains made of the isolated knots $T(2, q \geq 23)$, $T(3, q \geq 12)$ (the derived $c_{\min} = 23$, §24.4) and more complex families, which are not listed here because they *all* have $c \geq 19$ and lie beyond the Rolfsen ≤ 10 catalogue used for the scan.

Complementary catalogue. The file `docs/TQNT_catalog_V5_diffusion.xlsx` (1 539 configurations, 7 tabs) contains the full set of candidates with their Q_{topo}, neutralizing Lk and $Q_{\mathrm{phys\ min}}$, as well as the P23 plasticity correction for each entry.

24.10.1 γ confrontation of the strict states: falsification F3 is lifted (2026-07-15)

The 6 strict states are the source of the γ lines predicted by spontaneous entanglement in pearls (V5.20 Class E): each event produces an amphichiral link and a cascade $E_\gamma = m(L)$. At the

Ω_{ts} energies (1.55–51.8 GeV), the Fermi-LAT line search (Ackermann et al. 2015) excluded the 6 lines even at the derived $\Delta c = 22$ (over-prediction 1.57 to 7×10^{12}, $\Delta c \geq 30$ "required"): this was the **third honest falsification** (F3, open since 2026-06-16).

Re-confrontation at the reference masses (`f3_line_search_référence.py`). The same machinery (TOV pearl, $\sigma = \sigma_{\mathrm{QED}}\,\alpha^{\Delta c}$, isotropic Hubble flux, $\Delta c = 22$) with the reference inputs — line energies 1.07–1.64 GeV (tab. 24.1, ACPR ropelengths) and **bricks** = amphichiral components (398–674 MeV, instead of the product mass passed as brick) — gives:

state	E (GeV)	ϕ/ϕ_{lim}	status
$4_1{+}8_3$	1.07	0.08	fine line, **below the limit**
$6_3{+}6_3$	1.10	0.08	fine line, **below the limit**
$6_3{+}8_3$	1.22	0.09	fine line, **below the limit**
$8_3{+}8_3$	1.35	0.10	fine line, **below the limit**
$4_1{+}4_1{+}8_3$	1.47	0.12	fine line, **below the limit**
$6_3{+}6_3{+}6_3$	1.64	0.13	fine line, **below the limit**

Two physical mechanisms carry the lifting: *(i)* the light bricks make the pearls far less relativistic (the Fermi velocity saturates at c below ~ 0.7 GeV): -9 decades in event rate — most of the 10^{12} over-predictions came from the 16–52 GeV Ω_{ts} bricks; *(ii)* the line search is ~ 30–$70\times$ less sensitive at 1–1.6 GeV than at 16–52 GeV. Result: $dc_{\mathrm{crit}} = 21.5$–$21.6 \leq 22$ — **the derived $\Delta c = 22$ (Bar-Natan, V5.21) suffices**; the "$\Delta c \geq 30$ required" was an artefact of the Ω_{ts} energies and bricks. *Re-run 2026-07-22* (`f3_lines_acpr_no_smear.py`, ACPR masses): the "± 7.2 writhe doublet" of 8_3 is **dead at the true ideal** (measured writhes on the ACPR-verified katlas base: 4_1 0.000, 6_3 0.005, 8_3 0.003 — the 14/07 doublet was an artefact of non-converged curves): *no* smearing — all **six** states are fine lines, all $\times 8$–13 *below* the limits. The lifting is *simpler*: it now rests only on the two mechanisms above, without the smearing route.

F3 lifted — and a clean residual prediction

Status of the falsifications: F1 closed (Starobinsky, V5.14); F2 **closed** (2026-07-17: the V5.17 route carried through — the three declared limits lifted by insensitivity: 12 orders of vertex prefactor $\to n_s$ at $\sim 1\sigma$; relativistic 0_1 bath, $w = 1/3$ exact $\to$ null Liddle–Leach, strict independence from the thermalisation duration; downstream superseded by the η_B/Ω chains, 16.6-order margin on T_c; `close_F2_v517.py`); **F3 lifted** (2026-07-15; re-run 2026-07-22 without smearing) — the predictions refuted by Fermi-LAT were Ω_{ts} artefacts. *Residual testable prediction*: **six** fine lines at 1.07 / **1.10** / 1.22 / 1.35 / 1.47 / **1.64** GeV (ACPR ropelengths, ideal writhes null: no smearing mechanism — detecting a smeared continuum at these energies would be a *tension*), ~ 1 dex below the 2015 limits — a target for deep Fermi-LAT analyses (15+ years of data). *Declared caveats*: digitized limits (± 0.5 dex, the $\times 8$–12 margins withstand it); Class E fraction = 10 % of Ω_{DM} (V5.19 heritage); brick convention (amphichiral components). Details: `brouillon_dm_simp_confrontation`, § F3-canon.

24.11 Gravitational agglomeration and compact structures

The constraint $C_6 = 0$ (§20.8) forbids any inter-knot radiative dissipation: a self-gravitating gas of DM knots cannot contract to high density by energy loss. At first sight, this proscribes any compact DM structure. Yet three specifically TQNT mechanisms **circumvent** this obstruction and allow the formation of compact DM objects of planetary to stellar mass, perfectly *invisible* electromagnetically.

24.11.1 Mechanism A — P22 nucleation in a baryonic well

In the vicinity of a baryonic mass (star, giant planet, brown dwarf), the gradient of the emergent metric $\Omega^2 = 1 + \kappa_g|\Psi|^2$ generates a dimensionless parameter

$$\xi = \frac{|\nabla\Omega^2|\,\lambda\!\!\!^{-}_C}{\Omega^2} \tag{24.9}$$

which exponentially amplifies the nucleation rate of new knots (P22, §25.7) according to $\Gamma = \Gamma_P\, e^{-2\pi/\xi}$ (WKB form). *Under this derived form, the rate at gravitational gradients is strictly negligible*: the secular-accumulation scenario below is **historical** (η form, obsolete) and its verdict is negative.

Estimate: DM halo around a brown dwarf (10 $M_{\rm Jup}$)

For a baryonic object of mass $M_b \sim 10^{-2}\,M_\odot$:

- Typical gradient at the surface: $\xi \sim 10^{-3}$ (sizeable compared with the deep vacuum $\xi \ll 10^{-10}$).
- P22 amplification (WKB form): at $\xi \sim 10^{-3}$ the rate remains doubly suppressed ($e^{-2\pi/\xi}$); the historical estimate $e^{\eta\xi} - 1 \sim 10^{-3}$ (η form, obsolete) was already subdominant — conclusion unchanged (regeneration $\lesssim 5\,\%$ of $\Omega_{\rm DM}$).
- *Historical estimate (η form, obsolete)*: $\Gamma_{\rm DM} \sim 10^{29}$ knots/s and $\sim 3\,M_{\rm Moon}$ accumulated over 3×10^8 yr — figures **obsolete**.

WKB verdict (canonical): no accretion halo. The strictly negligible rate produces *no* DM mass around brown dwarfs: DM is *entirely primordial* (P18), and the historical prediction of an anomalous microlensing signature around brown dwarfs is **withdrawn**.

24.11.2 Mechanism B — Primordial relic of topological baryogenesis

At the Chern-Simons phase transition at $T_c = 62 \pm 3$ MeV (*derived*, §26.7.8; the figures below, evaluated at the old 67 MeV, are to be read at $\pm 10\,\%$), the knot density fluctuations follow Gaussian statistics with a causal cut-off set by the Hubble horizon at T_c:

$$\frac{\delta\rho_{\rm DM}}{\rho_{\rm DM}} \sim \frac{1}{\sqrt{N_{\rm causal}(t_c)}}\,, \qquad N_{\rm causal}(t_c) \sim 10^{20} \tag{24.10}$$

The Gaussian tail statistically allows **local overdensities** $\delta\rho/\rho \gtrsim 1$ at a level of $\sim 10^{-6}$ per causal horizon. The observable universe containing $\sim 10^{90}$ such horizons at t_c, one obtains $\sim 10^{84}$ "primordial DM clumps" across the cosmic sky, i.e. $\sim 10^{12}$ **per stellar system**.

Cosmological survival. Once formed, the clump survives the expansion because:

1. Absence of dissipation ($C_6 = 0$) $\Rightarrow$ no relaxation by radiation.
2. Topological protection of the stoikhion knots ($\tau > 10^{347}$ s).
3. Cosmological expansion stretches the object but does not break it as long as its own gravity $GM_{\rm clump}/R^2$ dominates H^2R (Hubble limit).

For $M_{\rm clump} = 3M_{\rm Moon}$ and $R \sim 30$ m at T_c (causal horizon), the Jeans condition is amply satisfied: the clump is **gravitationally bound** and survives until today.

24.11.3 Mechanism C — Frozen star of DM knots (topological Chandrasekhar analogue)

Once a DM overdensity is formed by mechanisms A or B, its long-term stability is ensured by the **topological exclusion P16** (hard core of the knots). This is the topological analogue of the degenerate Fermi pressure that supports white dwarfs and neutron stars.

Topological Chandrasekhar mass. For DM knots of mass m_χ, the equilibrium between gravity and P16 exclusion gives:

$$M_{\rm Ch}^{\rm DM} \sim \frac{M_{\rm Pl}^3}{m_\chi^2} \tag{24.11}$$

For $m_\chi = 21$ GeV (*historical* median of the 117 candidates, Ω_{ts} masses; under the geometric law the dominant reservoir lies at 1.7–6.6 GeV — $M_{\rm Ch}^{\rm DM} \propto 1/m_\chi^2$ would be $\sim$ 10–150× larger, the conclusion is unchanged):

$$M_{\rm Ch}^{\rm DM} \approx \left(\frac{1.22\times10^{19}\ \text{GeV}}{21\ \text{GeV}}\right)^2 \times 1.22\times10^{19}\ \text{GeV} \approx 4\times10^{56}\ \text{GeV} \approx 7\times10^{29}\ \text{kg} \approx 10^4\ M_{\rm Moon} \tag{24.12}$$

Any DM mass below $M_{\rm Ch}^{\rm DM}$ is *stable* against gravitational collapse.

Equilibrium radius. For N DM knots bound by their own gravity and supported by P16, the characteristic radius is:

$$R_{\text{DM-star}} \sim \frac{\hbar^2}{G\, m_\chi^3\, N^{1/3}} \tag{24.13}$$

For $M = 3\,M_{\rm Moon}$, $m_\chi = 21$ GeV and $N \sim 6\times10^{48}$:

$$R_{\text{DM-star}} \sim 10^{-3}\ \text{m} \approx 1\ \text{mm} \tag{24.14}$$

that is, a **dark matter pearl** of about 1 mm in diameter and density $\rho \sim 10^{32}$ kg/m^3 ($\sim 10^{10}$ times the density of a neutron star), without the slightest EM emission.

Synthesis: three routes to compact DM objects

Mechanism	Site	Time	Mass	Size
A (P22)	around baryons	$\sim 10^9$ yr	$\lesssim M_{\rm Ch}$	$\sim$ km–AU
B (primordial)	cosmological	$\sim T_c$	arbitrary	30 m at t_c
C (Chandra DM)	post-formation	$\sim$ dyn. relax.	$\leq 10^4\,M_{\rm Moon}$	$\sim$ mm

The three mechanisms can combine: P22 (A) or a primordial clump (B) creates the overdensity; the final stabilization (C) gives it its equilibrium size. All three lead to **compact DM objects, electromagnetically invisible, but gravitationally detectable** by lensing, microlensing or astrometric perturbation.

24.11.4 Falsifiable observational signatures

These three mechanisms predict **falsifiable populations of compact DM objects** in the 2025–2035 decade:

- **Gaia, Roman, LSST**: anomalous astrometric perturbations (unexplained μas on nearby stars), the signature of a DM pearl (mechanism C) crossing the solar neighbourhood.
- **Microlensing (OGLE, including the high-cadence survey toward the Magellanic Clouds; HSC)**: the 2024–2026 limits *bound* the compact population at $f \leq 1\,\%$ of the halo *continuously* from $1.4\times10^{-8}\,M_\odot$ to the cap $M_{\rm Ch} = 0.50\,M_\odot$ [0.45–0.56]; open window $M \lesssim 5.3\times10^{-11}\,M_\odot$ — the discriminating channel is **Rubin/LSST** (DR1 2028).
- *(The "P22 halo around brown dwarfs" — mechanism A — is withdrawn: null WKB rate.)*
- **Gravitational waves**: merger of compact DM objects (pearls) into binary neutron stars — *merger* waveform deviated by the pearl.
- **Capture cataclysms**: if a DM pearl is captured by a black hole, the tidal energy released is zero in EM (DM invisible) but detectable in gravitational waves.

A prolonged non-detection would refute certain mechanisms. Specifically:

- If Gaia + Roman detect no DM pearls by 2030: exclusion of mechanism C at $M > 10^{20}$ kg in the solar neighbourhood.
- The range $M \gtrsim 1.4 \times 10^{-8}\, M_\odot$ ($\approx 3 \times 10^{22}$ kg) is *already* bounded at $f \leq 1\,\%$ (OGLE); LSST tests beyond these bounds and toward the open window $M \lesssim 5.3 \times 10^{-11}\, M_\odot$.

A positive detection would be a **signature of the primordial population** (kinetics of topological baryogenesis, mechanism B): mechanism A (P22 accretion) is *withdrawn* (null rate).

24.12 Bestiary: newly predicted particles

Beyond dark matter, TQNT predicts **167 masses** with no counterpart in the Standard Model. Here is a representative selection:

m (MeV)	n	s	c_n	Config.
2.8	1	0	4	4_1
14.8	1	1/2	5	5_1
59.8	1	1/2	7	7_1
118	1	1/2	7	7_7
176	1	1/2	9	9_1
222	1	0	7	7_4
636	2	1/2	16	$4_1 + 7_7$
3 362	3	0	20	$5_1 + 6_2 + 9_1$
10 242	3	1/2	22	$7_1 + 7_4 + 8_2$
101 414	3	0	25	$7_3 + 8_3 + 10_1$

Particles with $n = 1$ (single knot) are **topologically stable**. Their non-observation at the FAIR, BES-III or LHC accelerators would constitute a **refutation** of the single-knot approach.

24.13 Cross-validation against experimental constraints

TQNT makes six specific predictions about dark matter, all tested by 2018–2024 observations:

24.13.1 Cosmic abundance Ω_{DM}/Ω_b

- Planck 2018 measurement: $\Omega_{DM}/\Omega_b = 5.45 \pm 0.03$.
- TQNT-V4 prediction (Big Bang condensation kinetics, P18): $\Omega_{DM}/\Omega_b = 4.3$ (deviation $-20\,\%$).
- TQNT prediction (asymmetric formation cost, closed reconstruction *with no free parameter*, §26.7.5): $\Omega_{DM}/\Omega_b = 3.9 \pm 0.4$ at the old adopted $T_c = 67$ ($-29\,\%$ of Planck); at the derived T_c (62 ± 3 MeV, §26.7.8): ≈ 0.6 $[0.2, 1.3]$, the order of magnitude. The fine agreement 5.45 ± 0.05 (α_{cond}) was conditioned on the V5 mass law, superseded; P22 does not contribute (zero rate).

24.13.2 WIMP-nucleon cross-section (direct detection 2024)

The most sensitive experiments give limits at 90 % CL:

Detector	Mass window	σ_{SI} (cm^2)
XENONnT (2023)	5–200 GeV	2.6×10^{-47}
LZ (LUX-ZEPLIN)	9–300 GeV	9.2×10^{-48}
PandaX-4T	5–200 GeV	3.8×10^{-47}
SuperCDMS	0.5–10 GeV	1×10^{-44} (low mass)
DarkSide-50	1–10 GeV	1.5×10^{-44}

No signal detected, excluding classical WIMPs over three orders of magnitude in cross-section.

TQNT prediction: for a topological knot of complexity c, the WIMP-nucleon cross-section is suppressed by the Vassiliev factor α_{EM}^{c-18}:

$$\sigma_{DM-N}^{TQNT}(c) \approx \sigma_{EM} \cdot \alpha_{EM}^{c-18} \tag{24.15}$$

For $c = 19$: $\sigma \sim 10^{-27}$ cm^2; for $c = 20$: $\sigma \sim 10^{-29}$ cm^2; for $c = 30$: $\sigma \sim 10^{-49}$ cm^2. TQNT **predicts the absence of a signal** in the current detector window.

$Q_{\mathbf{phys}}$ bipartition clause. Equation (24.15) applies without restriction to the *dominant* DM reservoir (isolated knots $T(2, q \geq 23)$, $T(3, q \geq 12)$: $c_{\min} = 23$ derived, §24.4), whose c can be arbitrarily large. For the catalogue §24.10 (secondary sub-population, Q_{phys}-neutral links), c is bounded ($c \leq 27$) and (24.15) applied alone would give $\sigma \gg \sigma_{\text{LZ}}$. The closure of the photon exchange channel then comes from the topological parity symmetry:

$$Q_{\text{phys}}(L) = 0 \text{ exactly} \quad \Longrightarrow \quad \langle L \,|\, J_\gamma^\mu \,|\, L \rangle \equiv 0 \tag{24.16}$$

identically at all orders in α_{EM}. The WIMP–nucleon coupling is then governed by the subdominant channels (gluonic, gravitational, Z^0 exchange), with cross-section $\sigma \lesssim 10^{-55}$ cm$^2 \ll \sigma_{\text{LZ}}$. Proposition 17.1 thus remains valid *a fortiori* for these configurations.

Empirical verification on the V5 catalogue. An exhaustive scan of the 117 catalogue candidates (§24.10, script `scripts/V5_alpha_top_k135/catalogue_117_LZ_scan.py`) reveals a *strictly binary* bipartition of Q_{phys}:

- **53 candidates with $Q_{\text{phys}} = 0$ exactly** (6 STRICT pure amphichirals + 47 REALIZABLE with integer Lk-balancing), all LZ-safe by (24.16); they constitute the **secondary stable DM sub-population**.
- **64 candidates with $|Q_{\text{phys}}| \approx 0.048$**, excluded by (24.15); identified with the **transient P22** population (§25.7).

No intermediate value of Q_{phys} appears in the catalogue; this discrete signature confirms the categorical nature of the mechanism (γ channel open or closed, with no grey regime).

24.13.3 Bullet Cluster (1E 0657-558, Clowe et al. 2006)

Observation: two galaxy clusters colliding at $v \approx 4\,700$ km/s, where the baryonic gas (X-ray) is *slowed* by electromagnetic friction while the dark matter (gravitational lensing) passes through *intact* with a separation ~ 25 kpc.

TQNT: knots with $c > 18$ have $\sigma_{EM} \approx 0$ by eq. (24.15), hence are not slowed by the gas. The separation is *automatic*, with no free parameter. **Decisive argument for particle DM: fully compatible.**

24.13.4 Galactic rotation curves (Rubin et al.)

Every spiral galaxy exhibits $v(r) \approx$ constant at large r (instead of Keplerian $1/\sqrt{r}$). This signals the presence of a halo of non-baryonic matter with an approximately NFW profile.

TQNT: the halos are formed by Big Bang kinetic condensation (chap. 26), and the DM is *entirely primordial* (P18): under the WKB form of P22 (§25.7), the nucleation rate at gravitational gradients is *strictly zero* — no halo regeneration, no correction to the profile. The predicted profile is therefore standard NFW, **indistinguishable from ΛCDM** on rotation curves (consistent with the epistemological position §B.6).

24.13.5 CMB constraints, Planck 2018

- $\Omega_{DM}h^2 = 0.1200 \pm 0.0012$: 1 % precision.
- $\sigma_{DM-N}/m_{DM} < 10^{-25}$ cm^2/GeV: supreme limit.
- $m_{DM} > 0.4$ MeV: to avoid excessive relic decay.

TQNT satisfies all the constraints: the minimum mass of a DM knot $T(2, 23)$ is $\sim$ 1.7 GeV $\gg$ 0.4 MeV; the cross-section is $\sim$ 20 orders of magnitude below the limit; and $\Omega_{DM}h^2$ emerges naturally from the condensation kinetics.

24.13.6 The WIMP crisis and the TQNT alternative

The non-discovery of WIMPs over three orders of magnitude in cross-section has constituted, since 2018, a crisis for standard cosmology. TQNT *provides the answer*:

Aspect	Classical WIMPs	TQNT
Formation mechanism	thermal freeze-out	topological condensation (P18)
Mass	$\sim$ 100 GeV–TeV	1.7 GeV–PeV
σ_{DM-N}	10^{-45} cm^2 (excluded)	10^{-27} to 10^{-49} cm^2
Stability	ad hoc symmetry (SUSY R-parity)	topological ($\tau > 10^{347}$ s)
Annihilation $\chi\chi \to f\bar{f}$	yes (searched for, NOTHING)	no (channel closed)
CMB compatibility	yes	yes
Bullet Cluster compatibility	yes	yes
Free parameters	≥ 5 (SUSY) or ≥ 3 (generic WIMPs)	**0** (inherited)

TQNT resolves the "WIMP crisis"

TQNT simultaneously derives:

1. The **existence** of dark matter (P18, inevitable).
2. Its cosmic **abundance** ($\Omega_{DM}/\Omega_b = 5.45$) — **reconstructed at the order of magnitude** (asymmetric formation cost, closed chain with no free parameter, T_c included: ≈ 0.6 $[0.2, 1.3]$ at the derived T_c; 3.9 ± 0.4 with the measured count at the old $T_c = 67$, §26.7.5).
3. Its **stability** ($\tau > 10^{347}$ s).
4. Its **absence in direct detection** (Vassiliev suppression).
5. The **Bullet Cluster** and the rotation curves.

Without introducing any new free particle, without any ad hoc mechanism. The 117 candidates of the V5 catalogue (§24.10) plus the isolated knots $T(2, q \geq 23)$ cover the entire expected observable DM spectrum.

24.14 Dark matter → stellar gas conversion?

Could the gravitational degradation cascade $K(c_n) \to \sum_i K_i(c_i \leq 18) + n_\gamma\,\gamma$ produce hydrogen gas from dark matter?

24.14.1 Baryonic yield

For a typical dark matter knot ($c_n = 25$, $m \approx 500$ GeV), fragmentation produces $\sim c_n/3 \approx 8$ light fragments (total mass ~ 4 MeV). The ratio:

$$f_H = \frac{m_p}{m_{\mathrm{DM}}} \approx \frac{940\ \mathrm{MeV}}{5 \times 10^5\ \mathrm{MeV}} \approx 2 \times 10^{-3} \tag{24.17}$$

99.8 % of the energy leaves as high-energy γ photons ($E_\gamma \sim m_{\mathrm{DM}}/5 \sim 100$ GeV). The baryonic fraction is tiny.

24.14.2 Fragmentation products

The fragments (simple knots 3_1, 4_1, etc.) are mainly free quarks that hadronize immediately ($< 10^{-24}$ s):

- u/d quarks (3_1) $\to$ protons, neutrons, pions
- Pions (8_1) $\to$ muons $\to$ electrons
- Final outcome: a few protons + electrons + dominant γ radiation

The formation of a proton (link $3_1 + 7_2 + \bar{7}_2$, 17 crossings) from the fragments is a **rare** event: it requires the simultaneous encounter of three knots in the fragmentation plasma.

24.14.3 Production rate per galaxy

The Bondi accretion rate (24.6) for Sgr A* ($M = 4 \times 10^6\ M_\odot$, $\rho_{\mathrm{DM}} \sim 100$ GeV/cm^3 at the galactic centre, $v \sim 200$ km/s) gives:

$$\frac{\mathrm{d}M_{\mathrm{DM}}}{\mathrm{d}t} \approx 10^{-3}\ M_\odot/\mathrm{yr}$$

Multiplying by $f_H \approx 2 \times 10^{-3}$:

$$\frac{\mathrm{d}M_H}{\mathrm{d}t} \approx 2 \times 10^{-6}\ M_\odot/\mathrm{yr} \tag{24.18}$$

that is, a **millionth** of the star formation rate of the Milky Way ($\sim 1.5\ M_\odot$/yr).

Over the whole observable universe ($\sim 10^{11}$ galaxies, $\langle M_{\mathrm{SMBH}} \rangle \sim 10^7\ M_\odot$), the gas accumulated over $t_U = 13.8$ Gyr represents $\sim 10^{-7}$ of the total baryonic mass.

Result: no stellar gas formation

The dark matter $\to$ hydrogen gas conversion is **negligible**:

- Baryonic yield $f_H \sim 2 \times 10^{-3}$ (almost everything in γ radiation).
- Rate $\sim 10^{-6}\ M_\odot$/yr per galaxy (a millionth of the SFR).
- Cosmic fraction $\sim 10^{-7}$ of the total baryons.

Nodal dark matter **does not form stellar gas**. The primordial hydrogen and helium gas comes exclusively from primordial nucleosynthesis (links formed at $T \sim 10^9$ K, $t \sim 180$ s).

24.14.4 Gravitational trapping: no electromagnetic signal

Since $r_{\text{crit}} \ll r_S$ by 14 to 20 orders of magnitude (§24.8), the degradation occurs **exclusively inside the horizon**. Gravitational time dilation ($g_{00} \to 0$ as $r \to r_S$) traps all electromagnetic emission:

- The cascade γ photons are emitted at $r \sim r_{\text{crit}} \ll r_S$: they cannot escape the black hole.
- From the outside, the dark matter knot appears "frozen" at the horizon (infinite gravitational redshift).
- The only observable effect is the **mass growth** of the black hole ($\Delta M \sim 10^{-3}\ M_\odot/\text{yr}$ for Sgr A*).

Prediction: total invisibility of dark matter

TQNT predicts a **strictly invisible** dark matter:

1. Interaction cross-section $\sigma = 0$ (neither EM, nor strong, nor weak).
2. No annihilation or decay signal ($\tau > 10^{347}$ s).
3. No γ signal from gravitational degradation (trapped inside black holes).

The only detectable effects are **purely gravitational**: rotation curves, gravitational lensing, CMB. This distinguishes TQNT from WIMP models, which predict detectable γ and antimatter signals.

24.15 Effect of the time dilation of astrophysical bodies

A massive body dilates local time: $g_{00} = -c^2(1 - r_S/r)$ with $r_S = 2GM/c^2$. The dilation at the surface is $\Delta\tau/\tau \approx GM/(Rc^2) = r_S/(2R)$:

Body	r_S (m)	R (m)	$h = r_S/R$
Earth	8.9×10^{-3}	6.4×10^6	1.4×10^{-9}
Sun	3.0×10^3	7.0×10^8	4.2×10^{-6}
White dwarf	1.8×10^3	6×10^6	3.0×10^{-4}
Neutron star	4.1×10^3	10^4	0.41

This dilation **does not affect** the stability of dark matter knots, for three independent reasons:

1. The CS barrier is topological. The instanton action $S_{\text{inst}} = k\pi$ does not depend on the metric (Witten's theorem [1]). The CS–metric coupling (P5) introduces a correction $\delta S/S = h \times \alpha_{\kappa_g} \sim h \times 10^{-40}$:

Body	h	Barrier correction
Sun	4×10^{-6}	4×10^{-46}
Neutron star	0.41	3×10^{-41}
Black hole	1	8×10^{-41}

The tunneling exponent ($2u \times k\pi = 5089$ for $u = 6$) is modified by **zero significant digits** in all cases.

2. Disruption is a gradient, not an absolute value. Time dilation (g_{00}) is an **absolute** effect (the value of the potential at a point). Tidal disruption is a **gradient** ($\partial^2\Phi/\partial r^2 \propto GM/r^3$). A uniform gravitational field, however intense, creates no tidal force: the knot falls freely without deformation (equivalence principle).

3. The effective mass correction is negligible. In general relativity, $m_{\text{eff}} = m\sqrt{1-h}$. For a neutron star ($h = 0.41$), m_{eff} decreases by 23 %, which increases r_{crit} by 19 %. But $r_{\text{crit}} \sim 10^{-10}$ m remains $\sim 10^{14}$ times smaller than the radius of the star. For non-compact bodies (planets, stars), the correction is $< 10^{-6}$.

Result: insensitivity to time dilation

The time dilation of an astrophysical body modifies **none** of the preceding calculations:

- CS barrier: topological, independent of g_{00}.
- Disruption: tidal force (gradient), not dilation (absolute value).
- Effective mass: correction $< 10^{-6}$ for any non-compact body.

Nodal dark matter passes through all astrophysical bodies (except black holes) without capture, without degradation, and without producing baryonic matter.

24.16 Topological origin of chemistry

Why do electrons bind only to protons (and to nuclei containing protons) to form the 92 natural elements? The answer lies in the **topological uniqueness** of the proton.

24.16.1 The proton: the only stable charged baryon

For a baryon to capture an electron and form an atom, three conditions are necessary:

1. **Charge** $Q > 0$: electromagnetic attraction with the electron ($Q = -1$).
2. **Effective stability** ($\tau_p \approx 8 \times 10^{33}$ yr, finite — cf. next subsection): surviving on chemical timescales ($\gg 10^{-8}$ s), met with $\sim 10^{49}$ orders of margin.
3. **Mass** $\gg m_e$: serving as the centre of mass (atomic nucleus).

Among the positively charged baryons of the TQNT spectrum:

Baryon	Config.	Q	τ	Atom?
Proton	$\mathbf{3_1{+}7_2{+}\bar{7}_2}$	**+1**	$\approx$**8**$\times$**10^{33} yr**	**YES**
Σ^+	$6_2{+}7_2$	+1	8×10^{-11} s	no (decays in 80 ps)
Δ^{++}	$3_1{+}5_2{+}6_2$	+2	$\sim 10^{-24}$ s	no (resonance)
Λ_c^+	$6_3{+}8_1$	+1	2×10^{-13} s	no (decays in 0.2 ps)
Σ_c^{++}	$4_1{+}5_1{+}7_1$	+1	$\sim 10^{-22}$ s	no (resonance)

All positively charged baryons **except the proton** decay before they can capture an electron (recombination time $\sim 10^{-13}$ s).

24.16.2 Proton stability: strong barrier, finite lifetime

The proton is the linking ground state. Its stability comes from *barriers* (and *not* from an exact baryonic conservation: χ is SU(2), only the $\mathbb{Z}_2$ parity is robust, and $u(7_2) = 1$ means the twist knot can be unknotted):

1. **Lightest of its class**: it is the lightest irreducible 3-component link with $Q = +1$; the easy relaxation channel (ΔLk) is closed.
2. **Optimal linking**: Lk $= +3$ is the energy minimum of the harmonic potential $E_{\text{tension}} = \beta_{\otimes}(\text{Lk} - \text{Lk}_{\text{opt}})^2$.

The only remaining decay (undoing the two 7_2) is suppressed by α_{EM}^{18}, hence a **finite** lifetime $\tau_p \approx 8 \times 10^{33}$ yr (and not ∞): in the vicinity of the Super-Kamiokande limit, **testable at Hyper-Kamiokande**.

24.16.3 Chemistry as a consequence of topology

Result: topological foundation of chemistry

The 92 natural chemical elements rest entirely on **a single link**: $3_1 + 7_2 + \bar{7}_2$ (proton). It is the lightest topological configuration that is positively charged and absolutely stable. All other charged configurations are eliminated by their instability ($\tau \ll 10^{-8}$ s).
The electron (3_1, charge -1) binds to the proton through the EM interaction: the same field $B = \tau_{\perp}(s)$ that confines the knot produces the long-range Coulomb potential. The neutron ($3_1 + 7_2 + \bar{7}_2$, Lk $= +2$, $Q = 0$) stabilizes nuclei through linking tension without modifying the charge.
Chemistry is a **direct consequence** of knot topology: $92 \times (Z + N)$ nuclei from a single type of link and two values of Lk.

Dark matter and chemistry. Dark matter knots ($c_n > 18$) form **no chemistry**: $\sigma_{\text{EM}} = 0$ (no electron capture), no strong nuclear interaction (no "dark" nuclei). Quantitatively: the polarizability $\alpha = 0$ (no electrons) implies a Van der Waals coefficient $C_6 = 0$ **exactly** (chapter 20). Without Van der Waals, no liquid/solid phase transition is possible: dark matter is a **pure gravitational gas**, with no chemical equivalent. Its halos remain spherical because $C_6 = 0$ forbids the radiative dissipation required for disc formation.

24.17 Vassiliev astro/primordial consolidation: six classes of compact objects (V5p19)

V5p19 catalogues six classes of compact objects predicted by TQNT and tests each against observations. From this emerges a **structural result** that cleanly separates what TQNT can predict from what it renders invisible *by construction*.

The six classes.

class	object	formation	verdict
A	dark matter pearls	late (post-recombination binding)	survives
B	black hole seeds (PBH)	late	survives
C	hybrid stars	late	survives
D	heavy proton (BBN plasma)	*direct primordial*	trivial null
E	strange knot (amphichiral)	late	survives
F	IMBH defects	late	survives

Demonstration: the Vassiliev suppression rule. The cross-section of a process creating a knot of complexity c is suppressed as $\sigma \sim \sigma_0\, \alpha_{\mathrm{EM}}^{c-18}$ (Vassiliev suppression, chap. 24). For class D (*direct primordial* creation of a heavy proton $c = 20$ from the plasma):

$$\sigma_D \sim \alpha_{\mathrm{EM}}^{2} \approx 5 \times 10^{-5}\, \sigma_0, \quad \text{abundance} \sim \alpha_{\mathrm{EM}}^{20} \approx 10^{-43}, \tag{24.19}$$

i.e. a signal 45 decades below the BBN limits: **trivial null, by construction**. By contrast, classes A, B, C, E, F are bound *after* recombination, by gravitation: there the knots are *already formed*, the Vassiliev suppression does not apply, and TQNT is **testable**. Hence the rule: *already formed objects ⇒ constrained by observation; primordial effects requiring the creation of high-c knots ⇒ invisible by construction.* This is a *structural signature* of TQNT, not a failure: 5/6 classes survive.

Five testable positive predictions (2026–2030).

1. Microlensing (Rubin/LSST, DR1 2028) → class A pearls — the *discriminating* channel of the dark sector (the old "survives Subaru ×5" was a mass-window artefact; OGLE 2024 bounds $f \leq 1\,\%$ continuously from 1.4×10^{-8} to $M_{\mathrm{Ch}} = 0.50\, M_\odot$);
2. Gaia DR4 → class C stacked envelope at 26σ (2027);
3. LISA → class F mergers, 0.05–12 events/mission, quantized total mass 2–6 $\times 10^5\, M_\odot$, no EM counterpart (requalification 2026-07-30: class F mergers fall *below* the ∼10 Hz wall of ground-based detectors — LVK structurally blind, the old "LIGO O5, 10^{-2}–1/yr" is obsolete);
4. JWST + Roman → class B AGN seed abundance, ±1 dex (2027+);
5. class D: *no path* (Vassiliev — by construction).

Honesty (no longer to be claimed). Class B *over-predicts* the abundance of the JWST "impossible" galaxies by +8 dex: TQNT does not "solve" the early galaxies. Class D being a trivial null (45 dex below the limits), TQNT predicts *no* distinct BBN signature. What survives: class A *bounded* by microlensing (OGLE: $f \leq 1\,\%$ of the halo continuously; the V5.19 "survives Subaru ×5" was a mass-window artefact), class E (lines lifted, F3; microlensing), class F requalified to LISA (LVK structurally blind), and the *Vassiliev astro/primordial pattern* as a validated structural signature. DOI: 10.5281/zenodo.20708582.

24.18 Periphery anomaly: the halo concentration–mass slope (V5.6)

The concentration–mass relation $c(M_h)$ of dark matter halos is a strong observable signature (Wechsler–Bullock: slope -0.10 ± 0.03, 135 SPARC galaxies). The V5.3 version (conformal mechanism alone) predicted a slope $+0.575 \pm 0.038$, i.e. a tension of $\mathbf{18.4}\sigma$: the pure conformal mechanism is *falsified* as the sole cause. Closure requires an additional mechanism — the **periphery anomaly** of the Wilson loops of $\mathrm{SU}(2)_{135}$.

Demonstration: $\delta_{\mathrm{framing}} = -3/8 \cdot n_0 = -0.599$. The Gauss phase accumulated by the χ condensate during hierarchical formation follows the Press–Schechter kinetics:

$$\tau_{\mathrm{form}}(M_h) \sim \sigma^{3/2}(M_h) \sim M_h^{-(n_{\mathrm{eff}}+3)/4}, \tag{24.20}$$

$n_{\mathrm{eff}} \approx -1.5$ being the Planck spectral index at galactic scales. The number of twists of the condensate satisfying $n_{\mathrm{twist}} \propto \tau_{\mathrm{form}}$, the periphery exponent is

$$\epsilon = -\frac{n_{\mathrm{eff}} + 3}{4} = -\frac{3}{8} \quad (n_{\mathrm{eff}} = -1.5). \tag{24.21}$$

With the calibration $n_0 \simeq 1.6$ (adiabatic dilution of the condensate from $T_c = 67$ MeV — the historical adopted value of the calibration; T_c now derived at 62 ± 3 MeV, §26.7.8 — to today, V5 §13.5),

$$\boxed{\delta_{\text{framing}} = n_0\,\epsilon = 1.6 \times \left(-\tfrac{3}{8}\right) = -0.599} \tag{24.22}$$

against the best empirical fit $\delta = -0.60$: agreement at $\mathbf{0.2}\,\%$. The mechanism is Atiyah's *2-framing* (1990) of 3-dimensional manifolds, already present in the quantization of Chern–Simons (Witten 1989, Reshetikhin–Turaev 1991).

Empirical closure. The combination (A) periphery anomaly + (C) $1/c$ expansion (Atiyah's 2-framing correction, $\Delta = \alpha_c/c_{\text{med}}$) brings the $c(M_h)$ slope from $+0.573$ (V5.3) down to $\mathbf{-0.106}$, in agreement with Wechsler–Bullock at $\mathbf{0.1}\sigma$ over the 135 SPARC galaxies (Lelli+ 2016). This is an *empirical* validation (commit `9c63393`): the periphery anomaly is not a fitted parameter but a derived consequence of the condensation kinetics and of Chern–Simons topology.

Chapter 25

Dark energy and vacuum energy

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25.1 The cosmological constant problem

In standard quantum field theory, the vacuum energy diverges: $\rho_{\rm vac}^{\rm QFT} \sim m_P^4 c^3/\hbar^3 \sim 10^{113}$ J/m^3, i.e. $\sim 10^{123}$ times the observed dark energy density $\rho_\Lambda^{\rm obs} \approx 5.26 \times 10^{-10}$ J/m^3 (Planck 2018 [12], $\Omega_\Lambda = 0.685$, $H_0 = 67.36$ km/s/Mpc).

TQNT resolves this problem in three steps, all governed by the integer $k+2 = 137$: the same integer that determines inflation (§26.4), baryogenesis and the fine-structure constant.

25.2 Step 1: topological cutoff (-93 orders)

In TQNT, only fluctuations forming topological knots with $c_n \geq 3$ crossings contribute to the vacuum energy (natural UV regulator). The mass scale is set by the electron (the simplest knot, 3_1):

$$\rho_{\rm vac}^{\rm nod} = \frac{m_e^4 c^5}{32\pi^2\hbar^3} \times e^{-\beta_\Lambda \times c_J(3_1)} \approx 2.21 \times 10^{20}\ \text{J/m}^3 \tag{25.1}$$

with $\beta_\Lambda = 0.914$ (topological Boltzmann factor) and $c_J(3_1) = 3.3$.

Justification of the cutoff. In TQNT, the only vacuum fluctuations that contribute to ρ_{vac} are those that form topological knots. The vacuum energy is the sum over virtual knots:

$$\rho_{\text{vac}}^{\text{nod}} = \sum_K \frac{m(K)^4 c^5}{32\pi^2\hbar^3} P(K)$$

where $P(K) \propto e^{-\beta_\Lambda c_J(K)}$ is the topological Boltzmann probability of forming the knot K through a quantum fluctuation. The dominant term is that of the simplest knot (3_1, $m = m_e$):

$$\rho_{\text{vac}}^{\text{nod}} \approx \frac{m_e^4 c^5}{32\pi^2\hbar^3} \times e^{-\beta_\Lambda c_J(3_1)}$$

Physically, $e^{-\beta_\Lambda c_J}$ is the exponential suppression of the probability of forming a knot with c_J crossings through a vacuum fluctuation: simple knots dominate. The ratio $\rho_{\text{vac}}^{\text{nod}}/\rho_\Lambda^{\text{obs}} \approx 4.2\times10^{29}$: the topological cutoff eliminates **93 orders of magnitude** ($10^{123} \to 10^{30}$).

25.3 Step 2: metric coupling α_{κ_g}

Postulate P5 ($\Omega^2 = 1 + \kappa_g|\Psi|^2$) couples the topological sector to the emergent metric. The dimensionless ratio:

$$\alpha_{\kappa_g} = \frac{\kappa_g}{\lambda\!\!\!^{-}_C(e^-)} = \frac{\kappa_g\, m_e c}{\hbar} = \frac{3.12\times10^{-53}\ \text{m}}{3.86\times10^{-13}\ \text{m}} = 8.08\times10^{-41} \tag{25.2}$$

measures the strength of the coupling between the gravitational scale ($\kappa_g = 8\pi G m_p/c^2$, a *length*; convention documented in eq. 16.3) and the quantum scale of vacuum fluctuations ($\lambda\!\!\!^{-}_C(e^-) = \hbar/(m_e c)$, consistent with the m_e^4 of the formula): a ratio of two lengths, *dimensionless*. *N.B.*: the $\lambda\!\!\!^{-}_C$ in the derivation of κ_g (eq. 16.2) is that of the *proton* — two distinct Compton wavelengths, now made explicit.

25.4 Step 3: non-perturbative suppression

25.4.1 The exponent $p = d/(d+1) = 3/4$

The Chern-Simons action (P10) is topological: it does not depend on the metric. At the quantum level, the modulus of the partition function $|Z_{\text{CS}}|$ is a topological invariant (Cheeger-Müller theorem [18]: $|Z_{\text{CS}}^{\text{1-b}}|^2 = \tau_{\text{RS}}$, Ray-Singer torsion). The pure CS sector has a *strictly zero* vacuum energy.

The non-zero vacuum energy comes from the coupling between the topological sector (χ, in $d = 3$ spatial dimensions) and the emergent metric (in $d+1 = 4$ spacetime dimensions). This coupling **breaks topological invariance**; the fraction of spacetime protected by topology is $d/(d+1)$:

$$\frac{\rho_\Lambda}{\rho_{\text{vac}}^{\text{nod}}} = c_{\text{grav}} \times \alpha_{\kappa_g}^{d/(d+1)} \qquad \text{(the } 1/(32\pi^2) \text{ is already in } \rho_{\text{vac}}^{\text{nod}}\text{; corrected 2026-07-14)} \tag{25.3}$$

Numerical check:

$$p = \frac{\log\!\left(\rho_\Lambda/\rho_{\text{vac}}^{\text{nod}}\right)}{\log\!\left(\alpha_{\kappa_g}\right)} = \frac{-29.62}{-40.09} = 0.739 \approx \frac{3}{4} \qquad \text{(deviation 1.5 \%)} \tag{25.4}$$

25.4.2 Gravitational central charge

Integrating out the matter sector over the emergent metric, the effective action induces a gravitational Chern-Simons term whose coefficient is the central charge:

$$c_{\text{grav}} = \frac{3k}{k+2} = \frac{405}{137} \approx 2.96 \tag{25.5}$$

This central charge is the same one that determines the latent heat of topological inflation: $\Delta c = c_{\text{free}} - c_{\text{grav}} = 6/(k+2) = 6\,\alpha_{\text{EM}}$ (§26.4).

25.5 Formula: topological dark energy

Result: topological dark energy

$$\boxed{\rho_\Lambda = \frac{c_{\text{grav}}}{32\pi^2} \times \frac{m_e^4 c^5}{\hbar^3} \times e^{-\beta_\Lambda\, c_J(3_1)} \times \alpha_{\kappa_g}^{3/4}} \tag{25.6}$$

$\rho_\Lambda^{\text{predicted}}$	$= 5.56 \times 10^{-10}$ J/m^3	(TQNT)
$\rho_\Lambda^{\text{obs}}$	$= 5.25 \times 10^{-10}$ J/m^3	(Planck 2018 [12], $H_0 = 67.36$): deviation $+5.9\,\%$
$\rho_\Lambda^{\text{obs}}$	$= 6.17 \times 10^{-10}$ J/m^3	(SH0ES 2022, $H_0 = 73.04$): deviation $-9.9\,\%$

The 123 orders of magnitude of the cosmological constant problem are resolved with no additional free parameter (β_Λ and k are already fixed by dark matter and the spectrum): the prediction falls **within the band of the Hubble tension** ($\rho_\Lambda^{\text{obs}} \propto H_0^2$, spread 17.6 %). The residual deviation with respect to either determination is *not significant* (2026-07-14 audit: structural noise of the formula $\gtrsim 15\,\%$ via β_Λ, γ_J, p; `audit_rho_lambda_amont.py`) — there is **no residual to explain**. Dimensional convention: $\kappa_g = 8\pi G m_p/c^2$ is a *length* and $\alpha_{\kappa_g} = \kappa_g/\lambda\!\!\!^{-}_C(e^-)$ a ratio of lengths, dimensionless (documented in eq. 16.3).

Numerical evaluation. The four factors are:

1. $c_{\text{grav}}/(32\pi^2) = 2.956/315.8 = 9.36 \times 10^{-3}$,
2. $m_e^4 c^5/\hbar^3 = 1.42 \times 10^{24}$ J/m^3 (electron energy density; *corrected* 2026-07-14, the previously printed value 6.83×10^{22} was a typo),
3. $e^{-\beta_\Lambda\, c_J(3_1)} = e^{-0.914 \times 3.3} = e^{-3.02} = 0.0489$ (topological cutoff),
4. $\alpha_{\kappa_g}^{3/4} = (8.08 \times 10^{-41})^{3/4} = 8.51 \times 10^{-31}$ (metric suppression).

Product: $9.36 \times 10^{-3} \times 1.42 \times 10^{24} \times 0.0489 \times 8.51 \times 10^{-31} = 5.56 \times 10^{-10}$ J/m^3.

Breakdown of the 123 orders.

Mechanism	Orders eliminated	TQNT source
Topological cutoff ($c_n \geq 3$)	-93	P1 (instability of nothingness)
Suppression $\alpha_{\kappa_g}^{3/4}$	-30	P5 (emergent metric)
Prefactor $c_{\text{grav}}/(32\pi^2)$	$+0.4$	P10 (Chern-Simons, $k = 135$)
Total	$\mathbf{-122.6}$	
Residual	~ 0	within the H_0 tension band (not significant)

25.6 Link with topological inflation

Dark energy and inflation share the same origin: the Chern-Simons level $k = 135$.

- **Inflation** ($T \sim 10^{31}$ K): the latent heat of the CS transition, proportional to $\Delta c = 6/(k{+}2)$, powers the exponential expansion (§26.4).
- **Dark energy** ($T \sim 2.7$ K): the topological residue of this transition, filtered by the $d/(d{+}1)$ protection and the charge $c_{\text{grav}} = 3k/(k+2)$, produces the present-day acceleration.

The hierarchy $V_{\text{inf}} \gg \rho_\Lambda$ is explained by the suppression $\alpha_{\kappa_g}^{3/4} \sim 10^{-30}$ of the residue relative to the initial latent heat: dark energy is the **fossil trace** of topological inflation.

25.7 Nucleation at boundaries — and the dissolution of the "5.6 % residual"

The residual deviation between the TQNT prediction ($\rho_\Lambda^{\text{TQNT}} = 5.56 \times 10^{-10}$ J/m^3) and the Planck measurement ($\rho_\Lambda^{\text{obs}} = 5.26 \times 10^{-10}$ J/m^3) amounts to 5.6 %. This section formalizes **topological nucleation at metric boundaries** (Postulate P22) — and records the double verdict of 2026-07-14: the resorption *mechanism* is **null** (calculation with no free parameter), and the *deviation* itself has **dissolved** under the upstream audit (not significant: Hubble tension band, structural noise).

25.7.1 Postulate P22: nucleation at metric boundaries

P22 — Nucleation at metric boundaries (WKB form, 2026-07)

At the boundary between a knotted region ($|\Psi|^2 > 0$, matter) and the vacuum ($|\Psi|^2 \to 0$), the gradient of the emergent metric $\Omega^2 = 1 + \kappa_g |\Psi|^2$ acts as a trigger for topological nucleation. The rate of spontaneous formation of a knot per unit volume and time follows:

$$\boxed{\Gamma(\mathbf{r}) \;=\; \Gamma_P \exp\!\Big[-\frac{2\pi}{\xi(\mathbf{r})}\Big], \qquad \xi(\mathbf{r}) = \frac{|\nabla\Omega^2(\mathbf{r})|\,\lambda\!\!\!^{-}_C}{\Omega^2(\mathbf{r})} \;=\; \frac{2a\,\lambda\!\!\!^{-}_C}{c^2}} \tag{25.7}$$

where Γ_P is the microscopic attempt rate (Planck scale, multiplied channel by channel by the P18 topological prefactors) and $\lambda\!\!\!^{-}_C = \hbar/(m_e c)$ the reduced Compton wavelength of the electron — *derived*: per species $\xi_m = 2a\lambda\!\!\!^{-}_C(m)/c^2$, the dominant channel is the lightest mass. The exponent $-2\pi/\xi$ is *derived* (charged-pair channel; $-4\pi/\xi$ for the thermal channel): via the CGQ identity $\mathbf{g} = (c^2/2)\nabla(\Omega^2 - 1)$, it *is* the Schwinger exponent $-\pi m^2 c^3/(qE\hbar)$, and the Unruh–Hawking Boltzmann factor $-mc^2/k_B T_U$ equals exactly $-4\pi/\xi$ (identities verified, `derive_eta_p22_test.py`). *Historical note*: the initial form $\Gamma_0\, e^{\eta\xi}$ ($\eta \sim 1$ never calibrated, Γ_0 free) is **superseded**: the WKB form eliminates *both* free parameters and promotes the Hawking/Schwinger unification from qualitative to *exact*.

Limiting regimes (unchanged in substance).

- *Deep vacuum* ($\xi \to 0$, far from any matter): $\Gamma \to 0$ (double exponential: at terrestrial g, $\ln(\Gamma/\Gamma_P) \sim -10^{28}$). Negligible residual rate.
- *Baryonic neighbourhood* ($\xi \sim 1$, CGQ skin): $\Gamma \sim \Gamma_P\, e^{-2\pi}$. Nucleation amplified at the "skin" of matter.
- *Horizon / critical field* ($\xi \gg 1$): $\Gamma \to \Gamma_P$, recovering *exactly* (and no longer qualitatively) the Schwinger rate ($\xi = 2$ at the critical field: $e^{-\pi}$) and the Hawking factor.

Epistemic consequence: P22 no longer has *any* free parameter; the reattribution of the dark energy residual has become a calculation *with no free parameter* — done: **null mechanism**; and the upstream audit has *dissolved* the deviation itself (next §).

25.7.2 Reattribution of the dark energy residual

The calculation carried out in §25.5 assumes a *uniform* nucleation of the vacuum. P22 corrects this assumption: what V3 attributes entirely to a vacuum density is in fact the sum of **two disjoint contributions**:

$$\rho_\Lambda^{\mathrm{TQNT}} = \underbrace{\rho_\Lambda^{\mathrm{vide}}}_{\text{true cosmological constant}} + \underbrace{\rho_{\mathrm{nucl,bord}}}_{\text{nucleation at boundaries (P22)}} \tag{25.8}$$

Estimate of $\rho_{\mathrm{nucl,bord}}$. The contribution of nucleation at baryonic boundaries reads:

$$\rho_{\mathrm{nucl,bord}} = \langle\, \Gamma(\mathbf{r})\, m_e c^2 \,\rangle_{\mathrm{cosmos}} - \Gamma_{\mathrm{vide}}\, m_e c^2 \tag{25.9}$$

where the average is taken over all of cosmological space. This contribution is *not* a cosmological constant: these are real knots, created and destroyed at the boundary of matter. It must be subtracted before comparison with the observable $\rho_\Lambda^{\mathrm{obs}}$.

The calculation with no free parameter — done, and negative (2026-07-14). The target was $\rho_{\mathrm{nucl,bord}} = \rho_\Lambda^{\mathrm{TQNT}} - \rho_\Lambda^{\mathrm{obs}} = 0.30 \times 10^{-10}$ J/m^3 (5.6 %), numerically suggested by $\Omega_b \approx 5\,\%$. Since the WKB form, Γ no longer has any parameter: the calculation was *carried out* (`p22_dark_energy_noknob.py`) with a *majorizing* scheme ($f_{\mathrm{vol}} = 1$, $\tau = t_{\mathrm{Hubble}}$, best boundary in the universe). Verdict: on the reference emergent metric (gravitational, P5), ξ is negligible everywhere (hadronic skin 1.4×10^{-36}, neutron star 10^{-17}, solar BH horizon 1.3×10^{-16}, halos 4×10^{-81}): $\ln \rho_{\mathrm{bord}} \leq -4.8 \times 10^{16}$ — **strictly zero**. One would need $\xi^* \approx 0.02$ ($a^* \sim 2 \times 10^{27}$ m/s^2 sustained cosmologically): nothing provides it. The historical hypothesis "$\xi \sim 1$ at the baryonic skin" required $\Delta\Omega^2 \sim 2.2 \times 10^{-3}$ at the edge of the hadron; gravitation gives 3×10^{-39} (**a 36-decade gap**, which the form $\Gamma_0 e^{\eta\xi}$ concealed behind its two free parameters).

2026-07-14 assessment — neither mechanism nor problem: the "residual" is dissolved

Double verdict: *(i)* the surface contribution $\rho_{\mathrm{nucl,bord}}$ is **null** under the WKB form with no free parameter (the resorption mechanism falls); *(ii)* the **upstream audit** (`audit_rho_lambda_amont.py`, draft `brouillon_audit_rho_lambda`) shows that the 5.6 % deviation *was never significant*: the prediction falls *within the band of the Hubble tension* (+5.9 % vs Planck, −9.9 % vs SH0ES; spread 17.6 %) and below the structural noise of the upstream formula (β_Λ known to $\gtrsim 15\,\%$; γ_J to one digit; 0.08 % on the 3/4 exponent would suffice). The real prediction — remarkable — is the **order of magnitude** (123 orders resolved). The falsification candidate "F4" has therefore *dissolved*: neither mechanism nor problem. The $\xi \gg 1$ regimes of P22 (*exact* Hawking/Schwinger) are intact; P22 no longer has *any* cosmological application.

Observational consequences.

- **No regeneration of the dark-matter halo.** Under the WKB form, the rate at gravitational gradients is *strictly zero*: dark matter is **entirely primordial** (P18). (The former prediction of a halo permanently regenerated at the baryonic skin, carried by the $\Gamma_0 e^{\eta\xi}$ form, is obsolete.)

- **Schwinger pair production.** The threshold $E > E_{\text{Schwinger}} \approx 1.3 \times 10^{18}$ V/m, already invoked (§19) as the breaking of the topological vacuum into $3_1 + 3_1^*$, is identified with the regime $\xi \gtrsim 1$ of equation (25.7).

- **Hawking radiation as the upper limit.** The thermal flux leaving a black hole is the limit $\xi \to \xi_{\text{horizon}} \gg 1$, where Γ becomes macroscopic.

25.8 Cross-validation with cosmological observations

TQNT makes seven specific predictions about dark energy, all confronted with the 2018–2024 data.

25.8.1 The "cosmological constant problem": 85 orders of magnitude

The quantum vacuum density naively computed by quantum field theory with a Planck cutoff:

$$\rho_{\text{vide}}^{QFT} \sim \frac{E_{Pl}^4}{16\pi^2} \approx 1.4 \times 10^{74}\ \text{GeV}^4 \tag{25.10}$$

exceeds the observed dark energy density:

$$\rho_\Lambda^{obs} = (2.3\ \text{meV})^4 \approx 2.8 \times 10^{-11}\ \text{GeV}^4 \tag{25.11}$$

by a factor of $\mathbf{5 \times 10^{84}}$, i.e. **85 orders of magnitude** (convention $E_{\text{Pl}}^4/16\pi^2$; the Planck cutoff in mass-energy density gives the **123 orders** quoted at the beginning of the chapter — the *same* problem, two QFT conventions). Steven Weinberg (1989) calls this discrepancy the "worst problem of theoretical physics".

TQNT resolution. TQNT *circumvents the problem*: there is no colossal vacuum energy because:

1. The "vacuum" in TQNT corresponds to the trivial state $\Omega^2 = 1$, $h_g = 0$, with no mass or integrated energy.
2. Gravity being *emergent* (P19, G derived), there is no coupling of the vacuum to the background metric.
3. The observed dark energy comes exclusively from **nucleation at boundaries** P22, not from a vacuum density proper.

The factor 10^{85} disappears because the two calculations (ρ_{vide}^{QFT} and ρ_Λ^{TQNT}) *measure different things*: the former is a fiction of naive QFT (infinite sum of the zero-point modes of the vacuum), the latter is the *real surface contribution* at the baryon-vacuum boundaries.

25.8.2 Equation of state $w(z)$: Planck vs DESI 2024

- *Planck 2018 (CMB):* $w = -1.03 \pm 0.03$, compatible with pure Λ.
- *DESI 2024 (BAO):* $w_0 = -0.65 \pm 0.10$, $w_a = -1.27 \pm 0.40$, **a hint of dynamical evolution**.
- *Planck + DESI combined:* slight preference ($\sim 3\sigma$) for an evolving w.

TQNT prediction:

$$w(z) = -1 \quad \text{exactly, at every epoch} \tag{25.12}$$

(vacuum-residue theorem: $\rho_{\text{nucl,edge}} = 0$ strictly, and the residue neither dilutes nor withdraws — $dw/dz = 0$ by construction). **DESI's slight preference for an evolving w must therefore regress; any *confirmed* evolution $w(z) \neq -1$ would refute the theorem.** (The old V5 prediction of a $\delta w(z)$ carried by P22 nucleation is *obsolete*: the cosmological P22 rate is null.)

25.8.3 Hubble tension H_0

The tension has persisted for 15 years:

Method	H_0 (km/s/Mpc)	Source
Early CMB (Planck 2018)	67.4 ± 0.5	Primordial universe
SH0ES 2022 (late-time SNe)	73.0 ± 1.0	Local universe
Tension	**4.5** km/s/Mpc	**4–5**σ

TQNT position: P22 nucleation is cosmologically *null* (WKB form, no free parameter): it creates no bias between epochs. TQNT is pinned to $H_0 = 67.4$ (the CMB reading) by a fourfold wall (elimination + Birkhoff + elasticity + river): the local tension belongs to a distance-ladder systematic or a genuine local void in GR — it is one of the four *inherited* problems (with S_8, primordial lithium and the muon puzzle). The old reinterpretation as a "signature of P22 activity" is *obsolete*.

25.8.4 Spectral index n_s and tensor-to-scalar ratio r

Quantity	Measurement	TQNT
n_s (Planck 2018)	0.9649 ± 0.0042	0.9655 ($N_e \approx 58$)
r (BICEP+Keck 2021)	< 0.036 (95 % CL)	≤ 0.01 (Vassiliev suppression)

Both TQNT predictions are compatible with the observations.

25.8.5 Comparison with other models

Aspect	Λ-CDM	Quintessence	**TQNT**
Free Λ?	yes, fitted	replaced by ϕ	no, derived (P22)
85-orders problem	unresolved	unresolved	**circumvented**
Evolving $w(z)$	no ($w = -1$)	yes (ad hoc)	no: $w = -1$ ($\rho_{\text{nucl,bord}} = 0$)
H_0 tension	unexplained	ad hoc	not resolved (the prediction falls *within* the band)
Dark matter	ad hoc	ad hoc	**derived** (P18)
Free parameters	1 (Λ)	≥ 2 ($V(\phi)$)	0

Dark energy: what TQNT derives — and what it does not solve

TQNT simultaneously derives:

1. The **existence** of dark energy (P22, inevitable).
2. Its **value** $\Omega_\Lambda = 0.685$, predicted *within the band of the Hubble tension* (+5.9 % Planck / −9.9 % SH0ES; with no significant residual).
3. $w = -1$ **exactly** (theorem; DESI must regress — any confirmed evolution would refute).
4. The **Hubble tension** H_0: the explanation by continuous P22 creation is *obsolete* (null rate; the falsification candidate "F4" dissolved at the upstream audit).
5. The absence of the **85-orders problem** (trivial vacuum).
6. The CMB parameters n_s, r (cf. topological inflation).

Without introducing any new ad hoc field, without a free potential, without tuning a cosmological constant.

The same P22 mechanism unifies: **dark energy** ($\xi \ll 1$) + **Schwinger pair production** ($\xi \sim 1$) + **Hawking radiation** ($\xi \gg 1$).

25.8.6 Falsifiable predictions for the decade 2025–2030

- *ROMAN Space Telescope measurement (2027+):* $w(z = 0.5\text{–}1.5)$ with 0.01 precision — TQNT requires $w = -1$ exactly; any confirmed evolution would refute it.
- *DESI Phase 2:* tight constraint on w_a — same verdict: the current preference for an evolving w must regress.
- *Euclid (ongoing):* radial halo profiles not strictly NFW (γ_{P22} correction).
- *CMB-S4 (2030+):* measurement of n_s to 10^{-4}, r to 10^{-3}.
- *JWST deep halo observations:* evolution of Ω_{DM}/Ω_b with z — TQNT predicts *no* evolution (null P22 regeneration: constant primordial ratio).

Open points.

- Rigorous derivation of $p = d/(d+1)$ via the effective coupled CS + metric action.
- Equation of state: $\rho_{\text{nucl,bord}} = 0$ strictly $\Rightarrow$ TQNT predicts $w = -1$ *exactly*; any confirmed deviation $w(z) \neq -1$ (DESI+) would be a *tension* for TQNT, not a P22 success.
- (η is *eliminated*: the exponent $-2\pi/\xi$ is derived and reproduces Schwinger *exactly* — the former "independent calibration of η" is moot.)

Chapter 26

Kinematics of the topological Big Bang

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26.1 Principle

In TQNT, the Big Bang is a **topological transition**: the field χ passes from the trivial state (no knots) to a state populated by knots and links. The instability of the void (P1) triggers this transition. The temperature T determines the available energy $E \sim k_B T$: a knot of mass m can only form if $E > m$.

26.2 Chronology

t (s)	T (K)	E (MeV)	TQNT event
10^{-43}	10^{32}	10^{31}	**Planck era.** $h \sim 1$. The field χ fluctuates. No defined time (g_{00} undetermined).
10^{-36}	10^{28}	10^{27}	**Topological inflation.** Knots form and unravel. The energy released by the topological transitions fuels the exponential expansion.
10^{-12}	10^{15}	10^{14}	**Electroweak breaking.** The knots W, Z, H, t freeze when $T < m/k_B$. The most unstable ones ($\tau < t_H$) decay immediately.
10^{-6}	10^{12}	10^{11}	**Topological QCD transition.** The irreducible links (proton = $3_1{+}7_2{+}\bar{7}_2$) stabilize. The complex knots *born decoupled* from the thermal background ($c \geq 23$, derived: $\Gamma/H(T_c) < 1$, §24.4) → dark matter. **Birth of G and κ_g** (a single step, §26.3).
1	10^{10}	10^{9}	**Nucleosynthesis.** The neutrons (Lk = +2) convert into protons (Lk = +3) by topological β-decay (change of Lk). Ratio $n/p \to {}^4$He.
180	10^{9}	10^{8}	**End of nucleosynthesis.** Primordial abundances frozen.
10^{13}	3000	0.26	**Recombination.** The electrons (3_1) bind to the protons. The CMB is released.
4.4×10^{17}	2.7	2.3×10^{-4}	**Today.** $\Omega_{\rm DM}/\Omega_b \approx 5.4$ (Planck; TQNT asymmetric formation cost: ≈ 0.6 at the derived T_c, order of magnitude, §26.7.5). The dark knots dominate because they are the *symmetric* relic (never annihilated).

26.3 Evolution of κ_g and of the gravitational constant

The chronology above has a precise gravitational side (consolidated from the metric matching, §17.7):

Phase	**Gravitational state**
$t < t_c$ (Planck, inflation, wave background)	Only the **rigidity** exists ($M_{\rm Pl}$, a property of the $k = 135$ algebra); gravitation is the *global* metric response of the χ wave background (expansion; $R + aR^2$ sector with $a = (k{+}2)^5/6$ in $M_{\rm Pl}$ units — the "k bridge"). κ_g and G are **undefined**: no localized wells, no Newtonian force.
χ condensation ($T_c = 62 \pm 3$ MeV *derived*, §26.7.8; $t_c \approx 1.6 \times 10^{-4}$ s)	**Birth in a single step**, directly at the final values: $\kappa_g = 3.1201 \times 10^{-53}$ m, $G = 6.6710 \times 10^{-11}$ m^3 kg^{-1} s^{-2} — their ingredients ($12/7$, α^{18}, $\sqrt{\sigma_\chi}\,\mathcal{S}$) are fixed by topology and by the condensation itself. The duration of the step is that of the condensation (the domain/string epoch of the Ω reconstruction).
$t > t_c$	**Constant.** No epoch evolution (the n/p abundance does not enter: the neutron's linking tension is *state* energy, which gravitates in the source mass without touching the coupling) and no per-particle variation (universality — the 19/07 discriminant). Prediction: $\dot{G}/G = 0$ strictly (LLR bound $\lvert\dot{G}/G\rvert < 1.5 \times 10^{-13}$/yr trivially satisfied); any measured drift would refute the single step.

The 10^{-36} hierarchy is thus born *with* matter: before t_c, the wave background perturbs the

metric with no topological suppression (the α^0 rung of the matching ladder); afterwards, every knot counts α per crossing — knot gravity is weak because its flux is closed, and it is so *from the start*, at the value it has kept ever since.

26.4 Topological inflation

26.4.1 Chern-Simons phase transition

The SU(2) CS theory at level $k = 135$ undergoes a confinement/deconfinement phase transition at the scale:

$$\boxed{\Lambda_{\rm CS} = \frac{E_{\rm Pl}}{\sqrt{k+2}} = \frac{E_{\rm Pl}}{\sqrt{137}} = \sqrt{\alpha_{\rm EM}}\, E_{\rm Pl} \approx 1.04 \times 10^{18}\ {\rm GeV}} \tag{26.1}$$

This scale emerges naturally as the ratio of the Planck energy to the square root of the CS level: $\sqrt{1/137} = \sqrt{\alpha_{\rm EM}}$.

Central charges. In the deconfined phase ($T > T_c$), the 3 generators of SU(2) are free: $c_{\rm free} = \dim({\rm SU}(2)) = 3$. In the confined phase ($T < T_c$), the central charge is $c_{\rm CS} = 3k/(k+2) = 405/137 \approx 2.96$. The difference:

$$\Delta c = c_{\rm free} - c_{\rm CS} = \frac{6}{k+2} = \frac{6}{137} = 6\,\alpha_{\rm EM} \approx 0.044 \tag{26.2}$$

26.4.2 Latent heat of the transition

The energy released by the transition is the latent heat:

$$\mathcal{L} = \frac{\Delta c}{12} \times \frac{\pi^2}{30} \times T_c^4 \tag{26.3}$$

that is, as an energy density:

$$V_{\rm inf} = \mathcal{L} \approx 3.6 \times 10^{107}\ {\rm J/m^3}\,, \qquad V_{\rm inf}^{1/4} \approx 3.6 \times 10^{17}\ {\rm GeV} \tag{26.4}$$

This scale is consistent with GUT-type inflation ($V^{1/4} \sim 10^{16}$–10^{18} GeV).

26.4.3 Number of e-folds

The topological transition proceeds as a **cascade**: each type of knot K freezes at its own temperature $T_{\rm fo}(K) = m(K)/43$. The total number of e-folds is determined by the nucleation time $\Delta t = 1/(\alpha_{\rm CS} \times H_{\rm inf})$, that is:

$$\boxed{N_e^{\rm total} = \frac{1}{\alpha_{\rm CS}} = k + 2 = 137} \tag{26.5}$$

Derivation. The topological phase transition lasts a time Δt determined by the nucleation rate of the "bubbles" of confined phase. In CS theory, this rate is:

$$\Gamma_{\rm nucl} = \alpha_{\rm CS} \times H_{\rm inf} = \frac{H_{\rm inf}}{k+2}$$

The transition is complete when $\Gamma_{\rm nucl} \times \Delta t = 1$, i.e. $\Delta t = (k+2)/H_{\rm inf}$. The number of e-folds during this interval is:

$$N_e = H_{\rm inf} \times \Delta t = H_{\rm inf} \times \frac{k+2}{H_{\rm inf}} = k + 2 = 137$$

The total number of e-folds is **the inverse of the fine-structure constant**.

26.4.4 CMB predictions

The observables of the cosmic microwave background depend on N_*, the number of e-folds remaining at the moment when the pivot scale $k_* = 0.05$ Mpc^{-1} leaves the horizon. The standard cosmological calculation gives $N_* \approx 57$ for $V^{1/4} \sim 10^{17}$ GeV.

The topological cascade produces a **plateau potential** (a multitude of small steps). The slow-roll parameters give:

Spectral index.

$$\boxed{n_s = 1 - \frac{2}{N_*} = 1 - \frac{2}{57} = 0.9649} \qquad \text{(Planck 2018 [12] : } 0.9649 \pm 0.0042\text{)} \tag{26.6}$$

In agreement with Planck (compatible).

Tensor-to-scalar ratio.

$$\boxed{r = \frac{12}{N_*^2} = \frac{12}{57^2} \approx 0.004} \qquad \text{(BICEP/Keck [14] : } r < 0.036\text{)} \tag{26.7}$$

Compatible with the current limits, testable by CMB-S4 and LiteBIRD (sensitivity $\sigma_r \sim 10^{-3}$).

Result: topological inflation

The Chern-Simons phase transition at level $k = 135$ predicts:

- Inflation scale: $V^{1/4} = E_{\text{Pl}} \times \sqrt{\alpha_{\text{EM}}} \approx 10^{18}$ GeV.
- Total number of e-folds: $N_e = k + 2 = 137 = 1/\alpha_{\text{EM}}$.
- $n_s = 0.965$ (Planck: 0.9649 ± 0.0042).
- $r \approx 0.004$ (testable by CMB-S4).

All the inflation parameters are determined by $k = 135$: **no free parameter**.

26.4.5 Extended validation on 12 cosmological observables

Beyond n_s and r, TQNT makes predictions on the whole set of CMB and large-scale-structure observables, all tested against Planck 2018, BICEP+Keck 2021 and DESI 2024.

Observable	TQNT ($k = 135$)	Observation
α_{EM}	$1/(k+2) = 1/137$	1/137.036 (CODATA)
N_e total	$k + 2 = 137$ e-folds	> 60 (sufficient)
N_* (pivot)	~ 57	57 ± 3
n_s	$1 - 2/N_* = 0.9649$	0.9649 ± 0.0042 *EXACT*
r	$12/N_*^2 \approx 0.004$	< 0.036 (BICEP+Keck)
$\alpha_s = dn_s/d\ln k$	$-2/N_*^2 \approx -6.2 \times 10^{-4}$	-0.005 ± 0.013
A_s	1.89×10^{-9}	$(2.105 \pm 0.030) \times 10^{-9}$
Ω_k	~ 0 (after 137 e-folds)	0.0007 ± 0.0019
f_{NL} local	~ 0.01 (slow-roll)	-0.9 ± 5.1
$V^{1/4}$	$E_{Pl}/\sqrt{k+2} \sim 10^{18}$ GeV	consistent Lyth
η_B	$\sim 6 \times 10^{-10}$	$(6.1 \pm 0.1) \times 10^{-10}$
n_M/n_γ	~ 0 (no 0D def.)	$< 10^{-29}$ (limit)

Inflation and the flatness problem. Without inflation, $\Omega_k(t_{BBN}) \approx \Omega_k^{\text{obs}} \times 10^{60}$ (extreme fine-tuning of the Big Bang). With $N_e = 137$ e-folds, the exponential expansion automatically dilutes the curvature by a factor $e^{2N_e} \sim 10^{118}$, more than amply sufficient to reproduce the observed flatness ($\Omega_k \approx 7 \times 10^{-4}$).

Horizon problem. The size of the causal horizon at the moment of decoupling (CMB) is $d_H \approx c/H \approx 200$ Mpc, much smaller than the angular coherence distance of the CMB (~ 14 Gpc). With 137 e-folds, all the regions of the CMB were causally connected *before* inflation, explaining the isotropy observed at $\Delta T/T < 10^{-5}$.

26.4.6 Comparison with the other inflation models

Model	Free param.	Predicted n_s	Predicted r
Higgs inflation	1–2 (ξ_H, m_t)	adjustable	$\sim 10^{-3}$
Starobinsky R^2	1–2	~ 0.965	$\sim 3 \times 10^{-3}$
Chaotic ϕ^2	1 (m_ϕ)	~ 0.967	~ 0.13 (**excluded**)
Natural inflation	1–2	variable	~ 0.03 (**limit**)
TQNT	**0**	**0.9649**	**0.004**

TQNT: the only inflation model with zero free parameters

TQNT is the *only framework* that predicts:

1. The **existence** of inflation (Chern-Simons transition at $k = 135$).
2. The predicted number of e-folds ($N_e = 137 = 1/\alpha_{EM}$).
3. The spectrum ($n_s = 0.9649$, compatible with Planck).
4. The scale ($V^{1/4} \sim 10^{18}$ GeV).
5. The **complete chain** inflation → baryogenesis → DM → dark energy.

With no ad hoc inflaton field, no free potential.

All these predictions share a **single integer** $k = 135$ which also controls α_{EM}, Ω_Λ, Ω_{DM}/Ω_b, η_B and φ_{CP}. A precise measurement of a single observable (e.g. r at 10^{-4} via CMB-S4) would simultaneously test the whole framework.

Falsifiable predictions 2025–2035.

- *CMB-S4 (USA, 2030+):* $\sigma_r \sim 10^{-3}$, direct measurement of the $r = 0.004$ predicted by TQNT.
- *LiteBIRD (JAXA, $\sim$ 2032):* $\sigma_r \sim 10^{-3}$, independent confirmation.
- *SPHEREx, Roman:* measurement of f_{NL} at $\sigma \sim 1$.
- *SKA + LiteBIRD combined:* global TQNT consistency test on 5 independent observables.
- *If $r > 0.01$ or n_s deviates from 0.9649 ± 0.001:* refutation of the current TQNT.

26.4.7 Synthesis: the unifying role of $k + 2 = 137$

Observable	TQNT formula	Value
Fine-structure constant	$\alpha_{\text{EM}} = 1/(k+2)$	$1/137$
Inflation scale	$\Lambda_{\text{CS}} = E_{\text{Pl}}/\sqrt{k+2}$	10^{18} GeV
Number of e-folds	$N_e = k + 2$	137
Latent heat (inflation)	$\Delta c = 6/(k+2)$	0.044
Grav. charge (dark energy)	$c_{\text{grav}} = 3k/(k+2)$	2.96
CP phase (baryogenesis)	$\varphi = \arg(V_{3_1}(e^{2\pi i/(k+2)}))$	5.7×10^{-4}

26.5 Why dark matter exists

During the cooling, **all** knot types form when $k_B T > m(K)$. The question is: which ones **survive**?

1. **Single knots ($n = 1$)** with $c_n \leq 18$: topologically stable, visible (EM) → electrons, light quarks.
2. **Single knots ($n = 1$)** with $c_n > 18$: the Vassiliev suppression *begins* at $c^* = 18$; the *effective* reservoir requires $c \geq 23$ (derived: decoupling at birth + survival, §24.4) → **dark matter**.
3. **Irreducible links** ($n = 2, 3$): stable if the lightest of their class → proton, neutron.
4. **Reducible or excited links**: unstable (finite τ) → decay into stable particles.

Dark matter is inevitable: complex knots are **more numerous** than simple ones — and even more so than the historical asymptotics $N \sim 2.68^{c_n}/c_n$ claimed, which massively *undercounts* (exact counts up to $c = 19$: per-crossing ratios 4.7 → 6.28 at $c = 20$, still growing (verified 2026-07-15, Table 3 of Thistlethwaite AGT 2025; $N(20) = 1\,847\,319\,428$); the $2.68^c/c$ is a factor ~ 45 too low as early as $c = 19$). Under the reference asymmetric formation cost (formation Boltzmann factor included), the dark population is dominated by the *edge* $c = 23$–25 (§26.7.5); and the **dominance** $\Omega_{\text{DM}} > \Omega_b$ comes from the annihilation asymmetry (symmetric dark relic vs visible CP crumb), not from numbers.

26.6 Topological baryogenesis

26.6.1 The Chern-Simons action violates parity

The Chern-Simons action (P10):

$$S[\chi] = \frac{k}{4\pi} \int_{\mathbb{R}^3} \text{Tr}\big(\chi \wedge d\chi + \tfrac{2}{3}\, \chi \wedge \chi \wedge \chi\big) \tag{26.8}$$

is a **pseudo-scalar**: under the parity transformation $P\colon \mathbf{x} \to -\mathbf{x}$, the CS 3-form changes sign:

$$P\colon S_{\text{CS}}[\chi] \;\longrightarrow\; -S_{\text{CS}}[\chi] \tag{26.9}$$

This property is intrinsic: it follows from the pseudo-tensorial character of the exterior product of three 1-forms in odd dimension. The CS action therefore violates P *explicitly*, without requiring spontaneous breaking.

Consequence for C and CP. In TQNT, charge conjugation C is the mirror reflection $K \to K^*$ (theorem 11.6: $Q(K^*) = -Q(K)$). The Jones polynomial of the mirror is:

$$V_{K^*}(q) = V_K(\bar{q}) = \overline{V_K(q)} \qquad \text{for } |q| = 1 \tag{26.10}$$

So the Jones polynomial of the mirror is the **complex conjugate** of that of the particle. For a chiral knot ($K \not\simeq K^*$), $V_K(q) \neq \overline{V_K(q)}$, that is, $\mathrm{Im}(V_K(q)) \neq 0$.

Theorem 26.1 (CP violation by the CS invariant)**.** *Let $q = e^{2\pi i/(k+2)}$ be the root of unity of the CS level. For every chiral knot K:*

$$\mathrm{Im}\big(V_K(q)\big) \neq 0 \quad \textit{and} \quad \mathrm{Im}\big(V_{K^*}(q)\big) = -\,\mathrm{Im}\big(V_K(q)\big) \tag{26.11}$$

For every amphichiral knot: $\mathrm{Im}\big(V_K(q)\big) = 0$.

Proof. $V_{K^*}(q) = \overline{V_K(q)}$ by (26.10), so $\mathrm{Im}(V_{K^*}) = -\,\mathrm{Im}(V_K)$. If K is amphichiral, $V_K = V_{K^*}$, so $\mathrm{Im}(V_K) = -\,\mathrm{Im}(V_K) = 0$. □

26.6.2 CP-violating phase of the Jones polynomial

Let us define the **CP-violation parameter** of a knot:

$$\boxed{\varepsilon_{\mathrm{CP}}(K) = \frac{2\ \mathrm{Im}\big(V_K(q)\big)}{|V_K(q)|}} \qquad q = e^{2\pi i/(k+2)} = e^{2\pi i/137} \tag{26.12}$$

This parameter measures the quantum interference between the nucleation channels K and K^* in the CS path integral. It is **entirely determined** by the topology of the knot and the level $k = 135$.

Explicit calculation for the trefoil 3_1. $V_{3_1}(t) = t + t^3 - t^4$. At $q = e^{2\pi i/137}$:

$$\begin{aligned} V_{3_1}(q) &= e^{2\pi i/137} + e^{6\pi i/137} - e^{8\pi i/137} \\ &= 1.00628 + 0.000577\,i \end{aligned} \tag{26.13}$$

whence:

$$\varphi_{\mathrm{CP}}(3_1) = \arg\big(V_{3_1}(q)\big) = 5.74 \times 10^{-4}\ \mathrm{rad}\,, \qquad \varepsilon_{\mathrm{CP}}(3_1) = 1.15 \times 10^{-3} \tag{26.14}$$

CP phases of the torus knots (fermionic generations).

Knot	Generation	φ_{CP} (rad)	$\varepsilon_{\mathrm{CP}}$	$\varphi \times 137/\pi$
3_1	1st	5.74×10^{-4}	1.15×10^{-3}	0.025
5_1	2nd	2.82×10^{-3}	5.65×10^{-3}	0.123
7_1	3rd	7.72×10^{-3}	1.54×10^{-2}	0.336
9_1	— (no 4th: Z_3 saturation)	1.60×10^{-2}	3.21×10^{-2}	0.699
4_1	(amphichiral)	$< 10^{-16}$	≈ 0	0

The CP phase grows with the complexity of the knot: the heavier generations violate CP more strongly. The amphichiral knots have $\varepsilon_{\mathrm{CP}} = 0$ *exactly*, consistent with their status as particles equal to their own antiparticle.

26.6.3 Topological Sakharov conditions

The three Sakharov conditions [21] are satisfied:

1. B **violation**: the topological transitions (reconnections of the field χ) change the number of components of the links, modifying the baryon number.

2. C **and** CP **violation**: the CS action is P-odd (26.9); the phase $\varphi_{\rm CP} \neq 0$ for 16 of the 19 chiral knots (84 %) breaks C and CP simultaneously.

3. **Out of equilibrium**: the topological phase transition ($T \sim T_c$) is an irreversible process: knots form faster than they unravel at $T < T_c$.

26.6.4 Asymmetric nucleation and estimate of η_B

At the topological phase transition ($T = T_c$), the knots K and K^* condense out of the field χ. The nucleation amplitude of a knot K is proportional to $V_K(q)$ (the expectation value of the Wilson loop in CS theory). The interference between the direct channel and the conjugate channel produces an excess:

$$\frac{n(K) - n(K^*)}{n(K) + n(K^*)} = \varepsilon_{\rm CP}(K) \tag{26.15}$$

After the transition, the K-K^* pairs annihilate. Only the asymmetric surplus survives. The annihilation rate, dominated by the electromagnetic cross-section of the lightest particle (the trefoil $3_1 \to e^-$, mass m_e), determines the surviving fraction. The resulting baryon-to-photon ratio is:

$$\boxed{\eta_B = \frac{1}{3}\, \varepsilon_{\rm CP}(3_1)\; P_{\rm topo}(3_1)\; \left(\frac{m_e}{T_c}\right)^2} \tag{26.16}$$

where:

- $\varepsilon_{\rm CP}(3_1) = 1.15 \times 10^{-3}$: CP violation of the trefoil (exact calculation from $V_{3_1}(e^{2\pi i/137})$),
- $P_{\rm topo}(3_1) = e^{-3\alpha} = 2.72 \times 10^{-2}$: topological condensation probability ($\alpha_{\rm cond} = 1.20$),
- $(m_e/T_c)^2$: *annihilation-suppression* factor (suppression by matter/antimatter annihilation),
- 1/3: three brick-knots per baryon (the SM "quarks", Rule 0).

Dependence on T_c. The critical temperature of condensation is now *derived*: $T_c = 62 \pm 3$ MeV (topological tightening, §26.7.8); the scan below is kept as a sensitivity analysis (the ≈ 67 MeV scale appearing in it is the annihilation freeze-out $T_{\rm ann}$, tied to the vev — cf. the status box further down):

T_c (MeV)	η_B	Ratio to Planck
200	6.8×10^{-11}	0.11
100	2.7×10^{-10}	0.44
67	$\mathbf{6.0 \times 10^{-10}}$	**0.99**
50	1.1×10^{-9}	1.77

Result: matter/antimatter asymmetry

The Chern-Simons chirality breaking predicts:

$$\eta_B = \frac{1}{3}\,\frac{2\,\mathrm{Im}\big(V_{3_1}(e^{2\pi i/137})\big)}{|V_{3_1}(e^{2\pi i/137})|}\,e^{-3\alpha}\,\left(\frac{m_e}{T_c}\right)^2 \tag{26.17}$$

For $T_c \approx 67$ MeV:

$$\eta_B = 6.0 \times 10^{-10} \qquad (\text{Planck 2018 [12]} : 6.12 \pm 0.04 \times 10^{-10})$$

Over the range $T_c \in [50, 200]$ MeV, η_B stays within the interval $[10^{-10}, 10^{-9}]$: the **right order of magnitude** is obtained without any additional free parameter. *Update (V7, 2026-08):* the chain carried to the *derived* T_c (62 ± 3 MeV) gives $\eta_B = 7.1 \times 10^{-10}$, within $-13\,\%$ of Planck (thermal freeze-out refuted, leaching link named) — consolidated in the edition of confrontations (V7).

Consolidated mechanism: nucleation, no relic. The asymmetry is imprinted *at the condensation* of the field χ (K/K^* interference, exact trefoil $\varepsilon_{\mathrm{CP}}$), and the departure from equilibrium (Sakharov #3) *is the phase transition* itself. There is **no long-lived heavy relic**: the CP carrier is the trefoil. (An alternative periphery via out-of-equilibrium decay of a heavy baryonic link is *excluded*: mass capped at $n \leq 6$ knots, cascade lifetime $\lesssim 10^{-9}$ s $\ll$ Hubble time; cf. the baryogenesis draft.)

Honest status of T_c: order of magnitude, not "0.99×Planck"

The "$T_c \approx 67$ MeV" that hits 6×10^{-10} spot-on is *not* derived at first hand. It must be decomposed into two physically distinct scales, conflated in the formula:

- T_{trans} (nucleation) = deconfinement of χ, ~ 155 MeV, *first-hand*; by itself it gives $\eta_B \approx 1 \times 10^{-10}$, i.e. $\sim 0.2\times$Planck (a factor ~ 5 low);
- T_{ann} (freeze-out of the K-K^* annihilation, in the factor $(m_e/T)^2$) ≈ 67 MeV: this scale is tied to the reconnection cross-section, hence to the *vev* (weak scale), **not derived**.

So η_B is correct **at the order of magnitude** (a factor ~ 5 from the sole first-hand scale), *not* parameter-free in the strict sense: the coincidence at $0.99\times$Planck rests on $T_{\mathrm{ann}} \approx 67$ MeV, dependent on the vev. Consistent with the honest position of the framework (§B.6): correct structure (large and exact CP violation, out-of-equilibrium via phase transition), fine value not derived. The complete exploration (routes, cascade, reconnection cross-section, derivation of the vev by Ω_{ts} at $\sim 15\,\%$) is archived in the draft `brouillon_baryogenese_Tc`.

26.6.5 Consistency and interpretation

1. **Amphichiral knots**: $\varepsilon_{\mathrm{CP}}(4_1) = \varepsilon_{\mathrm{CP}}(6_3) = \varepsilon_{\mathrm{CP}}(8_3) = 0$ exactly. These knots are their own antiparticle: no asymmetry is generated, in complete consistency with theorem 11.6.
2. **Four independent factors**: formula (26.16) combines four ingredients determined separately:
   - the Jones polynomial (pure topology),
   - the CS level $k = 135$ (dark energy, chap. 25),

- the penalty $\alpha_{\rm cond} = 1.20$ (ratio $\Omega_{\rm DM}/\Omega_b$),
- the electron mass m_e (electromagnetic equilibrium formula).

No parameter is adjusted for baryogenesis: T_c is the only remaining degree of freedom.

3. **Common origin of** $k+2=137$: the same integer $137 \approx 1/\alpha_{\rm EM}$ simultaneously controls the CP violation ($q = e^{2\pi i/137}$), the dark energy ($c_{\rm grav} = 3k/(k+2)$) and the fine-structure constant.

26.7 Condensation kinetics and the ratio $\Omega_{\rm DM}/\Omega_b$

26.7.1 Topological freeze-out

At high temperature ($k_BT > m(K)$), the knots are "inflated" by the thermal energy and unravel into photons (free waves of χ). A knot survives if its destruction rate falls below the Hubble rate:

$$\Gamma_{\rm dest} = \frac{k_BT}{\hbar}\, e^{-m(K)/(k_BT)} < H(T) \tag{26.18}$$

The numerical resolution gives a universal freeze-out ratio:

$$x_{\rm fo} = \frac{m(K)}{k_BT_{\rm fo}} \approx 43 \qquad \text{(independent of } c_n\text{)} \tag{26.19}$$

Each knot freezes when k_BT drops to $m/43$.

26.7.2 Topological condensation probability

The probability that a fluctuation of the field χ forms a knot with c_n crossings is:

$$P_{\rm topo}(c_n) = N(c_n) \times e^{-\alpha\, c_n} \tag{26.20}$$

where:

- $N(c_n)$ = number of prime knots with c_n crossings ($N \sim 2.68^{c_n}/c_n$),
- α = **topological penalty per crossing**: the energy cost of forcing the field χ to cross itself one more time.

26.7.3 Determination of α

The frozen mass density of a knot type is $\Omega(c_n) \propto m(c_n) \times Y(c_n)$, where $Y(c_n)$ is the abundance per entropy at freeze-out:

$$Y(c_n) \propto P_{\rm topo}(c_n) \times \underbrace{x_{\rm fo}^{3/2}\, e^{-x_{\rm fo}}}_{\text{Boltzmann tail}}$$

The second factor is the Boltzmann suppression evaluated at freeze-out: it is the thermal occupation number at the moment of freezing. Since $x_{\rm fo} \approx 43$ is **universal** (independent of c_n, §26.18), this factor is **identical for all knots** and **cancels in any ratio**. The cosmic ratio therefore simplifies to:

$$\frac{\Omega_{\rm DM}}{\Omega_b} = \frac{\displaystyle\sum_{c_n>18} m(c_n) \times P_{\rm topo}(c_n)}{\displaystyle\sum_{c_n\le 18} m(c_n) \times P_{\rm topo}(c_n)} \tag{26.21}$$

Only the **mass** $m(c_n)$ (equilibrium formula) and the **topological probability**

$$P_{\rm topo}(c_n) = N(c_n)\, e^{-\alpha\, c_n}$$

enter. The parameter α controls the competition between the exponential growth of $N(c_n)$ and the exponential suppression $e^{-\alpha\, c_n}$.

The historical calculation (V5 era) — doubly conditioned. The original calculation used the FEM masses then the extrapolation $m \sim c_n^{6.65}$, and the Hoste-Thistlethwaite count [11] ($c \leq 16$) matched to $N \sim 2.68^{c_n}/c_n$; the scan then gave $\Omega_{\rm DM}/\Omega_b = 5.5$ at $\alpha = 1.20$ (Planck: 5.45 ± 0.28), with a convergent series. This result, long displayed as a success, was **doubly conditioned**: *(i)* on the V5 mass law (boost $m(23)/m(3) \sim 8 \times 10^5$ on the dark side under $c^{6.65}$ — the reference law is *geometric*, quasi-linear); *(ii)* on the $2.68^c/c$ undercount (the exact counts grow at $4.7 \to 6.28$/crossing up to $c = 20$: a factor ~ 45 as early as $c = 19$, and the matching was even decreasing, $N(17) < N(16)$).

26.7.4 Re-analysis under the reference inputs (2026-07-15)

The analyses are redone (`omega_ratio_reaudit.py`, `omega_true_Nc.py`, `omega_scan_canonique.py`) with: reference **geometric** masses ($M_{\rm topo} \propto \ell(K)$, torus knots, insensitive to the m_s demotion); **verified** counts (2026-07-15: exact ≤ 16; Burton 17–19 confirmed up to one correction, $N(17) = 8\,053\,393$; *new* $N(20) = 1\,847\,319\,428$, Thistlethwaite AGT 2025; extrapolation $r_\infty \in [6.5\,;10.4]$, *rigorous* lower bound $r_\infty > \lambda_{\rm alt} = (101+\sqrt{21001})/40 = 6.1479$, Sundberg–Thistlethwaite); $c_{\min} = 23$ **derived** (§24.4); $c_{\max} \simeq 92$ **sourced** (geometric wall + locking read on the lattice, $\kappa_{\rm dom} = 1.14$, Caselle-Nada). The α derived from the kinetics under geometric masses is 1.214–1.220 (the historical 0.076 % was conditioned on the V5 law: now an *order-of-magnitude* success).

α	r_∞=6.5	r_∞=8.0	r_∞=10.4
1.22 (derived)	2×10^{21}	1×10^{27}	3×10^{34}
1.79	**5.6**	1×10^{6}	3×10^{13}
1.94	6×10^{-4}	**4.4**	6×10^{7}
2.13	3×10^{-6}	1×10^{-5}	**4.7**

Re-analysis 2026-07-15: the symmetric class is structurally unable to produce 5.45

Under the reference inputs the series **diverges** (net rate $r_\infty e^{-\alpha} \simeq 1.8$): at the derived α, even the minimal cutoff $c_{\max} = c_{\min} = 23$ gives $\Omega = 9.0$, the sourced $c_{\max}$ ($\simeq 92$) gives $\sim 10^{21}$; and the thermal formation factor flips it to the other extreme ($\sim 10^{-4}$). The outcome is *bimodal*: 5.45 is reachable only on a knife edge — the "weighted sum on both sides" class is missing physics. Counts *verified* (ratio 6.28 at $c = 20$, exact bound $\lambda_{\rm alt} = 6.1479$): counting uncertainty is not an exit.

26.7.5 The asymmetric formation cost (reference 2026-07-16)

The missing physics is the **asymmetry of fates**: *(i)* the **visible side annihilates** (knot + mirror, $\sigma_{\rm EM}$) — only the CP crumb survives, $\eta_B \simeq 6.1 \times 10^{-10}$, the one baryogenesis (§26.7.9) computes; *(ii)* the **dark side** ($c \geq 23$, born decoupled) has Vassiliev-crushed annihilation: a **fully symmetric relic**. The ratio inherits a factor $1/\eta_B \sim 1.6 \times 10^9$. To this is added the **formation Boltzmann factor** $e^{-m(c)/T_c}$, *derived* from TQNT ingredients (χ-wave background, equiprobability = fixed point of the mixing by the χ dance [2nd law, §20.18], emergent T; the cost is the *wave entropy* the formation forgoes, counted at the *full* mass — the "melted tension" variant $\sigma(T_{\rm lock})$ is refuted numerically; `brouillon_boltzmann_tqnt`). The geometric step 70.2 MeV $\simeq T_c$ makes the net rate $6.5\, e^{-1.22} e^{-1.05} = 0.68 < 1$: the series **converges without any cutoff** ($c_{\max}$ moot; the energy is supplied by the condensate's *latent heat*, $E_{\rm dom}^{\rm latent} \sim \Lambda_\chi^4 V_{\rm dom} \approx 16$–$39$ GeV: no ceiling on the reservoir).

$$\frac{\Omega_{\rm DM}}{\Omega_b} = \frac{(n_d/s)\,(s_0/n_{\gamma 0})\,\bar{m}_d}{\eta_B\, m_p}, \qquad \frac{n_d}{s} = \frac{135}{8\pi^3 g_s \kappa_{\rm dom}^3}\, P_{\rm occ}\, \frac{\sum_{c\geq 23} N(c)\, e^{-\alpha c - m(c)/T_c}}{Z},$$

(Kibble–Zurek quench normalisation, second-order transition, Ising class confirmed on the lattice; $\kappa_{\rm dom} = 1.14$ sourced).

$\Omega_{\rm DM}/\Omega_b$ reconstructed at the order of magnitude — with no free parameter

Nominal: $\Omega_{\rm DM}/\Omega_b = 15.6$ ($\times 2.9$ of Planck 5.45); input envelope (no fitting): $\alpha \to 15$–23; T_c 60–75 MeV $\to$ 1.1–195 (dominant sensitivity, but T_c is *triply* constrained). g_s **DERIVED** (2026-07-17, `derive_gs_effectif.py`: entropy integrals, marginal μ/π nearly full): $g_s(T_c) = 15.2 \pm 0.4 \Rightarrow \Omega = 14.3$ ($\times 2.6$). All inputs are derived/sourced/measured. **Dominance mechanism identified**: dark matter dominates because it is the *symmetric relic* (never annihilated) while visible matter is only the *CP crumb* ($1/1.6 \times 10^9$). The historical 5.5 "worked" because the V5 mass boost and the undercounting numerically mimicked the missing (annihilation $\times$ formation-Boltzmann) product. Status: *order-of-magnitude* success — tightened on 2026-07-17 by the χ **string-network simulation** (Vachaspati-Vilenkin type, 110 592 domains, $\mathbb{Z}_3$ triality phases) + the fragmentation cascade:

$$P_{\rm occ} = \underbrace{\tfrac{8}{9}}_{\substack{\text{string supply}\\ \text{EXACT (piercing 24/81; geometry: nodal anchoring, §3.4)}}} \times \underbrace{f_{\rm frac} = 1 - e^{-2e^{-\alpha_{\rm cond}}} = 0.45}_{\substack{\text{knotted fraction per fragment}\\ \text{DERIVED (tree identity, } \Delta{\rm Lk}=\pm 1)}} \times \tfrac{1}{\lambda},$$

(*NB 2026-07-23*: the weight $e^{-\alpha_{\rm cond}}$ receives a candidate closed form from the spacetime weave-rail combinatorics — $\alpha = \ln(2+\sqrt{2}) = 1.2279$, the capture-window theorem, §3.4; the numerical chain here keeps its established values.) Where λ (final fragment size in ξ) is *derived* (2026-07-17, `derive_lambda_kinks.py`): the exact Nambu dynamics (left/right-mover decomposition, static IC at freeze-out, periodicity theorem for terminal fragments), applied to the embedded fragments of the measured network, is a *scale-invariant* cascade of measured coefficient $\lambda = (1.14 \pm 0.03)\,\ell_{\rm cut}$; the endpoint is *not* radiative: the measured scaling law of the radiation functional, $F(L) = 0.0675\,L^{0.56}$ (0.05 dex dispersion, `settle_race_weight.py`), shows that $\Gamma_a(\ell) \sim \ell^{0.56}$ stays below the required $2\pi\kappa_s \ln(\ell/D)$ at all scales $\lesssim 18\,\xi$ (the $18\,\xi$ crossing is only a *pre-smoothing* of large fragments): the cascade runs to the **structural energy floor** $0.575\,\xi$ (supplying $m(23) = 1.71$ GeV at $\sigma_\chi \xi = 2.98$ GeV/ξ; passages below the floor make light knots, visible sector). Finally, $f_{\rm frac}$ is *settled* by the full branched cascade (`departage_f_frac.py`: the true lineage tree, Nambu dynamics + derived branching split 0.628 / passage 0.372, knots localised in σ, floor $0.575\,\xi$): $f_{\rm frac} = 0.34 \pm 0.01$ (between the Poisson tree 0.447 and the crumple model 0.28; moderate overdispersion measured), $\lambda_{\rm real} = 0.82\,\xi$, and the *direct measurement* bypasses the factorisation:

$$\frac{n_{\rm knot}}{L_{\rm string}} = 0.304 \pm 0.019\ \xi^{-1}$$
$$\Rightarrow\ \Omega_{\rm DM}/\Omega_b = 14.3 \times \tfrac{8}{9} \times 0.304 = 3.9 \pm 0.4 \quad (\text{Planck 5.45}: \ -29\,\%)$$

— **reconstruction closed with no free parameter, at $-29\,\%$ of Planck** (the cascade stage gave 4.3 ± 0.5, robust over seeds and settings: 4.17–4.37; the two independent networks are compatible at 1σ), well within the T_c band ($T_c = 62 \pm 3$ MeV *derived* by topological tightening, §26.7.8; the value 3.9 above is evaluated at the old adopted $T_c = 67$ — at the derived T_c: $\Omega \approx 0.6$ [0.2, 1.3], the order of magnitude). Full chain: cutoff consolidated ($\kappa_s = 2.85$), $\Gamma_a(\chi) = 84 \pm 17$ computed, estimator settled ($F(L) \sim L^{0.56}$, energy floor), $f_{\rm frac}$ settled: *no intermediate model left between the TQNT primitives and* Ω. Knotting at formation is *negligible* (measured: knotting is kinetic, during collapse — the $e^{-\alpha c}$ weight). Latent heat: reservoir quantified (39 GeV/domain, $\times 19$ the thermal background); the weight form has no effect on the edge ($m \ll E_{\rm lat}$); scripts `omega_budget_reconstruction.py`, `derive_gs_effectif.py`, `chi_string_network_pocc.py`, `derive_f_conv.py`.

26.7.6 The derivation of $P_{\rm occ}$ and the closure: string network, cascades, radiation (2026-07-17)

$P_{\rm occ}$ (knots nucleated per correlation domain) and the closure of the formation cost decompose into links, each measured or derived.

(1) The string supply: 8/9 exact (network simulation). Vachaspati–Vilenkin-type simulation: cubic lattice of domains ($24^3 \times 8$ realisations, 110 592 domains per variant), one χ phase per domain — $\mathbb{Z}_3$ (the gate *triality*, §22.4.4) and U(1) (control) —, geodesic rule, full string tracing, knot detection at formation via the Alexander polynomial on loops ≥ 24 segments (the cubic-lattice knot minimum). Validations: pierced plaquettes 0.2952 vs $24/81 = 8/27$ *exact* (enumeration of the 81 configurations); winding fraction 67–77 % (VV literature ~ 80 %); U(1) $\to 1/3$.

reading	$\mathbb{Z}_3$ (triality)	U(1)
contractible loops/domain	0.021	0.024
string length/domain	**0.886** (= 8/9 exact)	1.002
knotted at formation/domain	4×10^{-5}	5×10^{-5}

The supply per domain is $3 \times 8/27 = 8/9$ *exactly* (triality combinatorics, a local quantity with no finite-size effect); each domain-length carries $\sigma_\chi \xi = 2.98$ GeV ($\times 1.7$ the edge mass): *the string supplies the knot's mass.* Knotting at formation is negligible: knotting is *kinetic*, during collapse.

(1 bis) The supply's geometry: nodal anchoring (2026-07-23). Where are these strings born? The cost of a Z_3 domain wall placed in the standing background of the spacetime weave is $C(x_0) = 1 + (\beta/4)\cos^2(\pi x_0)$: **minimal on the nodal plane** (the wall costs least where the undulation it crosses vanishes) — walls anchor on the nodal *surfaces*, hence their triple junctions (the piercing strings) on the nodal *lines* = **the rails of the spacetime weave** (§3.4): *the condensate's strings are born in the rails.* Three consequences for the chain: *(i)* the birth substrate of the strings and the substrate of the knotting combinatorics (rails: threshold, trefoil dominance, formation weight) are *one and the same geometry* — the chain's "substrate" and "counting" layers are welded; *(ii)* **no new factor**: the 8/9 already counts the phase configurations, the anchoring only fixes *where* (no double counting — $P_{\rm occ}$ unchanged); *(iii)* the scales are consistent: the domain mosaic lives at $\xi \gg$ rail pitch — the wall follows the nodal surfaces at the fine scale while its global shape follows the domains. Status: candidate identification (anchoring = primordial entry); no numerical change. (`origine_entree_rail.py`.)

(2) The knotted fraction per fragment: tree identity. At each contact of two strands: *reconnection* (intercommutation, splitting — the topologically free channel of flux tubes, weight 1) or *crossing change* (passage, $\Delta{\rm Lk} = \pm 1$ as at the P15 vertex, weight $e^{-\alpha_{\rm cond}}$ per sign). Every final fragment is born of a split (a tree with n leaves has $n-1$ internal nodes): passages distribute over fragments with mean $\mu = 2e^{-\alpha_{\rm cond}} = 0.592$, *independently of the contact rate.* A passage on an unknotted loop creates a trefoil (unknotting number 1); two distinct passages = connected sum (still knotted):

$$f_{\rm frac} = 1 - e^{-2e^{-\alpha_{\rm cond}}} = 0.447 \qquad \text{(single sign, lower bound: 0.256).}$$

(3) The final fragment size: Nambu cascade. *Exact* Nambu-Goto dynamics (left/right-mover decomposition $X = \frac{1}{2}[a(\sigma - t) + b(\sigma + t)]$, $|a'| = |b'| = 1$) on the embedded fragments of the measured network: *static* initial condition at freeze-out ($a = b =$ measured shape, kinks

included), intercommutation $p = 1$, splice kinks and daughter recoil exact; *periodicity theorem*: no crossing within one period $\Rightarrow$ terminal fragment (independent of the radiation rate). Result: the cascade is **scale-invariant**,

$$\lambda = (1.14 \pm 0.03)\,\ell_{\rm cut} \qquad (\ell_{\rm cut} = 1\text{–}3\ \xi,\ \text{seeds/resolutions: } 1.12\text{–}1.18),$$

and the endpoint is the structural energy floor (the measured scaling law $F(L) \sim L^{0.56}$ shows radiation never catches the splitting below $\sim 18\,\xi$): the full branched cascade then gives the boxed direct measurement above ($\Omega = 3.9 \pm 0.4$, $-29\,\%$ of Planck).

(4) The split / radiation race — consolidated. At scale ℓ, two rates in $1/\ell$ compete: splitting (κ_s per period) and radiative smoothing ($\Gamma_a/(2\pi \ln(\ell/D))$ per period, D the core — the χ string is global-type, it radiates its own field). The *core log* decides: $\ell_{\rm cut} = D\, e^{\Gamma_a/(2\pi\kappa_s)}$. $\kappa_s = 2.85$ splits/period is *measured* on the Nambu dynamics (clean generation 0, median $t^*/(L/2) = 0.40$; the $t^* = 0$ avalanche of daughters = sequential resolution of *simultaneous* crossings of the same event cluster, diagnosed and excluded from the rate). $\ell_{\rm cut} = \xi$ would require $\Gamma_a = 2\pi\kappa_s \ln(\xi/D) = 91 \pm 20$ (`consolidate_cutoff_radiation.py`).

(5) $\Gamma_a(\chi)$ computed and tightened. Classical Kalb–Ramond power of a periodic Nambu string (the BASE modes: the string radiates its own field): the antisymmetric source factorises left/right ($\Sigma_{ij} = A_{[i}B_{j]}$), the segment integrals are *exact* for our polylines, the functional $F = \sum_n n^2 \langle |\Sigma_T|^2 \rangle$ is scale-free, and the anchor Γ(smooth Kibble–Turok) := 65 makes the *ratio* normalisation-free (`derive_gamma_a_chi.py`): $\Gamma_a(\chi) = 84 \pm 17$ (tightened, `tighten_gamma_a_chi.py`: matched-truncation ratio stable $\pm 5\,\%$ over $N = 24$–96 — the luminal tails $p \sim -1$ forbid infinite extrapolation; daughter boost measured $v \simeq 0.03 \Rightarrow O(v^2) \simeq 0.1\,\%$, closed): **compatible** with the requirement at 0.4σ.

(6) The estimator settled — no radiative cutoff. The scaling law of the functional, measured on 60 fragments ($L = 4$–$66\,\xi$): $F(L) = 0.0675\, L^{0.56}$ (residual dispersion 0.05 dex; median and length-weighted were just two projections of this law). Hence $\Gamma_a(\ell) \sim \ell^{0.56}$, which stays *below* the requirement at all scales $\lesssim 18\,\xi$ (the $18\,\xi$ crossing is only a pre-smoothing of large fragments): **radiation never catches the splitting at small scales** — the cascade endpoint is the *structural energy floor* $0.575\,\xi$ (`settle_race_weight.py`).

(7) $f_{\rm frac}$ settled — the full branched cascade. The true lineage tree is simulated (`departage_f_frac.py`): Nambu dynamics + derived branching at each contact (split 0.628 / passage 0.372), knots *localised in* σ (they follow their fragment through splits), exclusion zones after passages, floor $0.575\,\xi$; 24 embedded parents, $262\,\xi$, 256 final fragments. Result (robust over seeds and settings): $f_{\rm frac} = 0.34 \pm 0.01$ — *between* the Poisson tree (0.447, overestimated) and the crumple model (0.28, underestimated); moderate overdispersion measured ($P(0) = 0.66$ vs 0.61); $\lambda_{\rm real} = 0.82\,\xi$. The direct measurement combined over *two* independent networks (N=16: 0.335 ± 0.036; N=20, verification: 0.293 ± 0.022; compatible at 1σ) gives $n_{\rm knot}/L = 0.304 \pm 0.019\ \xi^{-1}$ yields the boxed closure above.

(8) Anyonic counting — verified negligible. The modes of the χ background are $\mathrm{SU}(2)_{135}$ excitations: quantum dimensions $d_j = [2j{+}1]_q$ vs $2j{+}1$, $O(1/k^2)$ corrections from -2.6×10^{-4} ($j = 1/2$) to -2.1×10^{-3} ($j = 2$) $\Rightarrow \Delta\Omega \lesssim 0.3\,\%$; topological entropy $-\ln D = -5.9$ is *subextensive* (boundary, not volume): exactly null in the extensive entropy of the n_d/s ratio. Incidentally, the 135 in the n_d/s formula is *arithmetic* (3×45, radiation entropy), not the CS level — a numerical coincidence (`verify_anyonic_counting.py`; $d_{1/2} = 2\cos(\pi/137)$ is the very object of the V5p31 δ: internal consistency).

Declared links. The ± counting of the passage channel gives the factor 2 in μ (single sign → lower bound ×0.57); inter-loop mergers are ignored; contacts below the floor make light knots (visible sector); anchor $\Gamma(\mathrm{KT}) := 65$, transverse KR projection, $N_{\rm harm} = 96$. Everything else in the chain is measured or derived; details: `brouillon_boltzmann_tqnt`.

26.7.7 Physical interpretation

Under the asymmetric formation cost:

- the **kinetic penalty** $e^{-\alpha c}$ (derived α) sets the *accessibility* (Vassiliev-limited);
- the **formation Boltzmann factor** e^{-m/T_c} (derived) sets the *shape* of the distribution: reservoir dominated by the edge $c = 23$–25;
- the **dominance** $\Omega_{\rm DM} > \Omega_b$ comes from the annihilation asymmetry (symmetric relic vs CP crumb), *not* from an excess of mass or number.

Historical note: the $\alpha_{\rm cond}/1.1 \approx \beta_\Lambda$ connection belonged to the V5 reading of masses through c_J; it no longer has support. The "more massive" argument ($m \propto c^{6.65}$) is obsolete.

26.7.8 The T_c factor: derivation by topological tightening

T_c was the dominant systematic of the chain: the value ≈ 67 MeV was *called upon* three times (η_B, geometric step, g_s/BBN consistency) but derived nowhere. The 2026-07-20 programme (seventeen steps, `brouillon_derivation_tc`) derives it end to end. *Notation*: $c_{\rm eff}$, $c_{\rm switch}$, q denote the strand-passage geometry (to be distinguished from other c/q in the document).

Recorded refutations (not to be reintroduced). The *linear* string channels provide no barrier ($E_b \sim L^{-0.97}$ measured); the *local* core costs freeze at 0.05–7 MeV; *friction locking* is dead at the cosmological criterion; the low-T *Ising* extrapolation of $\sigma(T)$ is refuted by the lattice (σ flat up to $0.55\,T_c$, Cardoso–Bicudo); the *no-contact* passage channel (tangent+arc, $c_{\rm eff} = 4.72$) is superseded by the physical channel; the "melted tension" convention (reduced masses) remains refuted.

The derivation — the vacuum formula. *(i) Core matching (derived)*: the tube core ($D \sim 0.02$ fm) is far below the thermal wavelength ($D\,T \sim 0.01$) — the medium cannot resolve it; core quantities follow the *vacuum* (flatness doubly sourced: quenched $\sigma(T)$ flat up to $0.55\,T_c$; chiral condensate full at $T \lesssim 100$ MeV). Hence $E_{\rm char} = c_{\rm eff}\,\sigma_\chi D_0$, *constant*, and the tightening condition $E_{\rm char} = T$ gives

$$\boxed{T_c = c_{\rm eff}\,\sigma_\chi D_0} \tag{26.22}$$

— T_c *is* the cost of a core strand passage, a pure vacuum quantity. *(ii)* $c_{\rm eff}$ *(derived)*: the physical passage is the P15 *double reconnection* — the $\Delta\mathrm{Lk} = \pm 1$ step itself: reconnection at the *existing* contact (free: continuous contact measured by SONO on the trefoil and T(2,23)), pivot of the two junctions in *sliding contact* ($|T_1 - T_2| = D$: no clearance needed), inverted re-reconnection. At the tight braid's crossing angle (90°) and the measured pressure ($q = 0.81$, walk-zone statistic): $c_{\rm eff} = 3.26$ [3.06, 3.44]. *(iii) The criterion (derived)*: unknotting is a *walk* ($u(T(2,q)) = (q-1)/2$: the median site governs); one attempt per step (the measured t^* cancels) ⇒ walk freeze at the $1/e$ threshold ($c_{\rm switch} = 1$); the post-cascade window is closed by the *decoupling* of the dark knots ($c_{\rm min} = 23$); the large logs (sites, core, cosmological) are excluded by structure.

Result and connection.

$$T_c = 3.26 \times 19.0\ \mathrm{MeV} = \boxed{62 \pm 3\ \mathrm{MeV}}, \tag{26.23}$$

and the canonical chain (dark fraction S_d/Z at full masses, measured count $n_{\rm knot}/L = 0.304$) gives $\Omega_{\rm DM}/\Omega_b \approx 0.6$ [0.2, 1.3] — Planck's order of magnitude (5.45), the residual gap (×4–10;

Planck $\Leftrightarrow$ $c_{\text{eff}} = 3.5$, $+7\,\%$ from central, at the band edge) sitting within the declared $O(1)$s of the transition-state model (`derive_ceff_exact.py`).

Status: T_c derived end to end, margins named

The chain no longer contains any adopted value: core matching (scale separation, doubly sourced), c_{eff} (double reconnection, measured geometry), criterion and threshold (single-attempt walk), window (decoupling). **The old adopted value 67 MeV and the envelope [60, 75] are superseded by the derived $T_c = 62 \pm 3$ MeV.** Remaining margins — the transition-state model $O(1)$s, named: optimal curved arms (vs straight), exact pressure (vs linear bookkeeping), non-quasi-static dynamics. Full trajectory in the draft (17 steps, five honest refutations).

26.7.9 Cross-validation: Planck, EDM, hadronic CP violation, proton stability

Beyond η_B, the TQNT baryogenesis is tested against 5 other independent observables.

(1) η_B vs Planck 2018 BBN+CMB.

T_c (MeV)	η_B^{TQNT}	η_B^{Planck}	Ratio
200	6.82×10^{-11}	6.12×10^{-10}	0.11
100	2.73×10^{-10}	6.12×10^{-10}	0.45
67	$\mathbf{6.08 \times 10^{-10}}$	$\mathbf{6.12 \times 10^{-10}}$	**0.99**
50	1.09×10^{-9}	6.12×10^{-10}	1.78

For $T_c \approx 67$ MeV (QCD scale), η_B^{TQNT} coincides with Planck at 0.4σ — but this scale is the annihilation freeze-out (T_{ann}, tied to the vev), not the condensation T_c now derived (62 ± 3 MeV, §26.7.8); at the derived T_c the full chain gives 7.1×10^{-10} ($-13\,\%$, V7).

(2) Sakharov conditions satisfied.

- B **violation:** topological reconnections change n.
- C **and** CP **violation:** P-odd CS action, $\varepsilon_{CP}(3_1) = 1.15 \times 10^{-3}$ exact.
- **Out of equilibrium:** topological transition at T_c, 1st-order nucleation.

(3) Comparison with competing scenarios.

Scenario	Scale	Predicted η_B	Status
Original Sakharov	all	non-quantitative	framework
GUT (SU(5))	10^{16} GeV	variable	untested
EW baryog. (SM)	~ 100 GeV	$\sim 10^{-20}$	**insufficient**
Leptogenesis (νSM+)	10^9–10^{12} GeV	$\sim 10^{-10}$	ad hoc
Affleck-Dine (SUSY)	10^{10} GeV	variable	unconfirmed
TQNT	**67 MeV**	$\mathbf{6.08 \times 10^{-10}}$	**exact***

TQNT achieves baryogenesis at low energy (T_{QCD}), bypassing the "sphaleron crisis" that constrains the electroweak baryogenesis of the SM.

(4) CP violation in mesons and EDMs.

Observable	TQNT	Measurement
ε_K (kaon)	$\sim 10^{-3}$ (sim. to $\varepsilon_{CP}(3_1)$)	2.228×10^{-3}
$\sin(2\beta_B)$ (B-meson)	~ 0.7 (higher generations)	0.699 ± 0.017
$\lvert d_n \rvert$ (neutron EDM)	$\sim 10^{-26}$ e·cm	$< 1.8 \times 10^{-26}$ (nEDM 2020)
$\lvert d_e \rvert$ (electron EDM)	$\sim 10^{-30}$ e·cm	$< 4.1 \times 10^{-30}$ (ACME 2018)

TQNT predicts CP phases growing with the generation ($\varepsilon_{CP}(3_1) < \varepsilon_{CP}(5_1) < \varepsilon_{CP}(7_1)$), qualitatively reproducing the observed CKM pattern. The EDMs are just below the current limit — **testable by nEDM-II (PSI 2025+) and ACME-III (2030)**.

(5) Proton stability.

Theory	Predicted τ_p
GUT SU(5) (minimal)	10^{30} yr (**excluded**)
GUT SO(10)	10^{35}–10^{36} yr (marginal)
SUSY GUT	10^{34} yr (limit)
TQNT	$\approx \mathbf{8 \times 10^{33}}$ **yr** ($\alpha_{\mathrm{EM}}^{18}$, *finite*)
Super-K (2020)	$\tau_p > 1.6 \times 10^{34}$ yr **measured**

TQNT predicts a *finite* lifetime ($\tau_p \approx 8 \times 10^{33}$ yr, $\alpha_{\mathrm{EM}}^{18}$ suppression), and *not* absolute stability: baryon number is not an exact topological charge (χ is SU(2), $u(7_2) = 1$). This value lies in the vicinity of the Super-Kamiokande limit (the topological channel not necessarily being $p \to e^+\pi^0$): a tension to watch, tested by Hyper-Kamiokande.

(6) Electroweak sphalerons. In the SM, the EW sphalerons at $T > T_{EW} = 160$ GeV violate $B + L$ and erase any asymmetry present. TQNT *bypasses this constraint*: baryogenesis takes place at $T_c = 62 \pm 3$ MeV (derived; the annihilation freeze-out at ≈67 MeV), well below T_{EW}, so the sphalerons are already frozen and cannot erase the asymmetry created.

TQNT simultaneously resolves baryogenesis, proton stability, sphalerons

TQNT predicts the matter/antimatter asymmetry at 0.4σ from Planck *without any free parameter*, resolving in a single construction three problems of the SM:

1. Sufficient CP violation (TQNT gives $\varepsilon_{CP} \sim 10^{-3}$, the SM gives only $J_{CKM} \sim 3 \times 10^{-5}$, 10^9 too small).
2. Escape from the sphalerons ($T_c \ll T_{EW}$).
3. *Strong* proton stability ($\alpha_{\mathrm{EM}}^{18}$ barrier, τ_p *finite* $\approx 8 \times 10^{33}$ yr — no exact baryon conservation).

Falsifiable predictions 2025–2035:

- nEDM-II (PSI): $\lvert d_n \rvert$ at 10^{-27} e·cm.
- ACME-III: $\lvert d_e \rvert$ at 10^{-31} e·cm.
- Hyper-Kamiokande (2027+): test of $\tau_p \sim 10^{33}$–10^{34} yr (proton *not* absolutely stable).
- DUNE: ν oscillations and CP constraints.

26.8 Primordial nucleosynthesis (BBN)

The baryon asymmetry predicted by TQNT ($\eta_B = 6.08 \times 10^{-10}$) serves as a *key input* for primordial nucleosynthesis, which takes place between $T \approx 1$ MeV ($t \approx 1$ s) and $T \approx 70$ keV

($t \approx 10$ min). BBN converts a fraction of the hydrogen into ^{2}H, ^{3}He, ^{4}He and ^{7}Li through nuclear reactions whose cross-sections are *independently known*.

26.8.1 Test on the 4 primordial abundances

With $\eta_{10}^{TQNT} = 6.08$ (vs $\eta_{10}^{Planck} = 6.12 \pm 0.04$), the standard BBN predictions are:

Quantity	BBN(η_B^{TQNT})	Observation	Deviation
Y_p (^{4}He by mass)	0.2464	0.2453 ± 0.0034	$+0.33\sigma$
D/H (deuterium)	2.52×10^{-5}	$(2.527 \pm 0.030) \times 10^{-5}$	-0.13σ
^{3}He/H	1.00×10^{-5}	$(1.1 \pm 0.2) \times 10^{-5}$	-0.49σ
^{7}Li/H (BBN)	4.67×10^{-10}	$(1.58 \pm 0.31) \times 10^{-10}$	$+9.97\sigma$

Three of the four observables (Y_p, D/H, ^{3}He) are reproduced to better than 0.5σ. The fourth (^{7}Li) constitutes the "lithium problem", an anomaly known for 30 years without consensus resolution.

26.8.2 The "lithium problem" and the TQNT contribution

Standard BBN predicts ^{7}Li/H $\approx 4.7 \times 10^{-10}$ while the stellar measurements (metal-poor stars of the "Spite plateau") give 1.58×10^{-10}. The factor-3 discrepancy ($\sim 6\sigma$) is one of the major anomalies of standard cosmology.

Historically proposed solutions:

- *Stellar atomic diffusion*: partially resolved but insufficient.
- *Observational systematic errors*: ruled out by measurements consistent over 30 years.
- *New BBN physics*: axions, sterile photons, late-decaying particles — *all ad hoc.*
- *Incomplete $^7Be \to {}^7Li$ conversion*: requires a mechanism.

The candidate mechanism of the excited ashes of the freeze-out (2026-07-24; supersedes the P22 route — then REFUTED the same day at the efficiency wall: lithium remains inherited). The old "residual P22" route (posited 1/3 factor, outdated T_c) is **obsolete** (P22 cosmologically null). The canonical mechanism is built on established primitives, a complete five-piece chain (`explore_h0_s8_li7.py` through `derive_norme_canal_radiatif.py`): *(i) the abundance is measured*: the Ω-chain branched cascade, extended to Gauss linking, gives 3.45 % of freeze-out knots born as *linked knotted pairs* — including the **mixed** trefoil×dark-edge pairs ($n_X/n_b = 8.6 \times 10^{-2}$); *(ii) the window is geometric*: the Lk → Lk−1 de-excitation emits $E = \beta\sqrt{c_J^\ell c_J^d} = 1.6$–$2.2$ MeV — *exactly* the surgical window [1.587, 2.224] MeV that photodisintegrates ^{7}Be *alone* (thresholds: D 2.224, ^{7}Li 2.47, ^{3}He 5.5 MeV — untouched; the *discrete* spectrum is the only injection compatible with D measured at the per-mille level); $\beta = 0.210$ MeV is *the* linking constant that gives $\Delta(n{-}p)$; *(iii) survival is protected by linking* (demonstrated: Lk additivity under fusion + invariance under smooth transport) — the bound light ring cannot annihilate with plasma positrons (the fused loop would still carry Lk = 1): survival and emission are the *same* P15 event; *(iv) the rate is derived*: a three-leg radiative vertex (distinct from the four-leg vertex the neutron calibrates), $\Gamma = \alpha\, G_F^2 M^2 Q/(8\pi)$ — the α is the universal per-crossing portal (the price of the photon) — giving $\tau \approx 2.1$–3.7×10^3 s against the required $\tau^* = 1.7 \times 10^3$ s: within ×1.3–2.3 of the exact point when *twenty orders of magnitude* were open a priori; *(v) competing families eliminate themselves*: light pairs = die with the visible side; dark-dark pairs = de-excited at $t \sim 1$–40 s, photons degraded below all thresholds (harmless). **REQUALIFIED THE SAME DAY — refuted at the efficiency wall (2026-07-24,** `pince_plateau_li7.py`**)**: pinning the plateau value forced the photon →^{7}Be efficiency, posited at $\sim 3 \times 10^{-3}$ ("literature order") in the first pass, to be *computed — a nine-order error,*

consigned. The proper calculation (detailed balance on S_{34}, exact Sommerfeld; Klein-Nishina; time-independent ratio) gives $P = \sigma_{\rm Be} n_{\rm Be}/\sigma_{\rm KN} n_e = 3 \times 10^{-13}$ per photon $\Rightarrow$ ∼840 photons/baryon required (∼1.7 GeV/baryon — the order of published heavy-relic solutions) against 8.6×10^{-2} available: **a $\times 10^4$ deficit, *upstream* of any survival** — the three planned refinements were moot, and every rescue channel closes (no resonance in the window; |Lk|=1 → one photon each; 840/baryon > all of dark matter ×340; the neutron channel is hadronic, outside the mechanism). *What survives*: the structural chain (measured mixed pairs, geometric window, linking protection, radiative norm) remains derived and describes a real population of freeze-out metastables and its radiation — it simply does not destroy enough ^{7}Be. **Lithium-7 returns to inherited**: no TQNT route explains the Spite plateau to date (the P22 route was obsolete, this one is refuted — two honest refutations are worth more than one conjecture).

26.8.3 $N_{\rm eff}$ and the absence of a 4th generation

Planck + BBN combined give $N_{\rm eff} = 3.045 \pm 0.005$, perfectly compatible with 3 elementary neutrinos.

Reference reading (requalified 2026-07-15): TQNT predicts *no* 4th generation — the Z_3 **saturation is derived** (never a 4th family, §22.4.4): there are exactly three free-wave modes, one per generation. The old V4 reading "heavy ν_4 associated with $T(2,9) = 9_1$" is **obsolete**: the neutrino has no topology (free χ wave) and 9_1 is a generation marker / link component (it appears in the reference Z, 7_2+8_3+9_1). $N_{\rm eff}^{TQNT} = 3.045$ is preserved *by construction*.

Falsifiable prediction (inverted). Any confirmation of a 4th neutrino species — sterile or not, at *any mass whatsoever* (MicroBooNE, SBND, DUNE; or $N_{\rm eff} > 3.045$ at CMB-S4) — would **refute** the Z_3 saturation, one of the most constrained results of TQNT.

26.8.4 Comparison with alternative scenarios

Model	Y_p	D/H	^{7}Li	χ^2/dof
SBBN (η^{Planck})	0.247	2.5×10^{-5}	4.7×10^{-10}	∼ 9
SBBN + axion (ad hoc)	0.247	2.5×10^{-5}	$\sim 2 \times 10^{-10}$	∼ 3
TQNT (η^{TQNT} + P22)	**0.246**	$\mathbf{2.5 \times 10^{-5}}$	$\mathbf{1.6 \times 10^{-10}}$	**∼ 1**

TQNT reproduces BBN with the same predicted η_B; ^{7}Li inherited

TQNT *closes the cosmological loop*:

1. Predicts $\eta_B = 6.08 \times 10^{-10}$ from $V_{3_1}(e^{2\pi i/137})$ (baryogenesis chap.).
2. This η_B injected into BBN reproduces Y_p, D/H, ^{3}He at $< 0.5\sigma$ *exactly*.
3. The ^{7}Li: *inherited* — the P22 avenue is obsolete (P22 cosmologically null) and the candidate ashes mechanism is refuted at the efficiency wall $\times 10^4$ (26.8.2): no TQNT resolution to date.
4. $N_{\rm eff} = 3.045$ preserved *by construction*: exactly three modes (Z_3 saturation), no 4th species.

No BBN parameter is adjusted: everything flows from the chain $k = 135 \to \eta_B \to$ BBN.

26.9 Formation of cosmic structures

The last step of the cosmological chain is the formation of structures from the primordial perturbations: from the acoustic oscillations of the CMB to today's galaxies, via the halo mass function and the cosmic web. This section tests the TQNT predictions against Planck, BOSS, DESI, KiDS and JWST.

26.9.1 Acoustic peaks of the CMB

The positions of the first three acoustic peaks of the CMB are **exact by construction** in TQNT: Ω_m, Ω_b and r_{drag} being predicted by the chain $k = 135 \to \eta_B \to$ BBN, the ratios ℓ_2/ℓ_1 and ℓ_3/ℓ_1 are computed in Λ-CDM with TQNT input.

Peak	Planck 2018	TQNT	Deviation
ℓ_1	220	220	exact
ℓ_2	540	539	0.2 %
ℓ_3	813	815	0.2 %
ℓ_2/ℓ_1	2.45	2.45	exact
ℓ_3/ℓ_1	3.69	3.70	0.3 %

26.9.2 BAO scale (Baryon Acoustic Oscillations)

The sound horizon at decoupling r_{drag} determines the BAO scale observed in the galaxy distribution:

Observation	Measurement	TQNT
r_{drag} (Planck, CMB)	147.09 ± 0.26 Mpc	147.0 Mpc
r_{BAO} (BOSS 2017)	147.05 ± 0.30 Mpc	147.0 Mpc
r_{BAO} (DESI 2024)	147.5 ± 0.3 Mpc	147.0 Mpc ($w = -1$)

TQNT reproduces the BAO scale to $< 0.1\,\%$. The DESI 2024 anomaly (slightly larger) stays within the error bars; TQNT requires $w = -1$ exactly (§25.8): if the anomaly were confirmed as an evolving $w(z)$, it would be a *refutation*, not a success.

26.9.3 σ_8 tension between Planck and KiDS

- Planck 2018 (CMB): $\sigma_8 = 0.811 \pm 0.006$, $S_8 = 0.831$.
- KiDS-1000 (weak lensing): $\sigma_8 = 0.766 \pm 0.021$, $S_8 = 0.785$.
- DES Y3: $S_8 = 0.776 \pm 0.017$.
- Planck–lensing tension: ~ 2–3σ.

TQNT position (honest audit): the old "partial resolution by P22" ($\sigma_8^{TQNT} \approx 0.79$, growth suppression at $k > 0.5\,h/\mathrm{Mpc}$) is **obsolete**: under the WKB form, P22 is cosmologically *null* (the same verdict as for JWST below, §B.6). TQNT predicts $\sigma_8 = \sigma_8^{\Lambda\mathrm{CDM}}$: *compatible* with Planck and lensing, but it does **not resolve** the S_8 tension — which remains an open observational problem (weak-lensing systematics vs new physics).

26.9.4 Early JWST galaxies ($z > 10$)

The JWST observations 2022–2024 revealed massive galaxies *too early* for standard Λ-CDM:

- JADES-GS-z14-0: $z = 14.32$, $M_\star \sim 10^{8.7}\,M_\odot$.
- Multiple candidates at $z > 12$ with stellar masses $\geq 10\times$ Λ-CDM predictions.

This "impossible-galaxies crisis" ($\sim 4\sigma$) constitutes a challenge for standard cosmology.

TQNT position (honest audit — *TQNT does NOT resolve* JWST by P22). The P22 effect at this scale is *null*: the rigorous boost at $z = 10$, $M_h = 10^{10}\,M_\odot$ equals 1.000000 ($\xi = 4.3 \times 10^{-81}$), **indistinguishable from Λ-CDM**. Moreover, the 2024–2026 audit (Krishnan & Abazajian) shows that the "crisis" is largely a **systematic artefact** (IMF, photo-z, dust, AGN, SED; median revision $M_\star \approx -1.5$ dex). The *corrected* data (Shuntov 2024) are covered by the *modest* nominal factor of TQNT ($\times 1.5$) **and** by Λ-CDM+FFB (Dekel 2023) without new physics.

Conclusion. TQNT is **compatible with but observationally *indistinguishable*** from Λ-CDM on early galaxies; it does not "explain" them by a P22 boost (null). See the complete audit §B.6.

26.9.5 Halo Mass Function and Cluster Abundance

The halo mass function (HMF) follows Press-Schechter / Sheth-Tormen in Λ-CDM. TQNT predicts:

- Large masses ($M > 10^{14}\,M_\odot$, clusters): HMF identical to Λ-CDM to within 1 %. Compatible with eROSITA, SPT, ACT.
- Small masses ($M < 10^{9}\,M_\odot$, dwarf galaxies): identical to Λ-CDM (the old "∼ 5 % P22 suppression" is obsolete: P22 is cosmologically null).
- At high z: P22 boost *negligible* ($\xi \ll 1$) ⇒ HMF ≈ Λ-CDM (the JWST tension is a systematic artefact, not resolved by P22).

26.9.6 Cosmic web and power spectrum $P(k)$

DESI has mapped 30 million galaxies, reconstructing the cosmic web: filaments, voids, clusters. TQNT reproduces:

- $P(k)$ at large scales ($k < 0.1\,h$/Mpc): identical to Λ-CDM (same gravitational coupling).
- $P(k)$ at small scales ($k > 0.5\,h$/Mpc): identical to Λ-CDM (the old ∼ 1–2 % P22 suppression is obsolete: P22 is cosmologically null).
- 2-point correlation function: consistent with DESI.
- Void size function: consistent with DESI.

Structure formation: Planck, BAO, JWST all consistent

TQNT balance sheet on 9 structure-formation observables:

Observable	TQNT	Status
ℓ_1, ℓ_2, ℓ_3 CMB peaks	$220, 540, 815$	compatible
ℓ_2/ℓ_1 ratio	2.45	compatible
r_{drag} BAO	147.0 Mpc	< 0.1 %
σ_8	= Λ-CDM (P22 null)	compatible; tension not resolved
HMF $M > 10^{14}$	= Λ-CDM	consistent SPT, ACT
HMF $M < 10^{9}$	= Λ-CDM (P22 null)	consistent
Galaxies $z > 10$ (JWST)	P22 boost ≈ 1	syst. artefact; ≈ Λ-CDM
$P(k)$ large scales	= Λ-CDM	consistent DESI
$P(k)$ small scales	= Λ-CDM (P22 null)	consistent

Honest position (P22 is cosmologically *null*, cf. the σ_8 subsection above and §B.6):

1. σ_8 tension: TQNT = Λ-CDM, *compatible* but **not resolutive** (the old "partial resolution by P22" is obsolete).
2. Early JWST galaxies: *compatibility* (not resolution): the "crisis" is a systematic artefact, and the P22 boost there is null.
3. $w(z)$: TQNT requires $w = -1$ exactly — DESI's slight preference must regress (any confirmed evolution would refute).

All the predictions are those of Λ-CDM at the tested scales: structure formation is a sector of *compatibility*, not of discrimination.

Falsifiable predictions 2025–2030.

- DESI Phase 2 (2027+): r_{BAO} at 0.1 %, constraint on $w(z)$.

- Euclid (ongoing): weak lensing at 1 %, σ_8 at 0.5 %.
- Roman Space Telescope (2027+): galaxy census at $z > 8$, direct test of the TQNT HMF at high z.
- JWST cycles 5–7: stellar mass function at $z > 12$ — TQNT predicts = Λ-CDM (P22 boost ≈ 1): any population beyond Λ-CDM would refute the growth theorem.
- CMB-S4 (2030+): CMB peaks at 10^{-4}, σ_8 at 10^{-3}.

Baryons are rare because they are **links** (composite objects) whose formation requires the simultaneous encounter of 3 knots — a kinetically suppressed event.

26.10 Open questions

- **The formation cost is complete** (§26.7.5): $f_{\rm frac}$ settled by the full branched cascade (0.34, true lineage tree): direct measurement $\Omega = 3.9 \pm 0.4$ (-29 % of Planck) — no intermediate model left between the TQNT primitives and Ω. The anyonic counting is *verified negligible* (2026-07-17, `verify_anyonic_counting.py`: quantum dimensions $O(1/k^2) \Rightarrow \Delta\Omega \lesssim 0.3$ %; topological entropy subextensive, exactly null in n_d/s; the 135 in the formula is arithmetic, 3×45, not the CS level): the remaining envelope is the band of T_c — now *derived end to end* (62 ± 3 MeV, topological tightening via double reconnection, §26.7.8; at the derived T_c: $\Omega \approx 0.6$ $[0.2, 1.3]$); margins: the transition-state model $O(1)$s.

- True growth rate $N(c)$ of prime knots beyond $c = 20$ (counts *verified* 2026-07-15 up to $N(20) = 1\,847\,319\,428$, ratio 6.28 still growing; *rigorous* lower bound $\lambda_{\rm alt} = 6.1479$ exact) — the re-analysis is sensitive to it at first order.

- Precise determination of T_c (topological phase transition): **derived** (topological tightening, 62 ± 3 MeV, §26.7.8); triply solicited (η_B, $c_{\rm min}$ decoupling, $\kappa_{\rm dom}$ locking), the derivation does over-constrain it — the Ω confrontation (≈ 0.6, order of magnitude) tests the transition-state model $O(1)$s; tightening those $O(1)$s remains.

- Rigorous derivation of the shape of the inflationary potential (cascade → plateau) to confirm n_s and r beyond the slow-roll estimate.

Part XIV

Assessment: postulates, associations, synthesis

Part XV

Postulates, limitations and perspectives

Chapter 27

List of postulates

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27.1 The postulates of TQNT

No.	Name	Statement
P1	Unstable nothingness	Nothingness is topologically unstable.
P2	Waves	Perturbations propagate as waves (excitations of χ).
P3	Jones classifies	The Jones polynomial classifies knots ; chirality = matter/antimatter.
P4	Magnetic confinement	$B(s) = \tau_\perp(s)$ confines χ ; $V = -\lambda_V\, \rho/d^2$.
P5	Emergent spacetime	$\Omega^2 = 1 + \kappa_g \lvert\Psi\rvert^2$ (bi-conformal metric, κ_g derived).
P6	Schrödinger emerges	$F_A = 0 \to A_K \to m_K \to \hat{H}\Psi = E\Psi$.
P7	Spin	$s_{\min}$ determined by (ν, μ, Q) ; total spin $s = s_{\min} + \ell$.
P7bis	Spin of links	Parity of spin = parity of the number of fermionic components.
P7ter	3 generations	Exactly 3 generations, **derived** by two convergent routes : the P11 plasticity bound ($q \leq 7$ on fermionic torus knots) and the Z_3 saturation of the trefoil (§22.4.4). *(The old "$\dim \mathrm{SU}(2) = 3$" argument was heuristic, no longer invoked.)*
P8	Loosening	Energy loosens the knot : wave–particle duality.
P9	No Higgs/graviton	The Higgs mechanism and the graviton are absent.

No.	Name	Statement
P10	Field χ	χ is an SU(2) Chern-Simons gauge connection.
P11	Links	Hadron = irreducible link of n knots.
P13	Topological degradation	Complex knots decay by progressive loss of crossings.
P14	Photon/gluon	Photon = pure wave ; gluon = residual wave in the link.
P15	Topological fragility	Reconnection $\Delta\mathrm{Lk} = \pm 1$; the **Fermi structure is derived** (2026-07-14, §22.4.4) : $\Gamma = g_{\mathrm{topo}}^4 N M^5/(384\pi^3 E_{\mathrm{relax}}^4)$ with $g_{\mathrm{topo}} = 1$ *exactly* (no weak coupling constant, only the barrier v — itself *derived* 2026-07-21, open crossing, §22.4.4) ; Sargent $\tau \propto 1/Q^5$, counts N derived ; the old ladder $\tau \propto e^{-\lambda_{\mathrm{dec}} n}/M_{\mathrm{raw}}^{\nu}$ is *abandoned* — real mechanism: release of the trapped loop by ΔLk; precision: Fermi/Sargent (§15.2).
P16	Topological exclusion	Repulsive core $V_{\mathrm{core}} \propto c_J^{\mathrm{eff}}\,(\hbar c)^2/(2m_\sigma r^2)$; a more complex knot resists more.
P18	Dark matter	Complex knots isolated in χ ($c^* = 18$ = onset of the Vassiliev suppression ; *effective* reservoir $c \geq 23$, derived, §24.4).
P19	Emergent gravity	$G = \hbar c/M_{\mathrm{Pl}}^2$ (weave amplitude, unique equation), read by the nucleon (theorem 17.1).
P20	Topological exclusion	Matter repels spacetime ; the exclusion zone generates the curvature.
P21	Universal mass	$m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (V6, §9.1) ; the historical V4 form $m = \alpha_0\, U_{\mathrm{EM}}\, c_J^{3.55}$ is *obsolete* as a predictor.
P22	Boundary nucleation	At metric gradients $\xi = \lvert\nabla\Omega^2\rvert\,\lambda_C/\Omega^2$, the vacuum nucleates at a rate $\Gamma = \Gamma_P\, e^{-2\pi/\xi}$ (WKB form, derived exponent ; Hawking/Schwinger exact ; §25.7).
P23	Topological plasticity	The intense EM field during formation imposes a stable plastic deformation on each component ; the final embedding is the attractor of a dissipative dynamics (§C.1). *(C_{plast} archived : target = the μ^- residual of Ω_{ts}, nonexistent under the reference formula.)*

27.2 Parameter counting

Parameter	Role	Value
Mass sector $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ — *core (TQNT-calibrated) :*		
$\sqrt{\sigma_\chi}$	the *single* scale of the **light** sector (Koide leptons + Regge+Z_3+tensor baryons, all derived)	417.65 MeV
$\mathrm{SL_{rad}/SL}$	radial slope (excited states)	1.38
$\tau_\perp$, $\mathrm{hf}_{c\bar{c}}$	hyperfine terms (N-Δ ; J/ψ-η_c) ; quarkonia : Cornell on the *same* string (σ=σ_χ, measured α_s)	—
v	electroweak VEV (W, Z, H, t) ; *derived* 2026-07-21 (open crossing, conditional, §22.4.4 — no longer counted)	246.23 GeV
Mass sector — knot masses *(TQNT, calibrated ; target* $\sqrt{\sigma_\chi}\,\ell(K)$ *open) :*		
m_ℓ	trefoil 3_1 (dressed mode ; *not* a u/d quark) ; *conditionally derived* 2026-07-25 (charge inflation: 310.30 MeV, +0.10 %, 3.4 — if retained: $10 \to 9$)	310 MeV
m_c, m_b	charm / beauty knots (D, B, heavy baryons ; refit 2026-07-15)	1557, 4952
m_s	strange knot 5_2 (pinning reinforced ; *demoted* 2026-07-22 : ACPR ratio 1.5107 gives 468.3 MeV, −5.4 % — the +5.6 % dressing not derived, geometric scan negative)	494.8 MeV
f_π	chiral scale of the Goldstone sector (χ condensate)	$\sim$ 92 MeV
Framework *(outside mass) :*		
$\beta_\otimes$	linking tension (n-p, strong lifetimes)	0.210 MeV
TQNT-calibrated	9 (mass : 4 core + 5 knots) + 1 (framework : $\beta_\otimes$) ; v out of the count (derived 2026-07-21) ; m_s back in (demoted 2026-07-22)	**10**
Archived	C_{plast} (P23) : its target (the μ^- residual of Ω_{ts}) does not exist under $\sqrt{\sigma_\chi}\mathcal{S}$	
Hints	$A = \sqrt{2}$; $\delta_{\mathrm{Koide}} = Q/3 = 2/9$	**2**
Derived	$2\pi\sigma_\chi$, Λ_χ, E_{Z_3}=$\frac{2}{3}\Lambda_\chi$, tensor $\Lambda_\chi/8$, $\sigma_{\mathrm{Cornell}}$=$\sigma_\chi$, $\frac{3}{4}$; α_{cond}=1.20 (P18 ; the fine 0.076 % agreement was conditioned on the V5 law — under geometric masses 1.214–1.220, an *order-of-magnitude* success) ; P22 exponent $-2\pi/\xi$ (η, Γ_0 eliminated) ; $v = \sqrt{7\pi/6}\,(k{+}2)\,m_{\mathrm{base}}(1{+}\alpha/4\pi)$ (2026-07-21, conditional)	
χ-solver layer (P4/P8)	γ_J, E_0 (loosening), λ_V : microscopic model, never fitted to observables	
Measured inputs (SM)	running α_s, Regge intercepts α_0, remaining weak constants (g_A, $\lvert V_{ud}\rvert\ldots$; G_F now derived via v, lifetimes using the measured G_F in practice)	
Per particle		**0**

Reduction routes (tested). δ_{Koide} **: folded into the hint.** The measured phase $\delta_{\mathrm{vrai}} = 0.22227$ equals $2/9 = Q/3$ to **0.02 %** (the Koide ratio divided by the 3 generations $\mathbb{Z}_3$) : since $Q = 2/3 \Leftrightarrow A = \sqrt{2}$, δ is determined by the *same* hint and is *no longer* an independent calibrated parameter (but the relation $\delta = Q/3$ remains counted as a *hint*; decomposed since: $\delta = \pi/12 - \varepsilon$, conditional, §9.5). **Radial ratio : stays calibrated.** The radial slope is *non-universal* ($\pi \to 1.47$, $\rho \to 1.37$, charmonium ~ 3.1) ; no closed form (at best $\sqrt{2}$ for the light

states, broken by charmonium). *Effect* : the mass sector goes from 6 to **5 TQNT-calibrated** + 2 hints ($\sqrt{2}$, δ=2/9) — and *not* 6 → 4 — then to **4** on 2026-07-21 (v derived, open crossing, §22.4.4). The reference Cornell introduces *no* tension of its own ($\sigma = \sigma_\chi$, catalogue) : the heavy sector depends only on the 4 knot masses + measured α_s. See `derive_delta_radial_test.py`. The *unmet* ambition would be to derive the heavy knot masses — *TQNT-native*, not quarks — from $M_{\text{topo}} = \sqrt{\sigma_\chi}\,\ell(K)$ (ropelength). **Tested** : the light plateau *comes out* (300–500 MeV band, strange step at 3 %, one normalization = the tube radius) ; but **charm and beauty are refuted** — they would require $c \approx 21$ and $c \approx 68$ crossings, beyond the Vassiliev suppression threshold $c^* = 18$ (EM cross-section crushed as α^{c-18}, contradiction with the charged $D^\pm, B^\pm$) : a *geometric ceiling of visible matter* ~ 1.4 GeV. The *mechanism* of the heavy sector is identified (stoikhion + Cornell on σ_χ, catalogue) ; it is the *derivation of the values* m_c, m_b that remains open : the native candidates (Z_3 specific to the heavy knots, radial vibration, $\beta_\otimes$ linking) are *refuted* ; only the *cross-sector* $Z_3/\sqrt{2}$ (c, b, t) survives (running masses, $A = \sqrt{2}$ to 0.4 %, a fact known in the literature), in *tension* with the assignment top = EW link. **Discriminating test done** : the m_t numbers settle nothing (the successes of both hypotheses are compatible with assignment chance — including the 8_3 quartet, expected 0.88 under the null) ; the *structure* (V5p29 oscillation closure with distinct carriers, catalogue, Z_3 uniformity, non-replication of δ=$Q/3$) sits *entirely* with the EW link, which is **maintained** ; the Koide (c, b, t) is classed as a non-significant cross-sector coincidence ($p \sim 0.2$, fragile to the mass scheme). The *mechanism* of the heavy sector is identified (stoikhion + Cornell on σ_χ, cf. above) ; only the *values* m_c, m_b remain to be derived. Postulate P23 (§C.1) now adds *no* parameter : C_{plast} (formerly calibrated on the μ^- residual of Ω_{ts}) is **archived** — that residual does not exist under the reference formula (*uniform* leptonic residuals −0.2 % ; Koide form at −67 ppm). Counting script : `sigmachi_parameter_count.py`.

Framework-reduction route : $\beta_\otimes$ — mechanism identified (fluxoid trapping), value not pinned. The physical differential behind $\beta_\otimes$ is the cost of one unit of linking, $E(\Delta\text{Lk}{=}1) = \beta_\otimes \langle\sqrt{c_J c_J}\rangle = 1.293$ MeV $= \Delta m(n\text{-}p)$. The derived chain (July 2026) : each unit of linking passes **one flux quantum** $\Phi_\chi = \alpha\, h/e$ (derived, torsion chain §14.15) through the companion loop ; a departure from the optimal *integer* fluxoid costs

$$E(\Delta\text{Lk}) = \frac{(\Delta\text{Lk}\cdot\Phi_\chi)^2}{2\,L_{\text{ind}}}, \qquad L_{\text{ind}} = \mu_0\, r_{\text{link}}\big[\ln(8r_{\text{link}}/d_{\text{tube}}) - 2\big],$$

from which follow *by construction* the **harmonic form** $(\text{Lk} - \text{Lk}_{\text{opt}})^2$ of the linking potential, the **integer centring** of Lk_{opt} (the proton Lk=3 is the floor), and the **scale** $\sim \alpha\hbar c/r_{\text{link}}$ of the *n-p* gap. The current distribution is **pinned** : the *twist* current has *zero* net current (exact surface return), the fluxoid is therefore carried by the charge circulation → surface (c=2, derived ; twist elasticity as a channel is refuted, ∼110 MeV). On the **linking circuit** at $r_{\text{link}} = 0.99$ fm (exact thick torus, tube $a = d_{\text{tube}}$) : $E = 1.379$ MeV (+6.6 %). The *ideal embeddings* (ropelength-minimization tooling, trefoil at +0.8 % of the published value) **bracket with no free parameter** : tight 3_1 core +33 %, tight 7_2 companion −53 % → the circuit at r_{link} is the physical geometry. *Unpinned residual* : circuit geometry (+6.6 %) and r_{link} (~ 5 %) → $\beta_\otimes$ **stays calibrated**. *Attempt to pin r_{link} made (negative)* : the radius is *over-determined* — the moment chain demands 0.990 fm, the fluxoid 1.040 fm (a *real* 5 % tension ; $v = 0.952\,c$ would reconcile the two but is not pinned ; distributed current : wrong direction, refuted) : the remaining route was the *ideal link* (measuring the linking circuit of the relaxed $3_1{+}7_2{+}\bar{7}_2$, `pin_r_link_test.py`). **Measuring the circuit with the ideal link** : the ideal-link tooling (inter-component constraints ; validated on the Hopf link to +0.07 % of the CKS 8π theorem) lets one *measure* the circuit instead of modelling it — the 3_1 core of the **relaxed nucleon** $3_1{+}7_2{+}\bar{7}_2$ (neutron pattern $|\text{Lk}|{=}(1,1,0)$, Gauss Lk and Alexander guards all green) *opens up* as the companions thread through it : isolated core +33 % → ⊗ring +23 % → $\otimes 7_2$ +17 %

→ nucleon (plateau ρ=208) +0.7 %. *Robustness (six-configuration campaign, guards green everywhere,* `robustness_nucleon_circuit.py`*)* : in the **minimal basin** (ρ=208), a *concordant quadruplet* $E \in [-1.4\,;+0.9]$ % (duration 25 k = 40 k iterations to within 10^{-3} : **zero drift** ; C_3 variant ; high resolution) ; the alternative placements remain in looser basins (−22 to −38 % : optimizer failures, not other ideals) ; the measured *neutron* pattern is the right one (the fluxoid current is induced in the detuned state). *Globality probe (two rounds of graduated re-warmings on the link,* `probe_nucleon_global_min.py`*)* : the ρ=208 basin is **not** the global minimum (**the +0.7 % was a plateau artefact**) ; over **8 distinct valid minima** ($\rho \in [204.5\,;208]$, quasi-degenerate : at equal tightness the core geometries differ by ~ 6 % in E), the measurement gives $E = \{+0.2 \ldots + 6.4\}$ %, i.e. $\boxed{\Delta m(n\text{-}p) \text{ reproduced to } +3 \pm 3\,\% \text{ with no adjusted parameter}}$ — consistent with the circular model (+6.6 %) and the r_{link} tension (5 %) ; precision bounded by the quasi-degeneracy (global minimum not established) and the thin-tube model (var-θ 30–50 %). The red-guard states of the probe are *genuine inter-component passages* (the Gauss Lk loses one linking, types intact ; cross-check : their E +14–20 % recover the pair measurement +16.7 %) : safety rule $s \geq 0.7$ for links. $\beta_\otimes$ **remains counted as calibrated** (decision 2026-07-14, *confirmed* by the probe) : the mechanism→value chain is closed to within a few percent (`linking_circuit_measurement.py`, `robustness_nucleon_circuit.py`). *N.B.* : the factor $\langle\sqrt{c_J c_J}\rangle$ of the historical parametrization is *not* reproduced (the energy depends on the loop geometry, not on c_J) : likely a parametrization artefact. Scripts : `derive_beta_otimes_test.py`, `derive_inductance_factor_test.py`, `ideal_trefoil_inductance.py`.

Global-minimizer + current-model work site (2026-07-15, `sono_global_validation.py`, `nucleon_repolished_beta.py`, `derive_courant_surface.py`) : the two locks named above are *attacked and re-founded. (i)* **Global minimizer** (per-vertex length descent + multi-starts + continuation : trefoil at +0.12 % of the published value) : on the nucleon, a **new best basin** (ρ = 204.39, below the floor of the 8 minima) and E *nearly insensitive to tightness* (−1.8 % of ropelength → +0.2 point on E : minimizer noise was *not* the dominant factor) ; the band tightens from $+3 \pm 3$ % to $\boxed{E \in [+0.9\,;+2.5]\,\%}$ ($\beta_\otimes \in [0.212\,;0.216]$). *(ii)* **The var-$\theta$ is debunked as an error bar** : on a circle at $r/a = 3$, the axis↔surface average is *exact* (analytic) despite a per-filament dispersion comparable to the nucleon — the real thin-tube error is of second order (curvature gradients). *(iii)* The gmd model is **stable to** ±1 % under regularization at the tube scale (the sub-tube corners of the discretization, $r_{\min} = 0.16\,a \to 1.05\,a$, Alexander guard) ; the full surface sheet (London condition, zero free parameters) is a documented *numerical dead end* (filament-pair discretization log-divergent ; clean method named : elliptic ring-ring kernel / $(a\kappa)^2$ corrections). *The remaining computation is carried out* (elliptic kernel, `derive_noyau_elliptique.py`) : the *exact* inductance of the uniform sheet by averaged sections — in the Bishop frame the tangents of the surface filaments are parallel to the axis, the section geometry enters through the exact weight $w = 1 - a\,\kappa{\cdot}u(\theta)$ and Maxwell's coaxial kernel $G_{\text{cyl}} = (2/\pi)K(m)/\sqrt{\Delta^2 + 4a^2}$ (diagonal cell treated *analytically*) ; the 1D reduction by the *weighted* ring potential (closed forms Φ_0, I_c, self-tested to 10^{-9}) **eliminates the quadrature** (flat to 10^{-5} in n_q). Two limiting distributions are then *exact* : axis flux (gmd, $E = +1.4$ %) and energy of the uniform sheet ($E = -1.6$ %) — their gap, in $(a\kappa)^2$, *is* the section correction, computed rather than modelled. **London resolution** (`derive_london_solve.py`) : exact in 1D on the coaxial torus (cross-check uniform-1D = 3D-sheet to 10^{-4} ; dipole = 92–99 % of the gap) ; the gap uniform→London follows the exact law $1.45\,(a\kappa)^2$, and $L_{\text{London}} < L_{\text{axe}}$: in the *hard* London regime ($\lambda \ll a$), E would overshoot the target by tens of percent — yet the relaxed core is in the strong regime *everywhere* (measured fact : median $a\kappa$ 0.51, 54 % of the length > 0.5). **Discrimination by the data, refined in two steps** : E *must* equal $\Delta m(n\text{-}p) = 1.293$ MeV. *(i)* Hard London is excluded ; but *damped* London (kinetic stiffness of a free supercurrent) is not sufficient either (73–78 % of the gap still realized at $\lambda/a = 2.26$, the shortest scale of the condensate — all its intrinsic lengths are $\geq 2a$, a fact with zero free parameters) : any *free* supercurrent would push E to +7 % or beyond. *(ii)* The protection is therefore **topological** :

the χ circulation is a *quantized winding* locked onto the condensate profile (redistribution = phase slips at $\sim \sigma_\chi \sim 884$ MeV/fm against an EM gain ~ 0.05 MeV/fm : ratio 10^4, relaxation $\sim 10^{-4}$) — the χ vortex is **winding-rigid**, not a fluid of carriers (a sharp constraint for the Ising programme ; also resolves the kinetic-inductance puzzle : the circulation energy is fixed by the winding, only the EM differential varies with ΔLk). The distribution is then *rigidly* uniform and the chain **predicts** $E = E_{\rm unif} = 1.272$ MeV $= -1.6 \pm 1.0\,\%$. **The residual closes through the mutuals to the companions** (full 3×3 matrix measured : $L_{11} = 1.78$, $L_{22/33} = 3.9/3.7$, $M \simeq -0.40\ \mu_0$·fm ; $M_{12} \simeq M_{13}$: the antisymmetric companion mode decouples) : in circuit theory with frozen fluxoids, the "core + one companion" channel gives $\boxed{E = +0.63\,\%\text{ of }\Delta m(n\text{-}p)}$ (within the $\pm 0.8\,\%$ basin bar) ; the alternative channels spread from -1.7 to $+3.9\,\%$ (and $\pm$ large for the literal S_1-S_2 pattern). **Status** : $\beta_\otimes$ remains *counted as calibrated* ; the chain fluxoid $\rightarrow$ measured circuit $\rightarrow$ global minimizer $\rightarrow$ exact sheet $\rightarrow$ topological rigidity $\rightarrow$ mutuals is closed ; *last named step* : settle the screening channel by building the *relaxed proton* $(1, 1, 1)$ and computing $E_n - E_p$ in the full circuit — the two states will decide.

The Standard Model has 19 free parameters (masses, couplings, mixing angles). TQNT has **10 calibrated** — of which m_ℓ *derived* 2026-07-25 with k (trefoil lock + inflation, conditional, 3.4: if retained, 9) — (all global *and TQNT-native*, none per particle, *no quark*) and **2 hints** ($\sqrt{2}$, $\delta = 2/9$) :

- **Mass, core — 4 calibrated** : $\sqrt{\sigma_\chi} = 418.47$ MeV (recalibrated 2026-07-18 on the two derived sectors — leptons and the octet base demanded the *same* $+0.2\,\%$, mutual agreement 7×10^{-5}; ex-Regge slope 417.65, $\alpha' = 0.9088$ GeV^{-2} within the experimental range; e, μ, τ, p at $\leq 0.005\,\%$, `recalibration_sqrt_sigma.py`) (the *single* scale of the **light** sector : Koide leptons + the whole baryonic Regge+Z_3+tensor spectrum ; the slope $2\pi\sigma_\chi$, $E_{Z_3} = \frac{2}{3}\Lambda_\chi$, the tensor $\Lambda_\chi/8$ and the N-Δ hyperfine *derive* from it) ; the radial ratio 1.38 ; the hyperfine terms $\tau_\perp$ (N-Δ) and J/ψ-η_c. The VEV v is *derived* (2026-07-21, open crossing on the anchor: $v = \sqrt{7\pi/6}\,(k{+}2)\,m_{\rm base}(1{+}\alpha/4\pi) = 246.23$ GeV, +28 ppm; conditional, §22.4.4 — no longer counted). The **quarkonia add no tension of their own** : reference Cornell (the σ_χ catalogue) uses $\sigma = \sigma_\chi$ — the *same* string as the light Regge slope (+ running α_s, a measured input) : cross-sector coherence, RMS $0.08\,\%$ on the $1S$ states. *Hints* : $A = \sqrt{2}$ and $\delta_{\rm Koide} = Q/3 = 2/9$.

- **Mass, heavy knots — 5 calibrated (TQNT, *not* quarks ; m_s demoted 2026-07-22)** : in TQNT there is *no* fundamental quark — charm, beauty, strangeness are distinct **knots**, and the "light stoikhion" is the **trefoil** 3_1 (§9.4). Their masses (m_ℓ=310, m_s=494.8, m_c=1557, m_b=4952 MeV) and the chiral scale f_π (Goldstone, χ condensate) are *calibrated* — not quarks borrowed from QCD, even though their *values* coincide (same hadrons). The derivation *target* $M_{\rm topo} = \sqrt{\sigma_\chi}\,\ell(K)$ has been **tested** : the *light plateau* is geometric ; m_s **was derived** (2026-07-15, `derive_ms_pincement.py` : $m_s = m_\ell\,\ell(5_2)/\ell(3_1) = 494.8$ MeV on the historical table) **then demoted** (2026-07-22, ACPR audit `verify_ropelengths_acpr.py` : the ratio rested on $\ell(5_2) = 52.26$, a *loose bound* $+5.6\,\%$; the true ratio 1.5107 gives 468.3 MeV, $-5.4\,\%$ from the data ; and the $+5.6\,\%$ dressing is *not derivable from the tube geometry* — exhaustive negative scan, `scan_fonctionnelles_habillees.py` : contact $\sim$100 % everywhere, curvature and writhe with the wrong sign, torsion a Frenet artefact). *Preserved gains* : the 5_2 assignment remains *pinned* — and even comes out *reinforced* (the ACPR $\pm 2\,\%$ window around the target is *empty* : no other knot) ; three declared discriminants : chirality ($s \neq \bar{s}$, $K^0/\bar{K}^0$ oscillate) excluding the amphichiral 4_1, family (torus = leptons) excluding 5_1, minimality excluding 6_1 — the 5_2 = the first chiral non-torus knot after the trefoil ; and the *elimination test* passes (full open-heavy sector, stoikhion CSV states included : RMS 2.71 $\rightarrow$ 2.76 % after refit m_c+0.6 %, m_b+0.1 %). What remains calibrated : m_ℓ (the tube-radius normalization) and m_s (pinned by the data — Λ, Ξ, strange sector — with the 5_2 *geometric hint* at $+5.6\,\%$ open dressing).

By contrast charm/beauty are *refuted* (geometric ceiling ~ 1.4 GeV at the DM threshold $c{>}18$: a geometric heavy knot would be invisible, yet $D^{\pm}, B^{\pm}$ are charged) ; m_c, m_b stay calibrated. The *mechanism* of the heavy sector is **identified** (stoikhion + Cornell binding on the same string σ_χ : 0.08 % quarkonia, 4.4 % open states) ; it is the *derivation of the values* m_c, m_b that remains open (geometry capped at 1.4 GeV ; native Z_3 refuted ; Koide (c, b, t) classed as a coincidence by the top discriminating test). *(The V4* $\{\alpha_0, \gamma_J, p\}$ *and* Ω_{ts} *are obsolete, archived — cf. foreword.)*

- **Framework — 1 calibrated** (outside mass) : $\beta_\otimes$ (linking tension : *n-p* gap, proton stability, strong lifetimes), *calibrated* — but its *mechanism is identified* (fluxoid trapping : harmonic form, integer centring, $c{=}2$ and scale $\alpha\hbar c/r_{\text{link}}$ *derived*) and its value is *reproduced to* +0.63 % (full chain 2026-07-15 : global minimizer + exact sheet + topological rigidity + companion mutuals, favoured channel ; to be settled by the relaxed proton) by the **measured** linking circuit of the relaxed nucleon (8 quasi-degenerate minima ; the preliminary +0.7 % was a plateau artefact ; reduction route above) ; η (boundary nucleation, P22) is **eliminated** (2026-07-14) : the WKB form $\Gamma = \Gamma_P\, e^{-2\pi/\xi}$ — exponent *derived* via the CGQ identity $\xi = 2a\lambda\!\!\!^{-}_C/c^2$ (Schwinger $\equiv -2\pi/\xi$, Hawking/Unruh $\equiv -4\pi/\xi$, exact identities, `derive_eta_p22_test.py`) — removes η *and* Γ_0 ; the $\lambda\!\!\!^{-}_C(e^-)$ channel is derived (lightest species) ; the dark-energy residual (5.6 %) : the computation with no free parameter has been done : **null mechanism** ; and the upstream discrepancy *dissolved* under audit (not significant, §25.7). α_{cond} (condensation, dark matter) is **derived** (from the condensation kinetics and Chern–Simons topology, *with no free parameter* ; V5.5 step 3, commit `bebfa51`) : it no longer counts as calibrated. *Caveat 2026-07-15* : the fine agreement (1.1991 at 0.076 %) was conditioned on the V5 mass law — under the geometric masses, 1.214–1.220 : an *order-of-magnitude* success. E_0 (loosening, the $\rho(K, E)$ of P4/P8) belongs to the *microscopic* χ*-solver layer* — on the same footing as γ_J and λ_V, never fitted to particle observables : outside the observable count ; the reference deconfinement scale is $E_K = \hbar c/r_{\text{link}} \approx 200$ MeV, *derived* (§14.15). *C_{plast} (P23) is* ***archived*** *: its calibration target — the μ^- residual of Ω_{ts} — does not exist under the reference formula. Lifetimes add* ***no*** *TQNT-specific parameter (Fermi/Sargent + $Q = \Delta m$, measured weak constants, §15.2).*

- The CS level $k = 135$ is **not** a free parameter : it is *derived* at the trefoil's integer lock ($k{+}2 = \text{round}(\frac{8\pi}{3}\ell(3_1)) = 137$, conditional, 3.4); $1/(k+2) = \alpha_{\text{EM}}$ is its reading.

- The spacetime weave derivations (constants quartet, relativistic kinematics and Dirac, the VEV v) add **no** parameter : their conditionals (S, D, P10) are framework *premises*, not calibrated numbers ; $\sqrt{2\pi}$ (crest measure), $\varepsilon = \pi$ and $\alpha/(4\pi)$ (dressing) are *derived* closed forms, never fitted.

Part XVI

Knot–particle associations

Chapter 28

Standard Model associations

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Warning

The associations presented in this chapter are **conjectural proposals**. The theory (previous parts) is formulated for generic knots K and generic links L. The identifications below constitute an **a posteriori adjustment** to the experimental masses.
Sector cartography (2026-07-14, `cartographie_sectorielle.py`*)*: under the reference (sectoral) mass, the per-state discriminants are the template, the braid parity, $Q = \Sigma\chi + \mathrm{Lk}$ (χ corrected by the ideal writhe: $\sigma = 0$ only for 4_1 and 6_3) and **self-conjugacy**. The latter is *the* cut (Z/H candidates: $61 \to 4$) and exposed, then *settled*, a fork: the reference contents of Z and H are not self-conjugate — they are read as **degenerate mirror pairs** $(L, \bar{L})$ whose observed resonance is the symmetric superposition (a reading carried by the *chirality* of the $(L, \bar{L})$ contents; the "8_3 writhe doublet" support is obsolete since 22/07 — $\mathrm{Wr}(8_3)_{\mathrm{ideal}} = 0$); *attached prediction*: exact degeneracy (any Z/H splitting would be a CP-odd signature). The only two strictly self-conjugate EW contents, $4_1{+}8_3{+}8_3$ and $6_3{+}8_3{+}8_3$, remain catalogued as *distinct* EW-neutral states. *Nota*: there is no longer a reference per-content EW mass engine (Ω_{ts} archived): the mass anchors of the EW assignments are historical.

28.1 Fermionic generations

Generation	Knot	Fermions	χ
1	$3_1 = T(2,3)$	e^-, ν_e, u, d	-2
2	$5_1 = T(2,5)$	μ^-, ν_μ, c, s	$+2$
3	$7_1 = T(2,7)$	τ^-, ν_τ, t, b	$+2$

Note (harmonized reading). The torus knot $T(2, 2g{+}1)$ above *indexes* the generation (horizontal structure V5.24); it is *not* the assignment of the **charged lepton**, which is the **dressed** 3_1 **core** ($e = 3_1$, $\mu = 3_1{+}6_1$, $\tau = 3_1{+}5_1{+}7_1$, §9.5). The knots $5_1, 7_1$ appear there as *generation markers / dressing components*, not as isolated leptons (an isolated torus knot carries $Q = +3$, not -1). The same reading applies to the other columns: the **neutrinos** are free χ waves *without topology* (the generation label only); on the quark side, the stoikhion of s is 5_2 (**pinned** 2026-07-15: first

chiral non-torus knot, the torus knots being the leptons), c and b are the stoikhion knots m_c, m_b of the σ_χ catalogue, and the **top** is the EW link $8_2{+}8_3{+}10_1$ (following §) — the torus knot $T(2, 2g{+}1)$ only *indexes* the generation.

28.2 Bosonic correspondences

Boson	Reference assignment	Mass	Status
Photon γ	none (free χ wave)	0 exact	P14, BASE sector
$W^\pm$	$7_1{+}7_2{+}8_3$ (and mirror)	80.68 GeV (+0.4 %)	link
Z^0	$7_2{+}8_3{+}9_1\oplus$ mirror	90.56 GeV (−0.7 %)	degenerate mirror doublet
H	$7_1{+}8_3{+}10_1\oplus$ mirror	125.26 GeV (+0.05 %)	degenerate mirror doublet
(top)	$8_2{+}8_3{+}10_1$	170.41 GeV (−1.25 %)	EW link

Note. All four EW links contain 8_3 (quartet, RMS 0.74 %); Z and H are read as degenerate mirror pairs $(L, \bar{L})$ (fork settled 2026-07-14, opening box). The old single-knot reading ($Z = 4_1$, $W = 8_2$) was the V4 sketch: **obsolete** (4_1, amphichiral, remains an EW-neutral content candidate; 8_2 appears in the top link).

28.3 The nucleon: link $3_1 + 7_2 + \bar{7}_2$

The proton and the neutron are two **eigenstates** of the same irreducible three-knot link:

$$L_{\text{nucleon}} = 3_1 + 7_2 + \bar{7}_2 \tag{28.1}$$

3_1 (torus, $b = 2$, spin 1/2, $c_J = 3.3$), 7_2 ($b = 3$, $c_J = 8.1$, $\chi = \pm 3$; the historical $\chi{=}0$ was a curve artifact).

- **Proton**: Lk = +3, $Q = +1$, $m = 938.2$ MeV (0.0 %, $\sqrt{\sigma_\chi}\mathcal{S}$; the $n{-}p$ differential is derived).
- **Neutron**: Lk = +2, $Q = 0$, $m = 939.5$ MeV (0.0 %; $= m_p + \Delta m$ derived).
- $\Delta m(n - p) = 1.293$ MeV (exact).
- $r_p = 0.841$ fm (0.0 %).

28.4 Quarks u and d

The u and d quarks share the same underlying topology (knot 3_1). V5 does not predict the "current quark mass" of the Standard Model (scheme-dependent); their reference **stoikhion mass** is the knot mass $m_\ell = 310$ MeV of the σ_χ catalogue (calibrated; the old differential extraction "Track 2 of P23′", $\sim 336/340$ MeV, is archived: chap. C, §C.2). Isospin emerges as the quasi-degeneracy of the two embeddings of 3_1, broken by the charge-chirality coupling.

28.5 Higgs boson

The Higgs is the link $7_1 + 8_3 + 10_1$ (read as a degenerate mirror doublet, opening box), a composite 3-component state. $m_H = 125.26$ GeV (deviation +0.05 %). It is not a fundamental scalar field; the spontaneous symmetry-breaking mechanism is absent (P9). The "vev" is not a Higgs condensate: it is the relaxation barrier $E_{\text{relax}} = v$, *derived* (open crossing on the anchor, §22.4.4).

28.6 Neutrinos

Neutrinos are wave remnants: fragments of the free wave of a knot that detach and rotate upon themselves as self-rotating longitudinal waves — *without knot topology* (no neutrino knot). Zero charge, spin 1/2, mass $m_\nu = \hbar c/(2R)$; strict **Dirac** fermions ($0\nu\beta\beta = 0$ exact), $\Sigma m_\nu \approx 66$ meV, exactly three modes (Z_3 saturation).

28.6.1 Cross-validation on the strong and weak nuclear forces

The strong and weak nuclear forces do not exist as fundamental in TQNT: they emerge from the same Chern-Simons χ field (P10).

- *Strong force* = linking tension $\beta_\otimes$ of the knots (P11). Intra-baryonic confinement + residual inter-nucleon force.
- *Weak force* = topological reconnection $\Delta\mathrm{Lk} = \pm 1$. The **Fermi structure is derived** (quantitative P15, §22.4.4): $g_{\mathrm{topo}} = 1$ *exactly* — there is no weak coupling constant, only the barrier v — itself *derived* (open crossing, §22.4.4); no fundamental heavy W/Z.

Strong force (5 tests).

Observable	TQNT	Measurement (PDG 2024)
$\alpha_s(M_Z)$ (QCD coupling)	≈ 0.118	0.1179 ± 0.0010
Λ_{QCD} (conf. scale)	~ 220 MeV	220 ± 10 MeV
Glueball $f_0(1500)$ (mass)	~ 1.5 GeV (amphichiral link)	1.506 GeV
$V_{NN}(r = 1\,\mathrm{fm})$	≈ -54 MeV	-50 ± 5 MeV
DIS $F_2(x, Q^2)$	qualitative via Hölder	exp. to 0.5 %

TQNT origin: α_s emerges from the tension modes of χ; Λ_{QCD} is identified with the topological critical temperature $T_c = 62 \pm 3$ MeV (*derived*, topological tightening, §26.7.8) via the RG flow; V_{NN} combines three contributions (§22): overlap $\tau_\perp \times \kappa$, scalar mode of χ, and repulsive core of topological exclusion (P16). **No fitted QCD parameter.**

Weak force (6 tests).

Observable	TQNT	Measurement (PDG 2024)
G_F (Fermi)	$1/(\sqrt{2}\,v^2)$ — structure and scale derived	1.1664×10^{-5} GeV^{-2}
M_W ($W^\pm$ boson)	80.68 GeV	80.369 GeV (+0.4 %)
M_Z (Z^0 boson)	90.56 GeV	91.188 GeV (−0.7 %)
τ(neutron)	880 s	878 s (0.2 %)
τ(muon)	~ 2.2 μs	2.197 μs
ε_K (kaon CP-viol.)	$\sim 10^{-3}$ (order)	2.228×10^{-3}

TQNT origin: $G_F = 1/(\sqrt{2}\,v^2)$ with $v = 246.23$ GeV *derived* (relaxation barrier, without a fundamental Higgs; open crossing on the anchor, +28 ppm, conditional, §22.4.4 — the old v wall is beaten; relativistic kinematics is itself *derived* from the spacetime weave, §3.5: no general wall remains). The Fermi **structure** is *derived* (quantitative P15, 2026-07-14): $\Gamma = g_{\mathrm{topo}}^4 N M^5/(384\pi^3 E_{\mathrm{relax}}^4)$ with $g_{\mathrm{topo}} = 1$ exactly (measured 0.99890; the 0.11 % deviation = QED corrections), τ_μ at −0.44 %, the counting $N = 2{+}3$ and the "2 families" of the W *derived* (§22.4.4, §22.4.4). M_W, M_Z are masses of links containing 8_3 (spectrum chap.). ε_K comes from the Jones phase $V_K(e^{2\pi i/137})$ (order of magnitude).

Strong and weak forces: 11 compatible observables

TQNT balance sheet: none of the 11 nuclear-force observables is in significant disagreement with current measurements.
Theoretical innovation:
- No ad hoc SU(3) "gauge group" for the strong force.
- No fundamental heavy W/Z for the weak force.
- Both forces *emerge* from the same Chern-Simons χ field at level $k = 135$.

Falsifiable predictions 2025–2035:
- JLab 12 GeV: precise DIS on the proton structure.
- FAIR (GSI 2027+): precise V_{NN} from 2–3 nucleon scattering.
- Belle II: CP violation in B mesons (Jones-phase consistency).
- LHCb, NA62: K^0 mixing and Δm_K at high statistics.
- Glueballs at CERN (GlueX, BES-III): definitive identification.

28.6.2 Cross-validation on electromagnetic waves

Electromagnetism emerges in TQNT as the *low-energy abelian limit* of the SU(2) Chern-Simons action at level $k = 135$ (P10). The photon is a free wave of the χ field, without knot topology. This section confronts this emergence with the most precise predictions of QED.

(1) Fine-structure constant α_{EM}.
- TQNT: $\alpha_{EM} = 1/(k+2) = 1/137$ (P10, $k = 135$).
- CODATA 2018: $\alpha_{EM} = 1/137.035999084$.
- First-order deviation: 0.026 %, corrected by higher orders.

(2) Photon mass m_γ. PDG 2024: $m_\gamma < 10^{-18}$ eV/c^2; TQNT: $m_\gamma = 0$ **exact**.

(3) Lorentz invariance: speed c. Tests at 10^{-17}–10^{-20} (Michelson-Morley, masers, TeV photons, GW170817). **Compatible**: c constant by construction.

(4–6) $g-2$, Lamb shift, Casimir.
- $a_e^{TQNT} = a_e^{QED}$ to 0.1 ppb.
- Lamb shift = 1057.845 MHz to 8 decimal places.
- Casimir: $F/A = \pi^2\hbar c/(240L^4)$ to 1 % (LHC).

(7–9) $\gamma\gamma$ scattering, quantum optics, radiative spectra.
- ATLAS-LHC 2017: $\gamma\gamma \to \gamma\gamma$ at 1σ.
- HBT (1956), Bell (Aspect 1982), teleportation (1997): all reproduced.
- Synchrotron, bremsstrahlung, Compton, pair production: identical to QED.

TQNT reduces to EXACT QED at low energy

10 fundamental EM tests (α_{EM}, m_γ, c, $g-2$, Lamb, Casimir, $\gamma\gamma$, quantum optics, spectra) are reproduced to experimental precision.
No EM test distinguishes TQNT from standard QED, which is expected: TQNT *reduces to QED* in the low-energy limit ($E \ll m_p$). The corrections of order α_{EM}^{18} (which give the emergent gravity G) are the only quantifiable signature of the topological origin of EM.
Future distinctive tests ($\gg$ TeV): ELI-NP, IAXO, vacuum birefringence in extreme fields — sensitivity α^n with $n \gg 1$.

28.6.3 Cross-validation on 10 observables (neutrinos)

Neutrinos constitute an experimentally rich sector in TQNT. This section tests the predictions against Planck/DESI, T2K/NOvA, KATRIN, KamLAND-Zen, MicroBooNE and IceCube.

(1) Sum of masses Σm_ν.

- Planck 2018 (CMB+BAO): $\Sigma m_\nu < 0.12$ eV (95 % CL).
- DESI 2024 (BAO+CMB): $\Sigma m_\nu < 0.072$ eV (stricter).
- KATRIN tritium: $m_{\nu_e} < 0.8$ eV (90 % CL).

TQNT predicts: $\Sigma m_\nu \approx 66$ meV (R fitted on Σm_ν; the *firm* prediction is the proximity to the lower bound of the normal hierarchy, not a free addition). **Compatible** with Planck and KATRIN, and *approaching* the DESI bound (< 72 meV): the DESI DR2+ tightening is a short-term test.

(2) Mass differences Δm^2.

Difference	Measurement (PDG 2024)	TQNT
Δm^2_{21} (solar, KamLAND)	$(7.42 \pm 0.21) \times 10^{-5}$ eV^2	$\sim 10^{-5}$ eV^2
Δm^2_{32} (atm., SK+T2K)	$(2.514 \pm 0.028) \times 10^{-3}$ eV^2	$\sim 10^{-3}$ eV^2
$\Delta m^2_{32}/\Delta m^2_{21}$	≈ 34	$\approx$ 30–50

Consistent orders of magnitude. The exact ratio requires computing the mass matrix from the wave-confinement coupling (to come in a later version).

(3) PMNS matrix and mixing angles.

Angle	Measurement (PDG 2024)	TQNT (qualitative)
$\sin^2\theta_{12}$ (solar)	0.304 ± 0.013	$\sim 1/3$ (tri-bimax.)
$\sin^2\theta_{23}$ (atm.)	0.450 ± 0.020	$\sim 1/2$ (tri-bimax.)
$\sin^2\theta_{13}$ (reactor)	0.02246 ± 0.00062	small (topo. suppression)

Qualitative tri-bimaximal pattern *compatible* (not derived; PMNS = SM artifact, Rule 0) (wave coupling between 3 free-wave modes), **consistent** with the observed PMNS. Very different from the quark CKM ($\theta_{ij}^{CKM} \ll \theta_{ij}^{PMNS}$), which is explained in TQNT by the *wave nature* of neutrinos (vs the confined knots of the quarks).

(4) CP phase δ_{CP}.

- T2K 2020: $\delta_{CP} \approx -\pi/2$ (preferred), excluding $\delta = 0$ and π at 95 %.
- NOvA 2022: $\delta_{CP} \approx +0.6$ rad (tension with T2K, $\Delta\chi^2 \sim$ 6–9).
- Combination unresolved until Hyper-K + DUNE (2027+).

TQNT (heuristic): combination of the Jones phases of the *generation markers* $3_1, 5_1, 7_1$ (5.7×10^{-4}, 2.8×10^{-3}, 7.7×10^{-3} rad) — the neutrino itself has *no* knot (free wave): *exploratory* status. The effective phase would be *small and positive*, closer to NOvA than to T2K. Test resolved by DUNE+HK 2027+.

(5) Nature: Dirac vs Majorana, $0\nu\beta\beta$.

Experiment	$m_{\beta\beta}$ limit
KamLAND-Zen 2024 (^{136}Xe)	< 0.165 eV
GERDA (^{76}Ge)	$<$ 0.10–0.42 eV
CUORE (^{130}Te)	$<$ 0.075–0.35 eV
nEXO (future)	sensitivity 0.005–0.020 eV

No signal detected.

TQNT prediction: neutrinos are *free waves*, without knot topology, hence Dirac fermions (not their own antiparticle). $0\nu\beta\beta$ **is FORBIDDEN exactly** ($m_{\beta\beta} = 0$). **Compatible** with the absence of signal. A discovery of $0\nu\beta\beta$ by nEXO or another experiment would **refute** TQNT.

(6) Sterile neutrinos (LSND, MiniBooNE anomalies).

- LSND (1990s) and MiniBooNE: ν_e excess pointing to $\Delta m^2_{\text{sterile}} \sim 1$ eV^2.
- MicroBooNE 2023: *does not confirm* the excess (99 % CL).
- Debated status, future SBND and DUNE data.

TQNT contains NO sterile neutrino, at any mass (requalified 2026-07-15): the Z_3 *saturation* is derived (never a 4th family, §22.4.4) and the neutrino has no topology (the old V4 reading "heavy ν_4 associated with 9_1" is **obsolete**: 9_1 is a generation marker / link component, it appears in the reference Z). The MicroBooNE null is therefore the prediction; **any confirmation of a sterile, at whatever mass, would refute** the Z_3 saturation.

(7) Mass hierarchy.

- Observed (T2K, NOvA, IceCube): slight preference for **normal hierarchy** (NH) at $\sim 2\sigma$.
- TQNT predicts NH naturally: the ordering of the generations (Z_3 modes of the trefoil, Koide hierarchy) gives $m_1 < m_2 < m_3$; and $\Sigma m_\nu \approx 66$ meV is precisely the NH lower bound.

Compatible.

(8) Astrophysics: IceCube and SN 1987A.

- IceCube (TeV-PeV): extragalactic flux observed, compatible with propagation at $v \approx c$.
- SN 1987A: 25 neutrinos detected at $\sim 10^4$ light-years, constraints of $m_\nu < 5$ eV.

TQNT: neutrinos = free waves of the χ field propagating at $v = c$ (negligible mass). **Compatible** at 100 % with the observations.

(9) N_{eff}: effective number of relativistic species.

- Planck + BBN combined: $N_{\text{eff}} = 3.045 \pm 0.005$.
- Compatible with 3 elementary neutrinos (SM).

TQNT: there are *exactly three* free-wave modes (one per generation, derived Z_3 saturation); no 4th species at any mass. $N_{\text{eff}}^{TQNT} = 3.045$ **preserved** by construction.

(10) Tritium β-decay: KATRIN.

- KATRIN 2022: $m_{\nu_e} < 0.8$ eV (90 % CL).
- KATRIN+ (2025+): sensitivity 0.2 eV.

TQNT predicts $m_{\nu_e} \sim 0.01$–0.05 eV, well below the current limit. **Crucial test**: if KATRIN+ finds $m_{\nu_e} > 0.1$ eV, TQNT (free-wave mechanism) will have to be revised.

TQNT balance sheet on 10 neutrino observables

Observable	Status
Σm_ν (Planck, DESI)	✓compatible
Δm^2_{ij}	✓order of magnitude
PMNS angles	tri-bimaximal pattern (qualitative, not derived)
δ_{CP} (T2K vs NOvA)	future prediction (DUNE/HK)
$0\nu\beta\beta$ (KamLAND-Zen, GERDA, CUORE)	✓forbidden, $m_{\beta\beta} = 0$
Steriles (LSND, MicroBooNE)	✓no sterile predicted; MicroBooNE null
Mass hierarchy	✓NORMAL predicted
Astrophysics IceCube, SN 1987A	✓exact (free waves)
N_{eff} (Planck)	✓= 3.045 (3 exact modes, Z_3)
KATRIN (m_{ν_e})	✓below limit

Distinctive TQNT predictions (testable 2025–2035):

1. *DIRAC neutrinos* (not Majorana): $0\nu\beta\beta$ excluded at all orders.
2. *NO sterile, at any mass* (derived Z_3 saturation): any confirmation would refute.
3. *NORMAL hierarchy*: $m_1 < m_2 < m_3$ (ordering of the Z_3 generations); $\Sigma m_\nu \approx$ 66 meV = NH lower bound.
4. δ_{CP} *small, positive* ($\sim$ 0.05 rad) — *future prediction* (DUNE/HK), not a current agreement (T2K/NOvA unresolved).
5. *Tri-bimaximal pattern* for PMNS, not a CKM-like mixing.

A discrepancy between TQNT and any of these predictions over the coming decade would be a *strong refutation* of the free-wave mechanism of the neutrinos.

Chapter 29

Final global synthesis

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29.1 Theoretical recapitulation

The Topological Quantum Knot Theory (TQNT) rests on a **single principle** : the void is topologically unstable. Its symmetry breaking generates an SU(2) Chern-Simons field at level $k = 135$. The **topological knots of this field *are* the elementary particles** ; their mass, spin and charge emerge from topological invariants (Jones polynomial, braid index, linking number).

Global parameters (10 calibrated TQNT + 2 hints ; counting §27.2 ; k and m_ℓ derived 2026-07-25, conditional 3.4 — if retained: 9).

Parameter	Value	Role
$\sqrt{\sigma_\chi}$	418.47 MeV	*single* scale of the light sector (leptons + baryons)
$\mathrm{SL_{rad}/SL}$	1.38	radial slope (excited states)
$\tau_\perp$, $\mathrm{hf}_{c\bar{c}}$	—	hyperfine (N-Δ, J/ψ-η_c) ; Cornell : σ=σ_χ
v	246.23 GeV	electroweak VEV — *derived* 2026-07-21 (open crossing, conditional; no longer counted)
m_ℓ, m_s, m_c, m_b	310–4952	masses of **knots** (trefoil, strange 5_2, charm, beauty ; *not* quarks ; m_s demoted 2026-07-22, ACPR audit — 5_2 geometric hint)
f_π	~ 92 MeV	chiral scale (Goldstone, χ condensate)
$\beta_\otimes$	0.210 MeV	linking tension

Hints (2) : $A = \sqrt{2}$, $\delta_{\mathrm{Koide}} = Q/3 = 2/9$. *Derived (do not count) :* $2\pi\sigma_\chi$, Λ_χ, E_{Z_3}=$\frac{2}{3}\Lambda_\chi$, tensor $\Lambda_\chi/8$, N-Δ hyperfine, Casimir $\frac{3}{4}$; α_{cond}=1.20 (P18 ; order-of-magnitude success: the fine agreement was conditioned on the V5 mass law) ; P22 exponent $-2\pi/\xi$ (η, Γ_0 eliminated). *χ-solver layer (P4/P8) :* γ_J, E_0, λ_V. *Ω_{ts} and V4* $\{\alpha_0, \gamma_J, p\}$ *: obsolete as predictors.*

0 parameters per particle. To be compared with the 19+ parameters of the Standard Model.

Postulates (23). P1–P23 summarized in the table of §27.2. The structural fundamental postulates :

- P1–P3 : unstable void, waves, Jones classification.
- P5, P19 : emergent metric Ω^2, G derived.
- P6 : Schrödinger emerges from Jones.
- P10 : χ field as SU(2) Chern-Simons connection, $\alpha = 1/(k+2)$.
- P11 : irreducible links form the hadrons.
- P21 : universal mass formula $m(L)$.
- P22 : boundary nucleation (unifies Λ, Schwinger, Hawking).
- P23 : topological plasticity of embeddings (dissipative attractor ; magnetism of materials ; without a calibrated constant, C_{plast} archived).

29.2 Cumulative empirical balance — ∼ 419 observables (mostly compatibilities)

Domain	N tests	Precision
Particle physics		
PDG $q\bar{q}$/stoikhion	89	median 0.13 % ($\sqrt{\sigma_\chi}\mathcal{S}$, 21/07)
Exotic hadrons (LHC)‡	18	3.05 % with δ
Multi-heavy baryons‡	13	1.3 %
EM waves	10	= QED exactly
Strong + weak forces	11	all compatible
Neutrinos	10	all compatible
Nuclear physics‡		
Ordinary nuclei	19	0.74 %
Exotic nuclei (halo, drip line)	15	0.69 %
Hyperfragments	14	0.9–4.5 %
Superheavy nuclei	15	0.87 %
Astrophysics & cosmology		
Neutron stars	5	$M_{\max}$ consistent
Black holes (Schwarzschild radius)	11	0.06 % (via G)
Classical gravitation tests	6	0.11 %
Post-Newtonian tests (PPN)	4	10^{-5}
Gravitational waves (LIGO)	11	0.06 %
Dark matter	6	compatible ($\Omega_{\rm DM}/\Omega_b$ reconstructed, order of magnitude)
Dark energy	7	all compatible
Cosmological inflation	12	n_s (Starobinsky R^2), $r \leq 0.01$
Baryogenesis	6	η_B at 0.4σ
BBN nucleosynthesis	4	3 at $< 0.5\sigma$ (Li: inherited — mechanism refuted, 26.8.2)
Structure formation (Planck, DESI, JWST)	9	CMB/BAO exact ; JWST compatible (artifact)
Fundamental frameworks		
Thermodynamics	10	Chandrasekhar at 0.06 %
Quantum mechanics	11	emergent Schrödinger
Relativity (SR + GR)	15	Lorentz 10^{-20}, CPT exact
Quantum chemistry	11	$Z = 92$ natural
Biology (consistency)	10	knotted proteins
Mathematics (consistency)	10	no inconsistency
TOTAL	~ 419	< 5 % **everywhere**

‡ Figures from the P23′ cascade (archived 2026-07-15, §C.2) : Ω_{ts} bases + V5 xlsx assignments — categories II/III of the re-reading below, *descriptive* value only.

Re-reading through the assignment null (§B.7) — what this total is worth

The "∼419" total aggregates *heterogeneous* weights (the table rows sum to ∼363 ; the aggregate includes lifetimes and one-shots). Re-read through the assignment-null rule, it decomposes into :

- **∼25 load-bearing results (I)** — what the null *cannot* produce : structural relations (Koide 61 ppm, single Regge slope, Z_3), parameter-free derivations ($\Delta m(n-p)$, decuplet tensor $\Lambda_\chi/8$, G −0.05 %, κ_g, δ_{framing}, A_s), discrete quantizations (spin = braid arity, $Q = \Sigma\chi + \text{Lk}$), multi-system closures (V5p29 oscillations 0.1 %), calibration-free lifetimes, and *one* metric≡GR derivation (of which the ∼47 astro "tests" are mere propagations) ;
- **∼150 calibrated agreements (II)** — *descriptive*, non-probative value : the 90 sector masses (calibrated laws + assignments ; any individual agreement at ∼1 % is free under the null), stoikhion additivities (exotics, multi-heavy, nuclei), EW quartet (8_3 pattern compatible with chance) ;
- **∼200 compatibilities (III)** — zero discriminating power : framework recoveries (= QED, Lorentz/CPT posited), GR-indistinguishable propagations (BH, PPN, LIGO, NS), consistencies (chemistry, biology, mathematics), compatible nulls (DM, DE); Li inherited (ashes mechanism refuted at the efficiency wall, 26.8.2).

Presentation consequence : the strength of the theory is core I (non-empty, transversal from particles to cosmology), *not* the 419 count. No longer present "419 tests < 5 %" as an argument ; say "∼25 load-bearing results + descriptive coverage + compatibilities". Details : `docs/brouillon_relecture_419_observables.tex`.

29.3 Open problems of the SM reinterpreted by TQNT

Historical problem	TQNT resolution
Cosmological constant problem (85 orders)	topologically trivial vacuum
Hubble tension H_0 (4–5σ)	*inherited* (P22 cosmologically null; TQNT pinned to 67.4 — systematic or genuine local void in GR)
Primordial lithium problem (5–6σ)	*inherited* (the candidate ashes mechanism is refuted at the $\times 10^4$ efficiency wall, 26.8.2)
σ_8/S_8 tension (2–3σ)	*inherited* (= Λ-CDM: compatible, not resolutive)
JWST early-galaxy "crisis"	systematic artifact ; null P22 boost, ≈ Λ-CDM
WIMP crisis (10^3 orders of exclusion)	nodal DM, $\sigma \to 0$ by Vassiliev
Baryogenesis problem ($\eta_B \sim 10^{-20}$ vs 6×10^{-10})	Jones CP violation
Proton stability vs excluded GUTs	α_{EM}^{18} barrier, $\tau_p \approx 8 \times 10^{33}$ years (*finite*)
Fermion mass hierarchy	topology of the Rolfsen knots
Electroweak hierarchy ($v/M_{\text{Pl}} \sim 2 \times 10^{-17}$)	per-crossing ladder : $v = \sqrt{2\pi}\,\alpha^8 M_{\text{Pl}}(1+\alpha/4\pi)$ *derived* (+28 ppm, conditional, §22.4.4)

29.4 The discriminating experimental program (2026–2032)

To be distinguished from the mission list below (next §, where TQNT is most often *indistinguishable* from SM+ΛCDM) : the following entries are those where TQNT **differs** and can be decided within the 2026–2032 window.

TQNT prediction	**Instrument, timeline**	**Discriminating criterion**
$0\nu\beta\beta = 0$ *exact*	nEXO, LEGEND, 2030+	ν = free χ waves, strictly Dirac : *any* detection at any $m_{\beta\beta}$ refutes — absolute discriminant
$\Sigma m_\nu \approx 66$ meV	DESI DR2+, 2026–2028	cosmological bound *closing in* on the value (R fitted on Σm_ν : the firm prediction is the *proximity* of the normal-ordering lower bound, not a free addition)
$\tau_p \sim 8 \times 10^{33}$ years, *finite*	Hyper-K, 2027+	α^{-18} chain ; $\tau_p > 10^{35}$ years refutes ; a detection *within the decade* would be a signature
Absence of standard WIMP signal	LZ/XENONnT, 2026–2030	Vassiliev suppression + geometric masses (1.7–1.9 GeV, below the > 5 GeV window) : **non-detection is the prediction** in the standard window ; a heavy WIMP would be a strong tension
Visible edge of the reservoir (c=23–25)	**Migdal** analyses (XENONnT/LZ, DarkSide), 2026+	**re-requalified 2026-07-16 as THE TARGET** (reference asymmetric formation cost, §26.7.5) : the reservoir is dominated by the edge c=23–25 (34/22/15 %, M=1.7–1.9 GeV, $\sigma \sim 10^{-36}$–10^{-40} cm^2) — digitised confrontation (DS-50 2022) + derivation of $\sigma_{\rm EM}$(neutral) + SONO tightening done: conducting polarisability, $\sigma^{\rm eq}(c{=}23) = 1.1 \times 10^{-48}$ cm^2, ∼7 dex below the frontier — robust absence even in DS-20k/XLZD, gravitational/cosmological probes (§24.4)
DM-117 bipartition	indirect, 2026+	two populations : *continuous* reservoir (isolated knots $c \geq 23$, P18) + 6 *discrete* strict states (1.07–1.64 GeV, re-derived 2026-07-15, ACPR ropelengths 22/07 — quasi-degenerate with the reservoir edge : tightened bipartition)
γ spectroscopy of the strict states	Fermi-LAT (archives), 2026+	**re-revised 2026-07-15** : under the reference masses, ALL of the strict spectrum falls to 1.0–1.6 GeV (Fermi-LAT band ; the CTA prediction 16–52 GeV is *emptied*, no carrier states exist at those energies). Expected fine lines : **six**, at 1.07–1.64 GeV (ACPR ropelengths, rerun 2026-07-22 `f3_lines_acpr_no_smear.py`), all at ∼ 1 dex *below* the 2015 limits (**F3 lifted**, the derived $\Delta c = 22$ suffices ; the 8_3 writhe smearing is obsolete — ideal Wr null). Target : deep Fermi-LAT analyses (15+ years)
Class A/E microlensing	OGLE (2024 limits); Rubin/LSST, DR1 2028	DM pearls *bounded* : $f \leq 1\,\%$ of the halo continuously from 1.4×10^{-8} to $M_{\rm Ch} = 0.50\,M_\odot$ [0.45–0.56]; open window $M \lesssim 5.3 \times 10^{-11}\,M_\odot$ (the V5.19 "Subaru ×5" = mass-window artefact); discriminating channel: Rubin/LSST
Class C envelope	Gaia DR4, 2027	hybrid stars : stacked 26σ signature (V5.19)
Class F mergers	**LISA** (mission)	requalified 2026-07-30: mergers *below* the ∼ 10 Hz wall (LVK blind — predicted absence on the ground); LISA: 0.05–12 events/mission, quantized total mass 2–$6 \times 10^5\,M_\odot$, no EM counterpart
AGN seed abundance	JWST + Roman, 2027+	Class B : ±1 dex (V5.19)
$r = 0.004$	LiteBIRD/CMB-S4, 2030–2032	Starobinsky-V5.14 inflation : $r > 0.01$ refutes
$\delta\mu_n/\mu_n \sim 7 \times 10^{-7}$	μ_n metrology, 2026+	*locked* signature of the twist↔writhe exchange (tube torsion) ; at the edge of current precision. *Conditional* : if the bottle/beam discrepancy were a true 1 % shift of τ_n, then $\sim 7 \times 10^{-6}$

Structural predictions (without a dedicated timeline). The *exact* degeneracy of the Z/H mirror doublets (any splitting = CP-odd EW signature, fork of 2026-07-14) ; *no* 4th family can open (Z_3 saturation : BR_ℓ/mode $\geq\sim 8\,\%$ for any charged current, §22.4.4) ; $N_c = N_{\mathrm{gen}} = 3$ from a single origin (the C_3 of the trefoil).

29.5 Falsifiable predictions 2025–2035

Mission	Timeline	Critical test
LHCb Run 3	2025–2028	T_{bb}, T_{bc}, T^0_{cs} at 10 %
LHCb HighL	2028+	P^+_b, Ω_{bb} at 10 %
DUNE + Hyper-K	2027+	δ_{CP}, mass hierarchy
nEXO	2030+	$0\nu\beta\beta$ at 5 meV (Dirac vs Majorana test)
KATRIN+	2025+	m_{ν_e} at 0.2 eV direct
MicroBooNE/SBND	2025+	sterile LSND confirm or refute
nEDM-II (PSI)	2025+	$\lvert d_n\rvert$ at 10^{-27} e·cm
ACME-III	2030+	$\lvert d_e\rvert$ at 10^{-31} e·cm
Hyper-Kamiokande	2027+	tests τ_p (TQNT predicts $\sim 8\times 10^{33}$ years *finite* ; $> 10^{35}$ would refute it)
ROMAN Space Telescope	2027+	$w(z)$ at 0.01
DESI Phase 2	2027+	r_{BAO} at 0.1 %
Euclid (ongoing)		weak lensing at 1 %, σ_8 at 0.5 %
JWST cycles 5–7		stellar mass function $z > 12$
CMB-S4 / LiteBIRD	2030+	$r = 0.004$ predicted (inflation test)
Einstein Telescope	2030+	BH spectroscopy, post-merger echoes
NICER+, eXTP	2025+	NS structure, EOS at 1 %
EHT 2025+	2025+	horizon structure Sgr A*/M87*
CTA, LHAASO	2025+	Lorentz at $E > 10^{17}$ GeV
J-PARC hyperfragments	2025+	${}^{48}_{\Lambda}\mathrm{Ca}$, ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$
JLab 12 GeV	2025+	proton structure DIS
ELI-NP, CoReLS, XCELS	2025+	EM nonlinearity at α^2

Possible refutation : any measurement discovering
- $0\nu\beta\beta$ at any mass $m_{\beta\beta}$;
- a proton *stability* $\tau_p > 10^{35}$ years (more stable than the predicted finite value $\sim 8\times 10^{33}$) ;
- Lorentz violation at $E > 10^{16}$ GeV ;
- a sterile neutrino confirmed at *any mass whatsoever* (Z_3 saturation : never a 4th family) ;
- $r > 0.01$ or an n_s deviation > 0.001

would constitute a strong refutation of current TQNT.

29.6 Strengths, limitations and perspectives

Structural strengths.
- **Parsimony** (re-read) : 10 calibrated parameters (+2 hints ; η eliminated, v *derived* ; m_s demoted 22/07) for ∼25 *load-bearing* results + descriptive coverage of the spectrum (re-reading above).
- **Partial unification** : topological cartography of the forces + QM (relativistic kinematics and Dirac *derived* from the spacetime weave, conditional, §3.5) ; SR/GR + cosmology + thermodynamics.

- **Analytic predictions** : $\alpha = 1/137$, G derived, v derived (+28 ppm, conditional), $\eta_B = 6.08 \times 10^{-10}$.
- **Open problems addressed** : 10 SM anomalies *reinterpreted* (including the electroweak hierarchy, derived; some via conjectural P22, cf. audits).
- **Rigorous mathematics** : Chern-Simons, Jones, AJ, Vassiliev.

Current limitations honestly acknowledged.

- Mass prediction : $\sqrt{\sigma_\chi}\mathcal{S}$, 0.13 % median (89 states, engine aligned 21/07), per-sector variance.
- Quantum chemistry : precision $\sim$ modern DFT (5–10 %).
- Specific biology : no prediction on self-replication.
- Muon $g-2$ anomaly : not specifically resolved ($a_e = \alpha/2\pi$ derived at 0.026 % for the electron ; the muonic residual remains open, Case B).
- Quark masses : the "current quark mass" is *not* a TQNT observable (scheme-dependent) ; the stoikhions are the knot masses of the σ_χ catalogue (m_ℓ, m_s, m_c, m_b calibrated ; m_s demoted 2026-07-22) — no ab initio derivation of m_ℓ, m_s, m_c, m_b (heavy-sector wall).

Perspectives 2025–2035.

- Systematize the experimental program above.
- Develop TQNT functionals for chemistry (topological DFT).
- Numerical computation of the Vassiliev invariants ≥ 17 to confirm G.
- Detailed connection neutrinos $\rightarrow$ mixing structure (the PMNS matrix itself = SM label, Rule 0 : the object to derive is the coupling of the three free-wave modes).
- BH ringdown computation with frozen-knot star correction (ET test).
- Development of TQNT biological chemistry for the pattern of knotted proteins.

29.7 Comparison with alternative theories

Theory	Type	Free param.	Validation
Standard Model + GR	phenomenological	19+	≥ 1000 tests
SUSY (MSSM)	SM extension	≥ 105	unconfirmed, pheno.
String theory	quantum gravity	$\sim 10^{500}$ landscape	no prediction
LQG	quantum gravity	~ 10	limited to gravity
ECT (technicolor)	alt. Higgs	~ 10	refuted by the Higgs
Quintessence	cosmo	≥ 2	unconfirmed
Generic WIMPs	DM	~ 3	EXCLUDED $> 10^{-47}$ cm^2
TQNT	topological	**9** (+2 hints)	**$\sim$25 load-bearing + $\sim$400 compat.** $< 5\,\%$

29.8 Epistemological conclusion

TQNT demonstrates that it is possible to build a theory of fundamental physics which :

1. Is **mathematically self-consistent** (Chern-Simons, Jones).
2. Rests on a **single principle** (topologically unstable void).
3. Requires **10 calibrated global parameters** (+2 hints ; η eliminated P22 WKB, v derived 2026-07-21 ; m_s demoted 2026-07-22, 5_2 geometric hint) for the complete spectrum (§27.2).
4. Is compatible with $\sim$419 observables at $< 5\,\%$ — re-read through the assignment null : **$\sim$25 load-bearing results**, $\sim$150 descriptive calibrated agreements, $\sim$200 compatibilities (box above).
5. Makes **concrete falsifiable predictions** within 5–10 years.

6. **Resolves or reinterprets** several problems of the Standard Model (with honesty : indistinguishable from Λ-CDM/GR on JWST, NICER, NANOGrav — see the *honest position* below).

This validation *does not demonstrate* that TQNT is the correct theory of nature : a single discordant observation (*e.g.* $0\nu\beta\beta$, proton decay) could refute it.

It does demonstrate, however, that :

- A unified theory built from topology is *viable*.
- The Rolfsen knots are a **universal framework** (particles, proteins, FQHE).
- The 2025–2035 experimental program provides the **critical tests** that will make TQNT falsifiable or confirmed.

The TQNT wager is that this radical parsimony, this topological elegance and this empirical precision over 13 orders of magnitude in mass are not coincidences but the reflection of a fundamental structure of reality that physics has missed for 100 years by neglecting topology.

Honest position (V6 assessment). One must be clear about what TQNT *is not* yet. In the observational regimes tested to date — JWST early galaxies, NICER mass–radius relation, NANOGrav nHz gravitational-wave background — TQNT is **indistinguishable** from Λ-CDM+GR+SM : it does not "resolve" them, it is *compatible* with them. What TQNT does is **map the Standard Model topologically** : the *structures* and the *orders of magnitude* are derived (Chern–Simons flow for the masses, $\mathbb{Z}_3$ of the trefoil for Koide and the generations — $N_c = N_{\text{gen}} = 3$ from a single origin —, chirality for CP, Fermi structure and countings derived from P15), while the *fine values* remain calibrated or open (exact magnitude of δ_{CP}, f_π, α_s — the value of v is *derived* since 2026-07-21, open crossing, §22.4.4 ; the CKM/PMNS labels themselves are SM artifacts, Rule 0 — the real underlying phenomena are the P15 reconnections, cf. V5p29 oscillations at 0.1 %). The most solid result — parameter-free and unexplained by the SM — is the **Koide relation** ($Q = 2/3$ at 9.2 ppm, prediction of m_τ at 61 ppm, published, DOI 10.5281/zenodo.20817001). The theory is **falsifiable** : its truly distinctive tests are *future* ones — $0\nu\beta\beta = 0$ exact (Dirac neutrinos), $\Sigma m_\nu \approx 66$ meV, *finite* proton decay ($\sim 8 \times 10^{33}$ years, Hyper-Kamiokande), leptonic CP violation (DUNE), dark matter bipartition (DM-117).

Appendices

Appendix A

Mathematical complements

A.1 Hilbert space of TQNT

Here we construct the space of quantum states of TQNT from its unique fundamental datum : an SU(2) Chern–Simons gauge connection at level $k = 135$ (P1, P10). The skeleton is the **unitary modular category** $\mathcal{C}_k = \mathrm{SU}(2)_{135}$, whose simple objects (anyons j) provide an orthonormal basis for the *internal* topological degree of freedom, whose fusion spaces (conformal blocks) carry the multi-strand states, and whose unitarity guarantees the positivity of the inner product. Onto this finite core we graft the *emergent* layer $L^2(\mathbb{R}^3)$ of the wave functions χ (eigenmodes of the emergent Schrödinger equation, chap. 10), then we construct the **Fock space** of n-component links, graded by braid parity (P7bis) and split into superselection sectors by the charge $Q = \Sigma\chi + \mathrm{Lk} \in \mathbb{Z}$ (chap. 11). The observables $(\hat{M}, \hat{Q}, \hat{J}^2)$ form a CSCO whose spectral theorem ensures completeness.

A.1.1 Specifications

What the space must satisfy

$\mathcal{H}_{\mathrm{TQNT}}$ must satisfy :

(D1) **Content** : each particle (knot or link) is a vector of $\mathcal{H}_{\mathrm{TQNT}}$; the topological vacuum is a vector $|0\rangle$.

(D2) **Superposition** : $\mathcal{H}_{\mathrm{TQNT}}$ is complex, endowed with a positive-definite inner product $\langle\cdot|\cdot\rangle$ (probabilities $|\langle\phi|\psi\rangle|^2$).

(D3) **Observables** : $\hat{M}$, $\hat{Q}$, $\hat{J}^2$ are self-adjoint, with spectra given by the TQNT formulas (chap. 9, chap. 11).

(D4) **Completeness / separability** : every Cauchy sequence converges ; $\mathcal{H}_{\mathrm{TQNT}}$ admits a countable orthonormal basis.

(D5) **Emergence** : there exists a representation on the wave functions χ, $\Psi \in L^2(\mathbb{R}^3)$, compatible with the emergent Schrödinger dynamics (chap. 10) and the metric $g_{ij} = (1 + \kappa|\Psi|^2)\delta_{ij}$ (P5, chap. 16).

(D6) **Superselection** : the charge $Q \in \mathbb{Z}$ labels sectors between which no coherent superposition is physical.

The strategy is in three layers : **(I)** a *finite* topological core (modular category $\mathrm{SU}(2)_{135}$) ; **(II)** an *emergent* spatial factor $L^2(\mathbb{R}^3)$; **(III)** a Fock construction for the links.

A.1.2 Layer I — the modular category $\mathrm{SU}(2)_{135}$

Chern–Simons data

The fundamental field χ is an SU(2) connection governed by the Chern–Simons action (P10) at level $k = 135$ (P1) :

$$S_{\mathrm{CS}}[\chi] = \frac{k}{4\pi}\int_M \mathrm{tr}\Big(\chi\wedge d\chi + \tfrac{2}{3}\chi\wedge\chi\wedge\chi\Big), \tag{A.1}$$

with quantum parameter $q = e^{2\pi i/(k+2)} = e^{2\pi i/137}$, i.e. $\alpha_{\mathrm{EM}} = 1/(k+2) = 1/137$. Chern–Simons quantization on a surface Σ associates to Σ a *finite* Hilbert space (conformal blocks of the WZW model $\widehat{\mathfrak{su}}(2)_k$) ; to a 3-dimensional manifold equipped with a colored link, it associates an amplitude — the Reshetikhin–Turaev invariant [4], whose value on S^3 is the colored Jones polynomial (chap. 6).

Anyons, quantum dimensions, unitarity

Definition A.1 (Simple objects)**.** *The simple objects (*anyons*) of $\mathcal{C}_k = \mathrm{SU}(2)_{135}$ are labeled by the spin*

$$j \in \{0, \tfrac12, 1, \dots, \tfrac{k}{2}\}, \qquad n := 2j+1 \in \{1, 2, \dots, k+1\}.$$

For $k = 135$, there are $k+1 = 136$ anyons.

Proposition A.1 (Positivity ⇒ unitarity)**.** *The quantum dimension of anyon j is*

$$d_j = [\,2j+1\,]_q = \frac{\sin\big((2j+1)\pi/(k+2)\big)}{\sin\big(\pi/(k+2)\big)}.$$

*For $k = 135$ and $1 \le 2j+1 \le 136$, $d_j > 0$. The category $\mathcal{C}_k$ is therefore a **unitary modular tensor category** : its morphism spaces carry a reference positive-definite Hermitian form.*

The useful modular data :

$$S_{nn'} = \sqrt{\tfrac{2}{k+2}}\,\sin\big(nn'\pi/(k+2)\big), \quad n, n' = 1, \dots, k+1,$$
$$\theta_j = e^{2\pi i\, h_j}, \qquad h_j = \tfrac{j(j+1)}{k+2}, \qquad c = \tfrac{3k}{k+2} = \tfrac{405}{137} \approx 2.96.$$

The matrix S is real, symmetric and *unitary* : it provides the reference inner product of the topological core.

Fusion spaces (conformal blocks)

Definition A.2 (n-strand state space)**.** *To n colored strands $j_1, \dots, j_n$ crossing a spatial slice (S^2 with n punctures, total charge j_0), the TQFT associates the fusion space*

$$V_{j_1\cdots j_n}^{j_0} = \mathrm{Hom}_{\mathcal{C}_k}\big(j_0,\ j_1\otimes\cdots\otimes j_n\big),$$

finite-dimensional (Verlinde), endowed with the positive Hermitian form of proposition A.1.

Proposition A.2 (Topological core)**.** *The topological core*

$$\mathcal{H}^{\mathrm{top}} := \bigoplus_{n\ge1}\ \bigoplus_{j_0, j_1, \dots, j_n} V_{j_1\cdots j_n}^{j_0}$$

is a pre-Hilbert space whose completion is a separable *Hilbert space. For a single strand ($n = 1$, $j_0 = j_1 = j$), $\dim V = 1$: one obtains the orthonormal basis of anyons*

$$\{|j\rangle\}_{j=0}^{k/2}, \qquad \langle j|j'\rangle = \delta_{jj'}, \qquad \dim = k+1 = 136. \tag{A.2}$$

A.1.3 Knots as vectors

A knot K is a *closed* Wilson line of the field χ. In S^3, its closure makes it an *amplitude* (colored Jones $J_n(K;q)$), not a state. The **particle state** appears when the line *crosses* the slice : its internal content is then a vector of the topological core.

Definition A.3 (Vector of a knot)**.** *To a knot K (roping/periphery fixed) one associates*

$$|K\rangle = \sum_{j=0}^{k/2} c_j(K)\ |j\rangle \in \mathcal{H}_1^{\text{top}}, \qquad c_j(K) = \frac{S_{0,2j+1}}{\mathcal{N}_K}\,\frac{J_{2j+1}(K;q)}{J_{2j+1}(\bigcirc;q)}, \tag{A.3}$$

$\mathcal{N}_K$ normalizing $\langle K|K\rangle = 1$. The inner product is $\langle K|K'\rangle = \sum_j \overline{c_j(K)}\, c_j(K')$.

Remark

The map $K \mapsto |K\rangle$ is many-to-one (isotopic knots, or those equivalent by skein relations, give the same vector) and its image is in general **overcomplete** : the catalogue (chap. 6) provides a *frame* with Gram matrix $G_{KK'} = \langle K|K'\rangle$ computable from the colored Jones. The proper orthonormal basis remains that of the anyons (A.2).

Examples from the catalogue : the electron $= 3_1$, the neutrino = free χ wave (no knot), the proton $= 3_1 + 7_2 + \bar{7}_2$ (chap. 14). Simple knots are single-component vectors ; composites are links (§A.1.5).

Gram matrix of the catalogue (low-color projection)

For the six knots whose colored Jones is computable in closed form ($0_1, 3_1, 4_1, 5_1, 7_1, 9_1$; Rosso–Jones and Habiro formulas), one forms the vectors (A.3) and the Gram matrix. At the root of unity, the reduced colored Jones grows with n ; one first computes the low-color projection ($n \le N$), numerically safe in double precision.

(projection $N = 10$, moduli $|G_{KK'}|$)

	0_1	3_1	4_1	5_1	7_1	9_1
0_1	1.000	0.724	0.510	0.597	0.426	0.295
3_1	0.724	1.000	0.854	0.973	0.880	0.722
4_1	0.510	0.854	1.000	0.831	0.796	0.753
5_1	0.597	0.973	0.831	1.000	0.961	0.828
7_1	0.426	0.880	0.796	0.961	1.000	0.942
9_1	0.295	0.722	0.753	0.828	0.942	1.000

Proposition A.3 (Independence, low color)**.** *G is Hermitian, with unit diagonal, **positive definite** (eigenvalues $\{6.0\times10^{-7}, 2.7\times10^{-4}, 0.020, 0.250, 0.987, 4.74\} > 0$). The six vectors are linearly independent : the frame = **Riesz basis** on its 6-dimensional subspace. Script : `hilbert_gram_matrix.py`.*

Complete sum at ζ_{137} : quasi-orthogonality

The complete evaluation at the root of unity ζ_{137} requires cyclotomic arithmetic. The symbolically exact computation over $\mathbb{Z}[\zeta_{137}]$ is intractable (the Rosso–Jones and Habiro products generate exact integers with $\sim 10^{200}+$ digits) ; the rigorous equivalent is the **arbitrary-precision evaluation at** ζ_{137} (mpmath, 400 decimals), *numerically certified* : $[n]_q = \sin(n\pi/137)/\sin(\pi/137)$ to within 10^{-398}, and the Gram matrix is stable to 10^{-96} between dps= 400 and dps= 280.

(*complete* sum over the 136 anyons, moduli $|G_{KK'}|$)

	0_1	3_1	4_1	5_1	7_1	9_1
0_1	1.000	0.000	0.193	0.010	0.076	0.059
3_1	0.000	1.000	0.000	0.000	0.000	0.003
4_1	0.193	0.000	1.000	0.000	0.004	0.014
5_1	0.010	0.000	0.000	1.000	0.000	0.000
7_1	0.076	0.000	0.004	0.000	1.000	0.000
9_1	0.059	0.003	0.014	0.000	0.000	1.000

Proposition A.4 (Quasi-orthogonality over the 136 anyons)**.** *The complete matrix is Hermitian (max$|G-G^\dagger| = 0$) and* ***positive definite*** *with spectrum* $\{0.785, 0.993, 0.999, 1.000, 1.005, 1.218\}$*, condition number* $\kappa(G) \approx 1.55$*. All off-diagonal overlaps* ≤ 0.19 *; the toric family apparently collinear in low color (*$\langle 3_1|5_1\rangle = 0.97$ *at* N=10*) becomes* quasi-orthogonal *(*$\langle 3_1|5_1\rangle = 0.000$*). The catalogue forms a* quasi-orthonormal *family.*

Reading. The quasi-collinearity in low color was merely a lack of "spectroscopic resolution" ; the complete anyonic content reveals that the catalogue knots are *physically distinct* states.

A.1.4 Layer II — the emergent bridge to $L^2(\mathbb{R}^3)$

The topological core carries the *internal* information (anyon, charge, spin), not the spatial profile. The latter emerges from the Schrödinger layer (chap. 10).

Definition A.4 (Emergent spatial factor)**.** *For each knot* K*, the retraction* $\rho(K, E) = c_J(K)^2/(c_J(K) + E/E_0)$ *generates an emergent potential* V_K *on* $\mathbb{R}^3$*, and*

$$\hat{H}_K = -\frac{\hbar^2}{2m_K}\Delta_g + V_K, \qquad g_{ij} = (1 + \kappa|\Psi|^2)\delta_{ij},$$

admits eigenmodes $\Psi_{K,m} \in h := L^2(\mathbb{R}^3, \sqrt{g}\, d^3x)$*. The space* h *is a* separable *Hilbert space.*

Proposition A.5 (One-particle space)**.** *The state space of a single particle is the tensor product*

$$\mathcal{H}_1 = \mathcal{H}_1^{\text{top}} \otimes h \;\cong\; \mathbb{C}^{k+1} \otimes L^2(\mathbb{R}^3),$$

internal topological $\otimes$ spatial — *the exact analogue of* (*spin*) $\otimes\, L^2$*. The internal observables* $(\hat{M}, \hat{Q}, \hat{J}^2)$ *act on the first factor ; the kinetic energy and position on the second.* $\mathcal{H}_1$ *is separable.*

A.1.5 Layer III — the Fock space of links

Composite particles are links of n knots. The total space is a Fock sum over the number of components.

Definition A.5 (Topological Fock space)**.**

$$\mathcal{H}_{\text{TQNT}} = \overline{\bigoplus_{n\geq 0} \mathcal{H}^{(n)}}, \qquad \mathcal{H}^{(0)} = \mathbb{C}\,|0\rangle\,, \quad \mathcal{H}^{(1)} = \mathcal{H}_1, \quad \mathcal{H}^{(n)} = \mathsf{P}_n\big(\mathcal{H}_1^{\otimes n}\big),$$

where $|0\rangle$ *is the topological vacuum and* P_n *the statistics projector. The bar denotes the completion.*

Statistics = braiding. The exchange of two components is a braiding (R-matrix of $\mathcal{C}_k$), with phase tied to the twist $\theta_j = e^{2\pi i\, j(j+1)/(k+2)}$. For well-separated components of integer charge, the anyonic statistics reduces to a sign : P_n is the (anti)symmetrization according to the **braid parity** (P7bis), which reproduces boson vs fermion — and the spin-statistics connection is itself *derived* (the ribbon's 4π + the ribbon braiding–twist axiom, §3.5).

Charge superselection. The charge $Q = \Sigma\chi + \mathrm{Lk} \in \mathbb{Z}$ (chap. 11) commutes with all local observables :

$$\mathcal{H}_{\mathrm{TQNT}} = \bigoplus_{Q\in\mathbb{Z}} \mathcal{H}_Q,$$

with no coherent superposition between sectors (D6). The linking number Lk and the link type refine these sectors (harmonic potential, chap. 13).

A.1.6 Observables and spectral theorem

Definition A.6 (Fundamental operators)**.** *On $\mathcal{H}^{(1)}$, in the knot/anyon basis :*

$$\begin{aligned}
\hat{M}\,|K\rangle &= m(K)\,|K\rangle\,, & m(K) &= \sqrt{\sigma_\chi}\,\mathcal{S}(K) \quad \textit{(reference V6 mass, §9.1)},\\
\hat{Q}\,|K\rangle &= Q(K)\,|K\rangle\,, & Q(K) &= \Sigma\chi(K) + \mathrm{Lk}(K) \in \mathbb{Z},\\
\hat{J}^2\,|K\rangle &= j(j{+}1)\,|K\rangle\,, & j &= j(K).
\end{aligned}$$

On a link $L = K_1 \cup \cdots \cup K_n$, $\hat{M}$ takes the value $m(L) = \sqrt{\sigma_\chi}\,\mathcal{S}(L)$, where $\mathcal{S}(L)$ is the factor of the sector *of L (baryons Regge+Z_3, Goldstone GMOR, heavy stoikhion+Cornell, EW VEV; chap. 9 — the V4 form $\alpha_0\, U_{\mathrm{EM}}\, c_J^{3.55}$ is obsolete), $\hat{Q} = \sum_i \Sigma\chi_i + \mathrm{Lk}$.*

Proposition A.6 (CSCO)**.** *$\hat{M}, \hat{Q}, \hat{J}^2$ (and $\hat{J}_z$, $\hat{\mathrm{Lk}}$) are self-adjoint, with discrete real spectrum, and pairwise commuting : complete set of commuting observables. $\hat{M}$ is bounded below.*

Theorem A.1 (Completeness)**.** *$\mathcal{H}_{\mathrm{TQNT}}$ is a* separable *and* complete *complex Hilbert space. The joint eigenstates of the CSCO $\{\,|K;\Psi_{K,m}\rangle\,\}$ form a total set : their linear span is dense. Every self-adjoint observable admits a spectral decomposition.*

Proof (sketch). $\mathcal{H}_1^{\mathrm{top}} \cong \mathbb{C}^{k+1}$ is finite-dimensional (prop. A.2) ; $h = L^2(\mathbb{R}^3, \sqrt{g}\,d^3x)$ is separable ; so is $\mathcal{H}_1$ (prop. A.5). A countable direct sum $\bigoplus_n \mathcal{H}^{(n)}$ of separable spaces, completed, remains separable. The inner product is positive definite (unitarity, prop. A.1, and L^2 norm). The spectral theorem for the CSCO furnishes the common eigenbasis ; its totality follows from the maximality of the CSCO. □ □

A.1.7 Synthesis and points to complete

Layer	Space	Role
I — topological	$\mathcal{H}^{\mathrm{top}}$ $(\mathrm{SU}(2)_{135})$	anyon, charge, spin, fusion
II — spatial	$h = L^2(\mathbb{R}^3, \sqrt{g}\,d^3x)$	profile Ψ, dynamics
III — Fock	$\bigoplus_n \mathsf{P}_n(\mathcal{H}_1^{\otimes n})$	links, statistics, charge
$\mathcal{H}_{\mathrm{TQNT}} = \bigoplus_{Q\in\mathbb{Z}} \mathcal{H}_Q$, separable and complete (th. A.1)		

Points to complete

1. **Self-adjoint domain with emergent metric.** Δ_g depends on $|\Psi|^2$: the equation is nonlinear ; the L^2 layer described here is its linearization. Establish the essentially self-adjoint domain of $\hat{H}_K$ and the self-consistency ($\Psi \mapsto g \mapsto \Psi$).
2. **Knot frame.** Full rank and quasi-orthogonality established over the 136 anyons (prop. A.4, arbitrary precision certified to 10^{-96}). It remains to formalize the Naimark decomposition (passage frame → explicit orthonormal basis).
3. **Fock truncation.** The sum $\bigoplus_n$ is physically finite (anyonic content bounded by k, crossing number bounded) ; give the precise cutoff and the norm convergence.

4. **Braiding reduction $\to \pm 1$.** Justify quantitatively the passage from anyonic statistics (R-matrix of $\mathcal{C}_k$) to the braid parity P7bis for well-separated integer-charge composites.
5. **Periphery anomaly.** Reconcile the convention $q = e^{2\pi i/(k+2)}$ (colored Jones, this chapter) and $q' = e^{i\pi/(k+2)}$ (quantum group) ; fix the phase of the central charge $c = 3k/(k+2)$.
6. **Anyon $\leftrightarrow$ knot identification.** Specify the map $K \mapsto j(K)$ and the fusion content of each link of the catalogue (consistency with measured spin, charge, mass, chap. 14).

A.2 Table of the 20 Jones polynomials

Knot	**Jones polynomial $V_K(t)$**	**span**	$\lvert \det \rvert$
0_1	1	0	1
3_1	$t + t^3 - t^4$	3	3
4_1	$t^{-2} - t^{-1} + 1 - t + t^2$	4	5
5_1	$t^2 + t^4 - t^5 + t^6 - t^7$	5	5
5_2	$t - t^2 + 2t^3 - t^4 + t^5 - t^6$	5	7
6_1	$t^{-2} - t^{-1} + 2 - 2t + t^2 - t^3 + t^4$	6	9
6_2	$t^{-1} - 1 + 2t - 2t^2 + 2t^3 - 2t^4 + t^5$	6	11
6_3	$-t^{-3} + 2t^{-2} - 2t^{-1} + 3 - 2t + 2t^2 - t^3$	6	13
7_1	$t^3 + t^5 - t^6 + t^7 - t^8 + t^9 - t^{10}$	7	7
7_2	$t - t^2 + 2t^3 - 2t^4 + 2t^5 - t^6 + t^7 - t^8$	7	11
7_3	$t^2 - t^3 + 2t^4 - 2t^5 + 3t^6 - 2t^7 + t^8 - t^9$	7	13
7_4	$t - 2t^2 + 3t^3 - 2t^4 + 3t^5 - 2t^6 + t^7 - t^8$	7	15
7_5	$t^2 - t^3 + 3t^4 - 3t^5 + 3t^6 - 3t^7 + 2t^8 - t^9$	7	17
7_6	$t^{-1} - 2 + 3t - 3t^2 + 4t^3 - 3t^4 + 2t^5 - t^6$	7	19
7_7	$t^{-4} - 2t^{-3} + 3t^{-2} - 4t^{-1} + 4 - 3t + 3t^2 - t^3$	7	21
8_1	$t^{-2} - t^{-1} + 2 - 2t + 2t^2 - 2t^3 + t^4 - t^5 + t^6$	8	13
8_2	$1 - t + 2t^2 - 2t^3 + 3t^4 - 3t^5 + 2t^6 - 2t^7 + t^8$	8	17
8_3	$t^{-4} - t^{-3} + 2t^{-2} - 3t^{-1} + 3 - 3t + 2t^2 - t^3 + t^4$	8	17
9_1	$t^4 + t^6 - t^7 + t^8 - t^9 + t^{10} - t^{11} + t^{12} - t^{13}$	9	9
10_1	$t^{-2} - t^{-1} + 2 - 2t + 2t^2 - 2t^3 + 2t^4 - 2t^5 + t^6 - t^7 + t^8$	10	17

Part XVII

Limitations and perspectives

Part XVIII

Limitations and perspectives

Appendix B

Limitations and open questions

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This chapter honestly lists the current limitations of TQNT and identifies the necessary research directions.

B.1 What is derived

TQNT achievements

1. **Mass spectrum** : 89 $q\bar{q}$/stoikhion states at median 0.13 % ($\sqrt{\sigma_\chi}\mathcal{S}$, engine aligned 21/07) with 10 calibrated parameters + 2 hints (v derived, m_s demoted 22/07 ; 0 adjusted per particle, free assignment).
2. **Schrödinger equation** : derived from Chern-Simons via the emergence chain (CS $\to A_K \to \hat{A}_K \to$ WKB $\to$ Fermi $\to$ Schrödinger).
3. **Spin** : classified by (ν, μ, Q) + excitation ℓ (19/19 knots + 9/9 excited states).
4. **Charge** : $Q = \Sigma\chi + \mathrm{Lk}$, always integer for observable particles.
5. **Nucleon** : $3_1 + 7_2 + \bar{7}_2$, masses to 0.2 %, r_p to 0.2 %, $\Delta m(n{-}p)$ exact.
6. **Proton stability** : topological barrier $\alpha_{\mathrm{EM}}^{18}$ (P15) $\Rightarrow$ *finite* lifetime $\sim 8 \times 10^{33}$ years (no exact baryonic conservation).
7. **"Confinement"** : fractional charges ($\Sigma\chi + \mathrm{Lk}$ non-integer) are inaccessible in an isolated knot (the proton is the link $3_1 + 7_2 + \bar{7}_2$, not quarks).
8. **Dark matter**: isolated simple knots, $c_{\min} = 23$ *derived* (decoupling/survival, §24.4); Vassiliev-suppressed cross-section $\sigma \sim \alpha^{c-18}$; reservoir dominated by the edge $c =$ 23–25 (1.7–1.9 GeV, Migdal target — out of EM reach: $\sigma_{\mathrm{eq}} \approx 1.1 \times 10^{-48}$ cm^2, prediction of *absence*). *Abundance ratio reconstructed at the order of magnitude* (asymmetric formation cost, §26.7.5).
9. **Dark energy** : ρ_Λ predicted to the **order of magnitude** (123 orders resolved ; the old "5.6 % gap" is *dissolved* — within the band of the H_0 tension, below the structural noise : neither a residual to explain, nor fine precision to claim).
10. **Gravitation** : $G = \hbar c / M_{\mathrm{Pl}}^2$ (the weave amplitude, unique equation), read by the nucleon — emergent metric.
11. **Electroweak VEV** : $v = \sqrt{7\pi/6}\,(k{+}2)\,m_{\mathrm{base}}(1{+}\alpha/4\pi)$ *derived* at +28 ppm (open crossing on the anchor, conditional, §22.4.4) ; the electroweak hierarchy v/M_{Pl} follows.
12. **Relativistic kinematics and Dirac** : derived from the spacetime weave and the ribbon frame (conditional S/D/P10, §3.5) : Lorentz = symmetry of $\Box\chi{=}0$, spin-statistics (4π + ribbon category), Dirac forced (antiparticle = mirror) ; the EM sector closes onto QED at derived $\alpha = 1/137$.

B.2 What is postulated (not derived)

1. **Chern-Simons SU(2)** (P10). The gauge group is postulated, not derived from a more fundamental principle.
2. **Strong sector** (former "SU(3)$_C$"). The color SU(3) gauge group is *not* fundamental (antisymmetrization artifact, §22.1) ; its *charge* and its arity $N_c = 3$ are **derived** (pleroma, Fox $\mathbb{Z}_3$). What remains *postulated/exploratory*, and justifies its presence here : the **strong dynamics** (flux tube as a parafermion vortex, non-abelian dressing) and the microscopic realization of the condensate.

3. **The ontology status of "everything is χ"** : relativistic kinematics and Dirac are *derived* from the spacetime weave (conditional S/D/P10, §3.5) — what remains posited is the ontological premise itself, with the general 3D case of second quantization (the solvable instance is done, §3.6) and the non-abelian strong dynamics.

4. **The unique assignment and the baryon number** B (§B.7) : structural underdetermination ; B posited as input (triality lead open).

Removed from this list (resolved or obsolete) : κ_g is **derived** ($96\pi\,\alpha_{\rm EM}^{18}\,\lambda\!\!\!^{-}_C/7$, no longer a free parameter) ; the **truncation to 3 generations** is **derived** by two convergent routes (P11 plasticity bound $q \leq 7$; Z_3 saturation, §22.4.4 — the old argument "$\dim \mathrm{SU}(2) = 3$" was heuristic and is no longer invoked) ; the **screening factor** $f(n)$ (0.86/0.77) belonged to the V4 mass form, *obsolete* as a predictor (the reference formula $m = \sqrt{\sigma_\chi}\,\mathcal{S}$ makes no use of it) ; and the **value of** v, long the second general wall (five routes closed, certified null), is *derived* since 2026-07-21 (open crossing on the anchor, +28 ppm, conditional, §22.4.4); finally **relativistic kinematics** (the old "Lorentz wall") is *derived* from the spacetime weave since 2026-07-21 (conditional S/D/P10, §3.5).

B.3 Open questions

B.3.1 Ab initio derivation of the mass scales

The old question "derive $\alpha_0 = 8.16 \times 10^{-4}$ MeV" is *obsolete* : α_0 belonged to the V4 form ($m = \alpha_0 U_{\rm EM} c_J^{3.55}$), abandoned as a predictor. The equivalent reference targets are : $\sqrt{\sigma_\chi}$ (the single scale of the light sector — to be derived from the χ condensate/deconfinement) and m_ℓ (tube-radius normalization of the static mass $M_{\rm topo} = \sqrt{\sigma_\chi}\,\ell(K)$, not pinned ; the geometric ratio $\ell(5_2)/\ell(3_1)$ once gave m_s — demoted 2026-07-22, ACPR audit — by the ratio $\ell(5_2)/\ell(3_1)$). A closed-form scan with a *self-null* (2026-07-17, `scan_ml_sigma_forms.py`) has ruled out the arithmetic route: the $\sim 1\,\%$ candidates ($m_\ell = 3\Lambda_\chi/2$, $D = 2\pi\alpha\,\hbar c/\sqrt{\sigma_\chi}$, $\sqrt{\sigma_\chi} = 2\pi T_c$) all fall within the expected coincidence density ($\sim$ 1–2 simple forms per percent). And the "tube profile from the solver" route is *structurally settled* (2026-07-17, `derive_ml_tube_profile.py`): the microscopic layer ($V = -\Lambda\rho_J/d^2$) is **scale-invariant** ($E(sx) = E(x)/s^2$ exactly) and **supercritical** ($2m\Lambda\rho/\hbar^2 = 6.6 \gg 1/4$; in the transverse 2D plane, any $1/d^2$ attraction falls onto the tube): the spectrum is set by the UV cutoff (verified with the solver: $E_1 = -2.2 \to -78$ for $\varepsilon = 0.3 \to 0.1$, no limit). m_ℓ **is therefore the dimensional-transmutation constant of the microscopic layer** (the structural analogue of $\Lambda_{\rm QCD}$): not derivable from the dimensionless couplings; the target migrates to the *UV completion* — what physics fixes $D = 0.0214$ fm? The candidate identification that ties the core to the CS level, $D = 2\pi\hbar c/((k{+}2)\sqrt{\sigma_\chi})$ ($m_\ell = 313.6$ MeV, $+1.2\,\%$), has been *bench-tested* (`mecanisme_flux_cs.py`): the energetic variant (tube charged by the CS Gauss law, tension minimisation) goes the *wrong way* ($R \sim \sqrt{k}$, refuted); the Lüscher quantum width is a *log* structure, not a scale ($w \approx 0.32$ fm, refuted as the core); the phase-space cell (quantum $\pi/(k{+}2)$ on the transverse circle of radius λ_σ) is viable in *form*, but the data measure a cell count $N = 2\pi\lambda_\sigma/D = 138.6$ — the integer $k{+}2 = 137$ sits at $-1.2\,\%$ and the neighbours 138/139 fit as well: **no integer resolution** (one integer step $= 0.72\,\%$ on m_ℓ, comparable to its unquantified calibration uncertainty). The identification stays at coincidence level, slightly disfavoured. The "pin m_ℓ" route is *exhausted* (2026-07-17, `pincer_ml_plateau.py`): the error-bar fit of the stoikhion sector (25 states, $m_s = 1.596\,m_\ell$ tied, m_c/m_b co-fitted) gives $m_\ell = 288 \pm 11$ MeV ($\pm 3.7\,\%$ — limited by the *model* error of $2.7\,\%$, not by the data; $m_\ell \leftrightarrow m_c$ degeneracy, corr -0.58): the integer test would need $15\times$ better. *Honest caveat*: the calibrated 310 sits at $+2\sigma$ of the sector's own optimum (which prefers 288, with $m_c \to 1589$): the value of m_ℓ is anchor-dependent at $\sim \pm 4$–$7\,\%$ — to be kept in mind for everything derived from it (m_s, dark-edge masses). What remains is route (ii): deriving the

transverse-circle radius (λ_σ is posited, not derived). Structural curiosity: supercriticality implies *discrete* scale invariance (an Efimov tower, $s_0 = 2.52$, ratio $e^{\pi/s_0} = 3.48$) — an unexplored lead. *Update 2026-07-19* (matching draft, `derive_ml_koide_identification.py`): an *independent* path lands on the same value — the **Koide** identification $m_\ell = \frac{3}{4}\sqrt{\sigma_\chi} = 313.85$ MeV (the trefoil has only *one* mass scale: the stoikhion mass = the derived leptonic scale of the same knot) *passes* the stoikhion-sector elimination test (ΔRMS +0.08 pt, free constraint — the m_s protocol) and is degenerate to 0.12 % with the CS candidate above; jointly, the two conditions predict the ropelength $\ell(3_1) = 411/(8\pi) = 16.353$ (Abraham 16.372, −0.12 %) and the integer resolution $N = 137$ becomes *compatible* (137.16). **Strong candidate, not consolidated** (confrontation with the full light plateau required); if it consolidates, $m_\ell/m_{\text{base}} = 3/(4\,\mathcal{S}_{\text{octet}})$ is derived and the count drops to $8 + 2$.

B.3.2 Neutrino masses : the self-rotation curvature (V6 refinement)

The neutrino is a **wave relic** (P14) of the χ field : a fragment of free wave which, upon detaching from a broken knot, **rotates on itself**. This self-rotation *curves* the wavelet into a **non-closed loop** of radius R — an *intermediate* state between the photon (straight wave, zero curvature, exactly massless) and the knot (closed topology, full mass). The curvature carries a tiny mass :

$$\boxed{m_\nu = \frac{\hbar c}{2R}} \tag{B.1}$$

For $\Sigma m_\nu \approx 66$ meV (i.e. $m_\nu \sim 0.02$–0.05 eV per species), $R \sim 2$–10 μm : the wavelet is *almost* straight (hence almost massless), but not quite — which explains a **non-zero but tiny** mass, with no nodal structure (hence no topological confinement mass, no charge). **Dirac** neutrino (no Majorana mass : $0\nu\beta\beta = 0$ exactly).

Numerical zoom — study of limits. The master relation equivalently reads $R = (\lambda_C/2\pi)/2$ (the self-rotation radius equals half the *reduced* Compton wavelength), whence the full Compton wavelength $\lambda_C = 4\pi R$. The window is fixed *below* by the oscillation mass splittings (guaranteed : $\sqrt{\Delta m^2_{21}} = 8.68$ meV, $\sqrt{\Delta m^2_{31}} = 50.3$ meV) and *above* by DESI ($\Sigma m_\nu < 0.072$ eV, normal ordering favored) :

Regime	m [meV]	R [μm]	$\kappa = 1/R$ [m^{-1}]	λ_C [μm]
photon (degenerate limit)	0	∞	0	∞
lightest ν (NO, $m_1 \to 0 \Rightarrow m_2$)	8.68	11.37	8.8×10^4	142.9
guaranteed atmospheric ν (m_3)	50.28	1.96	5.10×10^5	24.7
heaviest ν under DESI (m_3)	51.03	1.93	5.17×10^5	24.3

Window: mass $0 < m_\nu \lesssim 51$ meV (DESI) [$\lesssim 450$ meV in KATRIN lab bound] ; radius 1.9 μm $\lesssim R < \infty$ (down to 0.22 μm in lab bound) ; curvature $0 < \kappa \lesssim 5.2 \times 10^5$ m^{-1} (up to 4.6×10^6 lab) ; Compton wavelength 24 μm $\lesssim \lambda_C < \infty$.
Physical consistency. The leptonic knot (electron, $m = 0.511$ MeV) has $R_e \approx 193$ fm ; the neutrino's self-rotation loop ($\sim$ 2–11 μm) is $\sim 10^7$ times *more open* $\Rightarrow$ tiny curvature $\Rightarrow$ "almost a free wave, almost massless", but never exactly zero (the oscillations forbid three zero masses). The Kurie plot bends only at $m \gtrsim 50$ keV ($R \lesssim 4 \times 10^{-3}$ μm) : very far from the window, hence compatible.

Status (refinement, not derivation). This curvature mechanism replaces the earlier lead ("topological seesaw", not completed). It is *qualitatively* founded (the neutrino as a self-rotating wavelet, §15.4 and Neutrinos section) and explains why $m_\nu \neq 0$ but tiny ; **but R (hence the exact value of m_ν) is not fixed from first principles** — R is fitted to Σm_ν. *Falsifiable* : $m_\nu \neq 0$ in the DESI window ($[59, 72]$ meV), $0\nu\beta\beta = 0$ (Dirac). *To note* : this self-rotation

structure concerns the neutrino *mass* ; it does *not* provide an invisible channel $\pi^0 \to \nu$ in collision strong enough for the *Muon Puzzle* (non-robust rate, cf. `brouillon_dynamique_collision`).

The motion on the weave and the existence interval (consolidated 2026-07-27, `neutrino_trame_intervalles.py`). The weave reading places the neutrino as the *third* regime of motion: light is a wave that *never entered* a rail ($\mathcal{S} = 0$, a spectator); the knot is an *entered*, locked wave (deficit = mass); the neutrino is an open self-rotating wave that **grazes the weave without entering it**. Two hitherto-posited facts follow: $Q = 0$ (never entered a rail ⇒ no winding ⇒ no charge; the neutrality is *structural*) and **weak interactions only** (never magnetised ⇒ neither strong nor EM; only the P15 reconnection remains). *Existence interval*: the **upper** bound is the *knotting threshold* — as soon as the curvature mass reaches that of the smallest knot, the trefoil $m_\ell = \frac{3}{4}\sqrt{\sigma_\chi} = 313.85$ MeV ($R_{\min} = \hbar c/2m_\ell = 0.314$ fm), the fragment *can* knot and leaves the regime; the **lower** bound is the photon limit ($R \to \infty$), the flat weave imposing *no* floor (set by the oscillations, $m_2 \geq 8.68$, $m_3 \geq 50.3$ meV). Hence

$$0 < m_\nu \leq m_\ell = 314 \text{ MeV} \quad \Longleftrightarrow \quad 0.314 \text{ fm} \leq R < \infty,$$

and the observed neutrino lives $\sim 6 \times 10^9$ *below* the threshold: the most open state just above the photon. *Nuances*: the knotting threshold is an energetic *possibility* — the *effective* nucleation also requires the Z_3 gate, never crossed by a neutral object (both converge: never knotted); only the upper bound is derived, the lower one remains observational; R is still fitted to Σm_ν (the *refinement* status is unchanged).

SN1987A check (consolidated 2026-07-28, `neutrino_sn1987a_check.py`). *Point of rigour* : SN1987A does not measure an "absolute delay" — the ν's *preceded* the optical light by ∼3 h for a reason unrelated to mass (neutrinos escape the core at collapse; light is only emitted when the shock reaches the surface, *shock breakout*) : an *emission* offset, not a *propagation* one. The observable constraining the mass is the **energy-dependent spread** of the burst, $\Delta t(E) = (L/c)\,\frac{1}{2}\,(m/E)^2$. For $m = 50.3$ meV and $L = 51.4$ kpc : absolute delay 0.12 ms at 7.5 MeV; spread over the detected range (7.5 → 40 MeV) $\Delta t = 1.1 \times 10^{-4}$ s — $\sim 10^5$ times smaller than the burst duration (∼13 s, intrinsic) : **invisible, hence compatible**. The calculation recovers the historical bound (spread = burst for $m \sim 17$ eV; refined analyses : 5.7 eV, Loredo–Lamb 2002) : the prediction (0.050 eV) is ∼110× *below* SN1987A and ∼9× below KATRIN — SN1987A *confirms* compatibility but does not *test* the value (fine test : DESI/Planck, KATRIN/Project 8). *Kinematic lock* : a "superluminal" reading of the 3 h lead ($(v-c)/c \approx 2 \times 10^{-9}$) is **triply excluded** — by the mass mechanism itself ($m = \hbar c/2R > 0 \Rightarrow v < c$ strictly; superluminal ⇔ massless ⇔ *no oscillation*, yet they are measured), by the weave (c = the meshing = the medium's speed; a weakly coupled χ wave is *transparent*, not fast — and the ν, with $R \sim \mu\text{m} \sim 10^{30}\ell_P$, sits on a multitude of teeth like any particle), and by the data ($|v-c|/c < 10^{-9}$).

Why self-rotation is necessary (and not optional)

One might think self-rotation decorative — a parameter to remove to "simplify". It is not : without it, the model is not *less precise*, it is **refuted**.

No rotation ⇒ no curvature ⇒ $R = \infty \Rightarrow m_\nu = 0$ *exactly* ⇒ straight wave ⇒ photon (spin 1). But the oscillations impose $\Delta m^2 \neq 0$ (two massive states : $\sqrt{\Delta m^2_{21}} = 8.68$ meV, $\sqrt{\Delta m^2_{31}} = 50.3$ meV). A zero-mass neutrino gives $\Sigma m_\nu = 0$: **excluded**.

Self-rotation does a **triple job**, not a decoration :

1. **Mass** — the curvature carries $m_\nu = \hbar c/2R$; without it $m = 0$.
2. **Spin** $\frac{1}{2}$ — the angular momentum of the wavelet rotating on itself *is* the spin ; a

straight wave is spin 1 (wrong quantum number).

3. **Stability** — the rotation stabilizes an *intermediate open* state between the photon ($R = \infty$) and the knot ($R \sim$fm, closed, massive lepton). A *static* loop is not a stable point : it would close into a knot or unwind into a photon. Without rotation, there is no "neutrino" configuration.

Bonus consistency. In the crossing heuristic $\sigma \sim \sigma_{\text{geom}}(r_N/R)^2$, $R \to \infty$ gives $\sigma \to 0$: a non-rotating neutrino would be *undetectable*. We detect them (Super-K, SNO, IceCube) $\Rightarrow$ the curvature is also what makes an interaction possible.

Honest nuance. Self-rotation does not make the numbers *more precise* (R remains fitted to Σm_ν) ; its added value is to make the model *viable* and to fix Dirac / $0\nu\beta\beta = 0$. But between "rotation" and "no rotation", there is no choice : the latter is dead ($m = 0$, spin 1, undetectable, unstable).

B.3.3 Crossing matter : topological bounce versus reconnection

In TQNT, a neutrino "collision" is *not* a mechanical shock : the wavelet has neither charge nor knot topology. At each encounter with a knot (baryon), the result is a **binary choice** :

- **topological reconnection** (P15) : the only detectable "weak event" (capture, scattering), which changes the linking — *rare* ;
- **bounce** : the wavelet exits intact, no change of linking — *almost systematic*.

The numbers below take as *anchor* the **measured** cross sections σ_ν (we do not derive G_F from a W : the weak force is a reconnection, cf. §15.3).

Mean free path $\lambda = 1/(n\,\sigma_\nu)$. With $n = \rho N_A$ nucleons/cm^3 :

Medium	ν	λ
water	solar ES ($\sim$10 MeV, $\sigma \sim 10^{-44}$)	176 light-years
rock	reactor IBD ($\sim$4 MeV, $\sigma \sim 10^{-43}$)	6.5 light-years
lead	reactor IBD ($\sigma \sim 10^{-43}$)	1.6 light-year

(the classic "$\sim$1 light-year of lead to stop half of them" is recovered).

Interaction probability when crossing a body ($P \approx n\sigma L$). Earth (diameter) : $P \approx 4 \times 10^{-11}$ (solar), i.e. **1 in** 2.4×10^{10} ; Sun (diameter) : $P \approx 10^{-9}$ (whence we *see* solar neutrinos) ; 10 m water detector : $P \approx 6 \times 10^{-18}$ (whence the kilotonne targets).

Bounce/reconnection decomposition. The geometric cross section of the nucleon core ($r_N \approx 0.84$ fm) is $\sigma_{\text{geom}} = \pi r_N^2 = 2.2 \times 10^{-26}$ cm^2. The *reconnection probability per encountered core* is therefore

$$P_{\text{reco}} = \frac{\sigma_\nu}{\sigma_{\text{geom}}} \approx \begin{cases} 4.5 \times 10^{-19} & (\text{solar } \nu) \\ 4.5 \times 10^{-18} & (\sim 10 \text{ MeV}) \\ 3 \times 10^{-13} & (\sim 1 \text{ GeV}). \end{cases} \tag{B.2}$$

Crossing the Earth, the wavelet "sweeps" $N = n\,\sigma_{\text{geom}} L \approx 9.4 \times 10^7$ nucleon cores : it **bounces off $\sim 10^8$ knots and reconnects with almost none** ($N \times P_{\text{reco}} \approx 4 \times 10^{-11}$). This is the topological image of the "ghost".

Why is $P_{\rm reco}$ so small ? (heuristic). The wavelet is spread over $R \sim \mu$m whereas the target knot is ∼fm : the overlap of the "reconnectable core" with the target is diluted by a factor $(r_N/R)^2$. Numerically, for the heaviest ν ($R = 1.9$ μm), $(r_N/R)^2 \approx 1.9 \times 10^{-19}$, whence $\sigma_{\rm pred} = \sigma_{\rm geom}(r_N/R)^2 \approx 4 \times 10^{-45}$ cm^2 — of the right *order* as the measured solar σ ($\sim 10^{-44}$, to within a factor ~ 2). **Honest** : this static overlap captures the *low-energy magnitude* but does *not reproduce* the rise $\sigma \propto E$ at the GeV (carried by the phase space of the P15 reconnection, not derived here). It is an image, not a derivation.

Validation against detectors. The adopted σ_ν agree with the standard formulas at the decade level : ES $\nu_e e$ at 10 MeV $= 9.2 \times 10^{-45}$ cm^2, IBD at 4 MeV $= 6.9 \times 10^{-43}$ cm^2, DIS at 1 GeV $= 0.68 \times 10^{-38}$ cm^2/nucleon. *Absolute rate* : for the solar ^{8}B flux (5.25×10^6 cm^{-2}s^{-1}) on the 7.5×10^{33} electrons of the Super-Kamiokande fiducial volume (22.5 kt), these σ give ~ 7 events/day, of the same order as the $\sim$ 15–25/day *measured* (ES, > 3.5 MeV). The transparency of the Earth (1 ν in $\sim 10^{10}$) and its *opacity* beyond ~ 40 TeV — observed by IceCube (2017) — are reproduced. The agreement is *order-of-magnitude* : the measured σ are an *input*, TQNT provides the *image* of them (bounce vs reconnection), not the fine value.

B.3.4 Neutrino transport in matter : attenuation, cascade, bounce potential

The crossing of a dense volume (Earth, star) is described by a **transport equation** (Boltzmann) for the flux $\Phi(E, x)$ along the chord x :

$$\frac{\partial \Phi(E,x)}{\partial x} = \underbrace{-\,n\,\sigma_{\rm tot}(E)\,\Phi(E,x)}_{\text{attenuation (reconnections)}} + \underbrace{n \int_E^\infty \frac{d\sigma}{dE}(E' \to E)\,\Phi(E',x)\,dE'}_{\text{re-injection at lower energy (cascade)}} \tag{B.3}$$

The first term removes the wavelets that *reconnect* ; the second re-injects them at lower energy (ν_τ regeneration) : the flux exits **spectrally softened**, the energy flowing *down*. "Input ≠ output" is therefore *exact*, but quantified.

Attenuation. Dominant term alone (mono-energetic) : $\Phi_{\rm out}/\Phi_{\rm in} = e^{-\tau}$, optical depth $\tau(E) = N_{\rm col}\,\sigma(E)$ with the Earth column $N_{\rm col} = \rho D N_A = 4.2 \times 10^{33}$ nucleons/cm^2 (diameter) :

E	$\Phi_{\rm out}/\Phi_{\rm in}$	absorbed
1 MeV (solar)	$1 - 3 \times 10^{-8}$	~ 0
1 TeV	0.97	2.9 %
10 TeV	0.74	26 %
100 TeV	0.05	95 %
1 PeV	10^{-13}	100 %

Opacity ($\tau = 1$) at $E \approx 34$ TeV : the Earth's absorption *measured by IceCube* (2017). At low energy (solar), input = output to within 10^{-8}.

Bounce potential (MSW). The $\sim 10^8$ coherent *bounces* (§B.3.3) sum their phase into a matter potential :

$$V = \sqrt{2}\,G_F\,n_e \;\Rightarrow\; \Delta M^2_{\rm mat} = 2EV, \tag{B.4}$$

i.e. $V \approx 1.1 \times 10^{-13}$ eV (mantle) to 5.0×10^{-13} eV (core). This is the **MSW** effect *measured* (solar neutrinos). TQNT reading : each bounce imprints a tiny phase ; their coherent sum shifts the *effective* mass **without exchanging energy** — a dephasing, not a loss. *Specific validation* : with this potential, the resonance $E_{\rm res} = \Delta m^2_{21} \cos 2\theta_{12}/(2\sqrt{2}\,G_F n_e) \approx 2$ MeV in the solar core ; the ^{8}B ($\gtrsim 6$ MeV) are *above* (strong suppression, $P_{ee} \approx 0.32$) and the *pp* (~ 0.4 MeV) *below* (vacuum regime, $P_{ee} \approx 0.55$) — exactly the energy-dependent transition *measured* by SNO and Borexino.

No continuous loss. The self-rotation is at the Compton frequency $\omega = mc^2/\hbar \approx 7.6 \times 10^{13}$ rad/s : it is the *rest* energy, a Lorentz invariant, *not* drained by translation. Without charge, no ionization dE/dx : the only "loss" is the *discrete* reconnection (rare), already counted in σ_{tot}.

What transport does not allow : creating heavier particles

It is tempting to imagine that a neutrino gets entangled with a "loosened" link at the center of the Earth to produce a denser particle. **Energy conservation forbids it** for usual neutrinos :

- producing anything requires $E_\nu \gtrsim 0.1$ GeV (threshold $\nu_\mu n \to \mu^- p \approx 0.11$ GeV) ; a solar ν (~ 1 MeV $= 10^{-3}$ GeV) is far below ;
- the center of the Earth is *cold* in particle language : $T \sim 6000$ K≈ 0.5 eV, very far from the ~ 160 MeV of deconfinement — the knots are *not* loosened there ;
- even at high energy (TeV), the inelastic collision produces *lighter* secondaries (the energy goes down, cf. the cascade term of (B.3)), never "denser" than E_ν allows.

Rigorously surviving from this intuition : the **attenuation** $e^{-\tau}$, the **cascade** spectrally downward, and the **MSW potential** as the sum of the bounces. The creation of heavy matter, no.

B.3.5 CKM and PMNS matrices

V6 update. The Cabibbo angles receive a *topological image* (reconnection distance cost), *not* a fundamental derivation (the CKM remains an SM artifact, Rule 0 ; $\lambda_{\text{Cab}} = \sqrt{m_d/m_s}$ uses SM masses) : $|V| = \lambda_{\text{Cab}}^D$ with $\lambda_{\text{Cab}} = \sqrt{m_d/m_s} = 0.224$ ($V_{ud} = \sqrt{1-\lambda_{\text{Cab}}^2}$, $V_{us} = \lambda_{\text{Cab}}$; the Wolfenstein hierarchy $V_{us} \sim \lambda_{\text{Cab}}$, $V_{cb} \sim \lambda_{\text{Cab}}^2$ is the hierarchy of re-knotting distances, §15.3). The **CP violation** is attributed to the *chirality* of the knots (amphichiral CP-symmetric, chiral CP-violating), predicting a leptonic CP violation (PMNS) testable at DUNE. *Still open* : the Wolfenstein coefficient A and the exact magnitude of the phase $\delta_{CP} \approx 65°$.

B.3.6 Relativistic covariance: derived from the spacetime weave (2026-07-21)

The scalar relativistic kinematics, the spin-statistics connection and the Dirac equation are *derived* from the spacetime weave and the ribbon frame (conditional S/D/P10, §3.5): the Lorentz group is the symmetry of $\Box\chi = 0$; the doublet comes from the ribbon's 4π; Dirac is the forced minimal representation (the antiparticle = the mirror); the emergent Schrödinger equation is the *slow limit* of that Dirac equation (Pauli reduction) — the solver is the rest-frame effective theory of a covariant-by-inheritance framework. *Remain*: the full 3D Polyakov computation and the multi-knot operator (second quantization is advanced: germ, crossing and path sum done, §3.6), and the non-abelian strong dynamics.

B.3.7 Strong resonances

The broad strong resonances (ρ, Δ, Σ^*, Ξ^*) are reproduced by the linking tension $\beta_\otimes \times N_{\text{stab}}$ (global TQNT parameter, not adjusted to lifetimes ; §15.2) to 0.56 decade (order of magnitude) ; the narrow states ($J/\psi, \Upsilon, \phi$ by OZI ; η' by the $U(1)_A$ anomaly) remain at 2–3 decades, suppressed by non-topological selection rules.

B.3.8 Dark energy : the "residual gap" is dissolved

The topological cutoff + $\alpha_{\kappa_g}^{3/4}$ suppression + central charge c_{grav} give ρ_Λ to the right order of magnitude (123 orders resolved). The "5.6 % gap", long treated as a residual to explain, is **dissolved** (2026-07-14, double verdict) : *(i)* the P22 resorption mechanism is *null* (computation without a free parameter, WKB form) ; *(ii)* the gap itself was never significant (upstream audit : the prediction falls *within* the band of the H_0 tension, +5.9 % Planck / −9.9 % SH0ES, below the structural noise $\gtrsim 15\,\%$). There is *neither* a residual to explain, *nor* fine precision to claim : ρ_Λ is an *order-of-magnitude* success.

B.3.9 The exponent $p = 3.55$: derived in V5 (Chern–Simons values)

V5 status. This limitation was formulated for the *historical V4 form* $m = \alpha_0\, U_{\text{EM}}\, c_J^{3.55}$, where the exponent $p = 3.55$ was numerically adjusted on 46 particles (Gaussian variational estimate : $p \approx 3$–5). In V5, the operator Ω_{ts} was *derived axiomatically* (archived derivation, seven axioms) and the value 3.55 appeared there as the effective exponent of the saturation factors. The operator being **abandoned** as a predictor, the question of $p = 3.55$ is closed with it ; what remains to be clarified is the fine physical interpretation of the two saturating norms (open point, archived derivation).

B.4 Testable predictions

Falsifiable predictions of TQNT

1. ***Finite* proton lifetime** : $\tau_p \approx 8 \times 10^{33}$ years (barrier $\alpha_{\mathrm{EM}}^{18}$, *not* ∞ ; baryon number is not an exact topological charge). Near the Super-K limit (1.6×10^{34} years) : Hyper-Kamiokande and DUNE will test this prediction.

2. **Pion structure** : the pion is a $K + \bar{K}$ construction (knot + mirror, GOLDSTONE sector, GMOR mass) — *not* a single core knot. (The old reading "π^0 and $\pi^\pm$ = same knot 8_1 with self-linkings $|w| = 0/1$" was a V4 sketch, **obsolete**.) EIC discriminant to be reformulated on the mirror construction (form factors of the $K\bar{K}$ dipole).

3. **Non-WIMP dark matter** : the candidates are complex isolated knots ($c \geq 23$, derived), with Vassiliev-suppressed cross section $\sigma \sim \alpha^{c-18}$ (*effectively* zero ; the reservoir is dominated by the *edge* $c = 23$–25 — the old "invisible branch" concentrated at $c_{\max}$, $\sigma \sim 10^{-183}$ cm^2, is *obsolete*, §26.7.5 ; at the edge: $\sigma_{\mathrm{eq}} \approx 1.1 \times 10^{-48}$ cm^2, ∼7 dex below the frontier). Direct detection experiments (LZ, XENONnT, and the Migdal analyses) should see **nothing**.

4. **Nucleon** $3_1 + 7_2 + \bar{7}_2$: the proton radius $r_p = 0.841$ fm and the $\Delta m(n{-}p) = 1.293$ MeV are fixed predictions (no adjustable parameter).

5. **Ratio** $\Delta m(\Sigma^- {-} \Sigma^+)/\Delta m(\Sigma^0 {-} \Sigma^+) = 4$: prediction of the harmonic linking potential (SM : 2.5).

6. **Absence of a 4th generation** : *derived* (P11 bound $q \leq 7 + Z_3$ saturation, §22.4.4) ; $9_1 = T(2, 9)$ is a generation marker / link component (it appears in the reference Z), not the carrier of a 4th family. Any confirmed 4th species (fermion or neutrino, at any mass) would refute.

7. **Predicted unobserved states** : the configurations without SM counterpart in the main document are the self-conjugate EW-neutral states ($4_1{+}8_3{+}8_3$ at 1.75 GeV reference, $6_3{+}8_3{+}8_3$ at 1.89 GeV — ACPR ropelengths 2026-07-22 — the old Ω_{ts} anchors 16.0/26.5 GeV are obsolete) and the 6 strict dark links (1.07–1.64 GeV, *rederived* 2026-07-15, ACPR 22/07, under the geometric law — the Ω_{ts} values 1.55–51.8 GeV are obsolete). (The old light bestiary "15, 60, 176, 636 MeV" pertained to the per-knot FEM masses, obsolete ; the over-generation is resolved by stability : 617/617 links of the desert [12–80 GeV] unstable — the *desert theorem*, §D.1.4.)

8. **Tensor/scalar ratio** $r \approx 0.004$: predicted by topological inflation ($k = 135$). Testable by CMB-S4 and LiteBIRD (sensitivity $\sigma_r \sim 10^{-3}$).

9. **No dark-matter** γ **signal** : TQNT predicts $\sigma = 0$ **and** no degradation photon (trapped in black holes). Any detection of a DM annihilation γ signal would falsify TQNT.

B.5 Synthetic comparison with the Standard Model

Criterion	Standard Model	TQNT
Free parameters	19	10 global ; 0 adjusted per particle (free assignment)
Particle mass	fitted	**predicted** ($< 5\,\%$)
Proton mass	lattice QCD	$0.0\,\%$ analytic ($\sqrt{\sigma_\chi}$ recalibrated)
$\Delta m(n{-}p)$	lattice QCD (~ 1 MeV)	**1.293 MeV exact**
Proton radius	measured	0.841 fm ($0.0\,\%$)
Confinement	not demonstrated	**topological**
Gravitation	GR (separate)	emergent (Ω^2)
Dark energy	Λ fitted	ρ_Λ predicted (order of magnitude, 123 orders)
Dark matter	WIMP (not found)	isolated knots $c \geq 23$ (derived)
Number of generations	postulated (3)	**derived** (P11 + Z_3 saturation)
Proton stability	$\tau_p \sim 10^{34}$ years (GUT)	$\tau_p \approx 8 \times 10^{33}$ years (α^{18}, finite)
Hierarchy m_e/m_t	unexplained	$m = \sqrt{\sigma_\chi}\,\mathcal{S}$ (sectors : Koide, Regge, VEV)

B.6 Confrontation with real data and honest audits (V6)

Three collider channels, three consistent nulls. On the real ATLAS data ($H \to \gamma\gamma$, 76 562 diphotons ; $ZZ \to 4\ell$, 16 events at 125 GeV = SM) and CMS (dimuon spectrum, 18 296 pairs : only J/ψ, Υ, Z stand out), **no positive signature of a TQNT particle distinct from the Standard Model**. The Higgs companion predicted by the multiplet interpretation is *below resolution* ($0.46 \ll 1.7$ GeV) and excluded in $\gamma\gamma$ by $\mu(\gamma\gamma) \approx 1$. TQNT is *observationally indistinguishable* from the SM in these regimes — by construction : its additional states are neutral (DM), degenerate (companions), or suppressed (Vassiliev).

Positive verifications (2026). The ledger also has its dated positive side: Ω_{cc} *observed* by LHCb ($\sim$3 727 MeV, *inside* the predicted bracket [3 715, 3 788]; declared chronology); $\Sigma_b(6097) = j1$ at $+0.02\,\%$; $B_J(5970)$ *identified* as the bottom 1D top ($+0.02\,\%$) and $B_{sJ}(6064)$ as the strange 1D top; the strange baryonic shift confirmed in sign and order; the forty years of H-dibaryon nulls *postdicted*; and the minimal entry door (a pair, Breit-Wheeler 1.022 MeV) *observed* (STAR 2021); and the 2024–2026 PDG blind test *re-run under frozen sector rules* (2026-07-23, `confrontation_blind_pdg_2026.py`): of 22 recent states (CMS/LHCb/Belle), **11 predicted with no per-state choice** (standard PDG molecular contents, derived bands, stoikhions ± 10 MeV) at a **median of** $0.35\,\%$ (max $1.26\,\%$; T_{cc} goes from $-3.0\,\%$ under free Ω_{ts} assignment to $+0.03\,\%$ at the DD^* threshold), 4 compatible, 5 *declared out-of-scope* (all-charm/$J/\psi\phi$ compression), 1 open — the June agreement ($0.28\,\%$) was reachable by $\sim 10^4$–10^5 equivalent assignments; this one has *nothing to scan.* Same date, two *absence* confrontations on real data: **direct detection** (`confrontation_direct_detection_2026.py`) — the reservoir window (1.07–2 GeV) sits below the mass thresholds of LZ 2024/XENONnT/PandaX-4T *and* $\sigma_{\rm eq} = 1.1 \times 10^{-48}$ lies below the absolute minimum of each curve; at the edge, DS-50 leaves 6.5–6.8 dex of margin (robust for DS-20k/XLZD); the June report's 44-decade spread is *closed from both sides* (ansätze A/B excluded by the data at $+10/+18$ dex, C ruled out by mechanism, the derived value inside the surviving window); and **AMS-02** (`confrontation_ams02_2026.py`) — zero reservoir contribution by two independent walls: kinematic (the positron excess lives at 8–800 GeV, a $M \leq 1.9$ GeV pair injects at $E \leq 1.9$ GeV — incapable whatever σv) and suppression ($\alpha^{2(c-18)} = \alpha^{10}$, $\sigma v \sim 10^{-47}$ cm^3/s, $\sim$20 dex below the low-bin $e^+/\bar{p}$ bounds); TQNT evades the classic "light thermal WIMP"

constraint *by construction* (abundance from formation cost, not from an annihilation freeze-out). None of these is a TQNT-only discovery; they are verified post/predictions in the sense of the assignment null test (§B.7).

Anomaly portfolio (2026). Twenty-eight *genuinely unexplained* observations confronted with no tuning (2026-07-23, `confrontation_anomalies_2026.py` + `calcul_gce_tqnt.py` + `audit_m3_planck*.py`, portfolio draft): the TQNT spectrum being a *theorem*, seven "new physics" anomalies receive a **falsifiable absence** prediction — bottle/beam τ_n (required $\mathrm{BR_{inv}}$ 1.2 % vs *exactly* 0: no dark state below m_n, the lightest sits at 1.07 GeV), X17/ATOMKI (nothing between m_e and m_π; MEG II 2024 null — *supported*), the Galactic-centre GeV excess (*double* exclusion: required σv 2.7×10^{-28} vs 10^{-47}, 19 dex, *and* the kinematic wall $E_\gamma < 1.9$ GeV vs a 2–3 GeV peak → pulsars; the only TQNT window there = the 6 fine lines), AMS anti-helium (a single confirmed cosmic anti-He^4 would refute nucleation baryogenesis), EDGES 21 cm (required σ 10^{-21} vs σ_{eq}, 27 dex; SARAS 3 — *supported*), the MiniBooNE sterile ($\mathbb{Z}_3$ saturation; MicroBooNE — *supported*), evolving dark energy (constant $w = -1$, *promoted to a theorem* 2026-07-24, `deduit_trio_cosmo.py`: the vacuum residue neither *dilutes* — it is the weave's own zero-point deficit, not matter — nor ever *withdraws*; by the river theorem its push is constant: $dw/dz = 0$ *by construction* — the DESI $w_0 w_a$ preference must regress); plus one *conditional* absence (cosmic birefringence $\beta = 0$, spectator light + no axion; LiteBIRD test). One **executed internal audit**: the June Planck "m=3 signal" (r∼2.3, "robust") survives the axis/mask/frequency tests (frequency *completed* 24/07: 100/143/217 GHz at $r = 2.55/2.25/2.54$, flat) but *is not an* m=3 — the controls show $m = 1, 2, 4$ *all* elevated ($p = 0$, m=1 above m=3) and the $\mathbb{Z}_3$ harmonic m=6 at only 1.23: generic low-m anisotropy of diffuse residuals, *reproduced at 100 GHz* (m=1: 2.67, m=3: 2.70, $m \geq 5$ normal; a selection artefact; anisotropic noise ruled out, II_cov); *no* $\mathbb{Z}_3$ signature, consistent with the framework's prediction (sub-horizon domains, zero imprint). The extension (24/07) adds **six anomalies explained in the strong sense** — the framework's mechanism makes the observed fact *expected*: the global eV-sterile tension ($\mathbb{Z}_3$ saturation: the lab will never converge, cosmology is right), lepton universality in $b \to s\ell\ell$ ($e/\mu/\tau = \mathbb{Z}_3$ modes of the *same* trefoil, $g_{\mathrm{topo}} = 1$ → $R_K = R_{K^*} = 1$ exactly; LHCb 2022 — *supported*), the muon g-2 ($a = \alpha/2\pi$, no new contribution → SM convergence; 2025 lattice — *supported*), KATRIN ($\Sigma m_\nu \approx 66$ meV → never a signal), the proton radius (0.841 fm canonical = the muonic value of the converging CODATA) and the massive JWST galaxies (P22 boost *null*: no new physics, systematics — V5.18 audit; *promoted to a corollary* 2026-07-24: mechanized growth + Birkhoff forbid any collapse shortcut — a $z > 10$ population beyond ΛCDM would refute the growth theorem, no longer just an audit); *five further absences* (gallium/BEST, DAMA/LIBRA — ANAIS/COSINE *supported* —, ANITA, large-angle CMB anomalies — *structural isotropy* 2026-07-24: the push is density-sourced, no preferred axis possible; new prediction: *non-replication* of the anomalies in independent probes of the same sky (LiteBIRD E-modes, lensing reconstruction, 21 cm; any significant cross-replication would refute) —, plus the reactor anomaly *resorbed as predicted*); and three declarations: the W mass *without content* (±0.4 % precision, assignment null test not passed: too loose to arbitrate CDF), FRB/GRB out of scope; Li-7 received and then *lost* its candidate mechanism the same day (excited ashes of the freeze-out, 26.8.2: refuted at the efficiency wall — 3×10^{-13} per photon computed, $\times 10^4$ deficit; the structural chain survives, the destruction falls short). Finally four *inherited and declared* mysteries: H_0, S_8 ($\sigma_8 = \sigma_8^{\Lambda\mathrm{CDM}}$ exactly), Li-7, the air-shower muon puzzle — where the framework has no mechanism, it says so.

Earlier empirical audits invalidated. No longer to be presented as successes : the JWST P22 boost (null at 81 decades), NICER M–R (non-discriminating, $\Delta R/R \sim 10^{-9}$), NANOGrav 15-yr (SMBHB preferred, $\ln B = -1.03$). Three honestly assumed falsifications : F1 (pure

inflation n_s, closed by Starobinsky), F2 (reheating; *closed* 2026-07-17, the V5.17 route carried through by insensitivity), F3 (Class E γ lines excluded by Fermi-LAT at the Ω_{ts} energies ; **LIFTED 2026-07-15**, re-run 2026-07-22) : the re-confrontation with the reference masses (`f3_line_search_référence.py`, then `f3_lines_acpr_no_smear.py` with ACPR ropelengths) shows the strict states all fall to 1.07–1.64 GeV, where the **six** fine lines are ×8–13 *below* the limits at the derived $\Delta c = 22$ ($dc_{\rm crit} = 21.5$–21.6 : the "$\Delta c \geq 30$ required" was an artifact of the Ω_{ts} energies and building blocks ; the light reference building blocks make the pearls less relativistic, -9 decades of rate). *Historical* : the 14/07 softening via the "8_3 writhe doublet (± 7.2)" is **obsolete** — at the true ideal (katlas base, ACPR-verified), $\mathrm{Wr}(8_3) = 0.003$: the doublet was an artefact of non-converged curves ; no smearing, and the lifting does *not need* it. Residual prediction : six fine lines at 1.07–1.64 GeV ~ 1 dex below the 2015 limits). The F4 candidate (resorption of the 5.6 % of dark energy by P22) has *dissolved* : the mechanism is null (computation without a free parameter, WKB form) *and* the gap was not significant (upstream audit : band of the H_0 tension, structural noise ; §25.7) — neither mechanism, nor problem.

Distinctive falsifiable tests (2026–2032). The only distinctive levers are *future* or *negative* : $0\nu\beta\beta = 0$ exactly (Dirac neutrinos ; KamLAND-Zen sees nothing, nEXO/LEGEND will decide), $\Sigma m_\nu \approx 66$ meV (in the DESI window $[59, 72]$ meV ; DESI-Y5/CMB-S4 will decide), leptonic CP violation (DUNE), bipartition of dark matter (DM-117). *On the LHCb side, dated*: H_{cc}^{++} at $4\,497 \pm 10$ MeV (30 μm displaced vertex, $0^+/1^+$ doublet of 75 MeV), $\Xi_{cc}^* = 3\,659$ (vs $\sim$3 690 QCD), B_0^*/B_1' centroid 5 665 (vs $\sim$5 700–5 750 models), $\Omega_b(1P) = 6\,297$–$6\,304$, $\Omega_{ccc} = 4\,782$ strict, $J^P(5970) = 3^-$; *on the astroparticle side*: no direct dark-matter detection at 1.7–2 GeV (DS-20k/XLZD — any detection would refute), six fine lines 1.07–1.64 GeV (Fermi-LAT 15+ yr), r (LiteBIRD 2032). **Clear position** : falsifiable, monitored, not falsified, with the Koide relation ($Q = 2/3$, 9.2 ppm) reinterpreted topologically ($r = \sqrt{2}$: *identified* with $d_{1/2}(k{=}2)$, the ribbon's quantum dimension — the same $\sqrt{2}$ as the ladders' excess — yet still a *hint*, not derived end to end). This is the honest assessment : TQNT *maps* the Standard Model topologically (structures and orders of magnitude), the fine values remaining calibrated or open.

Collision dynamics and Muon Puzzle (V6, honest failure). TQNT has been endowed with a *collision dynamics* (it had none) : destruction (deconfinement) → recomposition (P22 nucleation), followed by the χ wavelength ; the hadronization scale is the *confinement energy* ($\sim$160 MeV, *not* the mass — the mass $\sim c_J^{3.55}$ would give an absurd Hagedorn of 14 GeV), and thermal hadronization reproduces $K/\pi \simeq 0.14$. Applied to the *Muon Puzzle* (muon excess of cosmic showers, $\sim 8\sigma$), it produces a mechanism with a *measured* coupling (stimulated Bose recombination of the bosonic χ field, bias toward the charged/muonic channel = pion phase-space density $\langle f \rangle \sim 0.25$, HBT), *without a free parameter*. **But it fails quantitatively** : confronted with the *models* (the correct reference of Auger's z, and not the isospin 1/3 that the models already beat), $f_{\rm EM}^{\rm TQNT} \sim 0.29 \approx$ models ⇒ *no net excess* (to match Auger $z \simeq 1.3$ would require $\langle f \rangle \sim 0.53$, $\sim 2\times$ the HBT value). And the mechanism **is not new** : the topological charge protection and the thermalization $\gamma_s{=}1$ are *identical* to the ρ^0 effect and to the "strange fireball" proposal (Anchordoqui et al. 2016) *already present* in the models (SIBYLL 2.3d), which still underpredict. **Honest conclusion** : TQNT does not solve the Muon Puzzle ; the lasting gain is the *collision dynamics* itself (states → production), not a won anomaly (draft `brouillon_dynamique_collision`).

B.7 Scope and limit : the knot model as a theory of four observables

Scope thesis (V6 program)

The knot–particle map is a theory of **four observables**, all topological : *mass* ($m = \sqrt{\sigma_\chi}\,\mathcal{S}$, §9.1), *charge* ($Q = \Sigma\chi + \mathrm{Lk}$), *spin* (braid parity), *lifetime* (Fermi + $Q = \Delta m$, §15.3). It runs into *one* wall : the **baryon number** and the **unique assignment**, which are *not* derived. Delimiting this wall is a *positive* result. (Details : `docs/TQNT_V6_programme_synthese`.)

The leptonic tower is fixed by the charge, not chosen. Among the simple spin-$\frac{1}{2}$ knots (toric $T(2,n)$), *only* 3_1 gives $Q = \chi + \mathrm{Lk} = -2 + 1 = -1$: the toric $5_1, 7_1, 9_1$ have $\chi = +2 \Rightarrow Q = +3$, hence are *not* leptons. The electron, the muon and the tau are therefore **the same knot** 3_1 in three $\mathbb{Z}_3$ generations (§9.5). It is the charge — not a free choice — that determines the leptonic assignment. *Honest reservation* (consistent with §9.5) : the hierarchy via Koide is an entry *repackaging* ($r = \sqrt{2}$ postulated, then *carried* by the direction doubling, conditional, §9.5) ; the relation $m_0 \approx m_p/3$ remains a suggestive *numerical* coincidence, to be treated with the same caution as the other rational coincidences ; δ is decomposed $\pi/12 - \varepsilon$ (the optimum $2/9$ is its reading).

The baryonic wall — and its resolution proposed by triality. The baryon number B is encoded by *no* invariant of the knot *type* : π^+ (meson) and the neutron (baryon) are indistinguishable by type (3 knots, 3 chiral, $\mathrm{Lk} = 2$). This is the wall already flagged for proton stability (§15.3 and "proton stability") : *no* exact topological baryonic conservation. **Proposed resolution** (exploratory, consistent with the pleroma §22.1) : the *same* $\mathbb{Z}_3$ of the trefoil that gives the generations also gives the *color* (3 lobes = 3 colors) ; then $B = \mathrm{SU}(3)$ triality = occupation of the lobes — a $\mathbb{Z}_3$ charge that the bare catalog (multisets) does *not* encode, whence the invisibility of B by the type. *Status* : a lead, not established — it requires encoding the lobe occupation in the catalog.

Scope table (honest).

Derived (continuous knot invariants)	**Not derived** (discrete "quark" q. n.)
mass $\sqrt{\sigma_\chi}\,\mathcal{S}$ (median 0.37 %)	baryon number B
charge $Q = \Sigma\chi + \mathrm{Lk}$	meson/baryon/lepton distinction
spin (braid parity)	unique assignment (under-determined)
lifetimes (structurally)	fine values (x, δ_{CP}, f_π ; the vev is *out*, derived 2026-07-21)

Consistent with **Rule 0** (quarks are SM accounting artifacts) : what *counts* the quarks (the baryon number) has no reality in the knot *type*. The knot model is a theory of **masses, charges, spins and lifetimes** ; the baryon number must, as it stands, be *posited as input* (1 bit per particle) or derived from color triality (lead above). The whole V6 program converges toward this *single wall* — a positive result that precisely delimits the scope of the model.

Methodological rule: the assignment null

Rule (mandatory before any claim of agreement by assignment)

Before presenting as *evidence* a mass agreement ($\sim 1\,\%$) obtained by *choosing* a knot configuration:

1. **Enumerate** the accessible assignment space (e.g. 1 140 combinations of 3 knots among the 18 of the catalogue);
2. **Count** the number of configurations *expected* in the tolerance window under the null (random assignment);
3. If expected $\gtrsim$ observed: the agreement carries **no evidential weight** — record it as a *compatibility*, never as a success.

Calibration (top discriminant test). The rule was quantified on the seemingly most favourable case: (i) *one* mass agreement at $\pm 1.25\,\%$ — 4 combinations out of 1 140 fall in it, exactly the number expected by density: **any isolated agreement at $\sim 1\,\%$ is free** ($P \approx 1$); (ii) even a *shared motif* — the knot 8_3 common to the four EW states (W, Z, H, t) — is compatible with chance (knots completing the quartet: observed 1, expected 0.88 under the null); (iii) the Koide triplet (c, b, t) (1/20, expected $\sim$ 0.2–0.3 with scheme freedom) remains non-significant ($p \sim 0.2$); (iv) **the full EW quadruplet** (2026-07-17, `null_assignation_ew.py`): even the closure on *four* simultaneous masses (RMS $\leq 0.74\,\%$, free global calibration) is expected $\sim 8 \times 10^5$ times among the $\sim 3 \times 10^{12}$ triplet quadruplets of the catalogue ($\sim 3 \times 10^4$ with a shared knot imposed): the EW quartet is **confirmed as compatibility** — the numerical agreement carries no evidential weight; what survives is the *structure* (shared 8_3 — itself chance-compatible, cf. (ii) —, self-conjugation predictions; the "writhe doublet" invoked until 21/07 is obsolete, $\mathrm{Wr}(8_3)_{\mathrm{ideal}} = 0$). Declared scheme: Abraham N50K U_{EM} (the post-glitch calibrated scheme of the commits is not regenerable triplet by triplet). The general lesson: a multi-system closure carries weight only if the configurations are *already fixed* elsewhere (cf. corollary).

Corollary: only *structure* discriminates. The agreements that carry weight are those the assignment null *cannot* produce: **multi-system closures** (one same formula over N systems with configurations *already fixed*, e.g. the V5p29 oscillations at 0.1 %); **uniformity of the quantum numbers** within a family (the 3 leptonic Z_3 modes share everything but the mass); **replication of a relation** from one sector to another (e.g. $\delta = Q/3$: leptons yes, (c, b, t) no); **scheme coherence** (an agreement that depends on the pole/$\overline{\mathrm{MS}}$ choice is not physical). Conversely, the $\sim$ 419 compatible observables (synthesis chapter) must be read *through* this rule: a majority of compatibilities, whose real weight is what the null lets survive. Scripts: `top_discriminant_test.py` (calibration), exhaustive assignment scan (median 4, max 43 candidates per particle).

The global meta-null test (2026-07-27, `meta_test_nul_global.py`). The null-test rule is applied locally; the *cumulative* trials factor of the edifice (identifications, closed sets, scans) is quantified here as an order-of-magnitude estimate: for each load-bearing result, $p \approx T(2\varepsilon)^n$ with T *bracketed* (mechanism accepted / bare scan) and n independent confrontations; results derived from another or calibrated are excluded; the global look-elsewhere is bounded by the objective script count of the repository (635 $\to \times[158, 1905]$ attempts). **Resulting hierarchy**: *(1) the hard core* — V–P hyperfine ($p_{\mathrm{pess}} = 3.8 \times 10^{-11}$), recoupling ladder ($10^{-10}$), Koide ($2 \times 10^{-4}$), oscillations ($2.7 \times 10^{-4}$) — **survives any plausible trials accounting** (core alone, pessimistic, $\times 1905$: 3.9×10^{-25}); *(2) the conditionals* (v: $p_{\mathrm{pess}} = 1.7\,\%$; n_s, Fermi, G: 4–15 %; $E{=}\alpha m_\ell$) stand on their *mechanisms*, not on the bare numbers — the text's status is confirmed; *(3) the consistencies* ($k{=}137$ *alone*: $p \sim 0.64$; m_e $\pm 3\,\%$; Ω_{cc} alone; A_s, η_B) contribute almost nothing

statistically. Presentational consequence: the correct statement is "**four results beyond any trials factor, a cluster of mechanism-backed conditionals, dated predictions**" — the ∼250 observables are the *context*, not the argument. Declared limits (T bracketed judgements; the robust conclusion is the hierarchy).

Appendix C

Topological plasticity (P23): framework and status

Chapter contents

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Epistemological status of this chapter

This chapter gathers results stemming from a methodical exploration that followed the writing of TQNT-V5. Two types of results coexist and must be clearly distinguished:

- **Numerical results of the Ω_{ts} era** (historical; the operator is archived): strict Ω_{ts} reached $\sim 1.7\,\%$ RMS with optimal assignments (review archived outside the text); μ^- *was* the only persistent anomaly *of* Ω_{ts} (analysis archived outside the text) — under the reference formula this residual **does not exist** (uniform leptonic residuals; C_{plast} **archived**, §27.2); radial symmetry cancels plasticity for isolated knots (π^0 singleton consistent at $0.8\,\%$).
- **Proposed theoretical framework** (P23, radial plasticity): rigorous mathematical formulation (Biot-Savart integral, ξ_{BS} computable analytically), but *not quantitatively validated* as a universal predictive principle. The honest assessment of the time is archived outside the text.

The residual $\mu^- = -2.27\,\%$ can be *described* by a deformation $s_z(6_1) = 1.044$ calibrated on the particle itself, but this deformation is not *predicted* by a universal principle in the current state.

The chapter comprises:

- §C.1: mathematical formulation of P23 (status: theoretical framework, not quantitatively validated; C_{plast} archived).
- §C.1, paragraph (C3): programme for deriving from P23 the phenomenological QM corrections (η-η' mixing, spin-orbit, Coulomb, isospin) — status: theoretical promise, calculation to be carried out.
- §C.2: assessment of the P23′ extension (Track 1 negative; Track 2 cascade **archived** 2026-07-15 to `Archives_recherches/archive_p23prime_cascade.tex`); requalified predictions

of the black-hole sector.
- critical review of the mass table, methodological notes (kaons, μ^- residual), assessment of the exploration: *archived outside the text* (Ω_{ts} validation machinery).

The **reference** results born of this thread (taxonomy of composites, unified $\tau_\perp$ hyperfine) have their own chapter (chap. D).

C.1 Postulate P23: topological plasticity of knots

C.1.1 Statement

P23 — Topological plasticity

During the formation of a link $L = \{K_1, \ldots, K_n\}$ (production, decay, interaction), the intense local electromagnetic field imposes on each component K_i a *non-elastic plastic deformation* characterized by a vertical scale factor $s_i \in \mathbb{R}^+$. The observed state is the **attractor** of the dissipative dynamics:

$$\boxed{\frac{\mathrm{d}s_i}{\mathrm{d}t} \;=\; F_i(\mathbf{s}, L) \;=\; \sigma(L)\,\gamma(L)\,(\mathrm{cr}_i - \overline{\mathrm{cr}}) - k_p\,(s_i - 1)} \tag{C.1}$$

where:
- $\sigma(L) = +1$ if L is pseudoscalar ($J = 0$), $\sigma(L) = -1$ otherwise ($U(1)_A$ anomaly).
- $\gamma(L) \;=\; C_p / \sum_j U_\kappa U_{\tau_\perp}(K_j)$ is the amplitude, inversely proportional to the total curvature-torsion energy of the link; C_p is a universal scale constant.
- $\mathrm{cr}_i - \overline{\mathrm{cr}}$ is the complexity deviation of component K_i from the link average (distribution by topological complexity).
- k_p is the reference anchoring stiffness. Together with C_p above, these are the two *internal* constants of the P23 formalism; they only appear through the ratio $\gamma(L)/k_p$ in the attractor, so a single combination is effectively constrained by experiment. This combination was summarized by $C_{\mathrm{plast}} = -2.92$ MeV, calibrated on the residual of μ^- *of* Ω_{ts} — a residual that **does not exist** under the reference formula (uniform leptonic residuals $-0.2\,\%$, absorbed by the 2026-07-18 $\sqrt{\sigma_\chi}$ recalibration; Koide form at -67 ppm): C_{plast} is **archived** and no longer appears in the count (§27.2).

The stable attractor $\mathbf{s}^* = \{s_i^*\}$ determines the effective embedding of each component:

$$s_i^* \;=\; 1 \;+\; \frac{\sigma(L)\,\gamma(L)}{k_p}\,(\mathrm{cr}_i - \overline{\mathrm{cr}}) \tag{C.2}$$

The observed mass is then evaluated by Ω_{ts} with the deformed geometries:

$$m_{\mathrm{obs}}(L) \;=\; \Omega_{ts}\big(L\,;\,\mathbf{s}^*\big). \tag{C.3}$$

C.1.2 Physical origin: radial superposition of the intrinsic fields

The intrinsic magnetic field (P4) as driver. Each knot carries an intrinsic magnetic field $B(s) = \tau_\perp(s)$ wound around its flux tube (P4, §2.10). The radial plastic force on a point $\mathbf{r}$ of the tube is:

$$\mathbf{F}^{\mathrm{rad}}(\mathbf{r}) \;=\; \oint_L \nabla_\perp B(s)\,\mathrm{d}s\,, \qquad L = \bigcup_i K_i \tag{C.4}$$

integrated along the union of the components.

Cancellation by symmetry for the isolated knot. For a *single* component ($n = 1$), the field $B = \tau$ is a *closed* circulation along a closed curve K_1. The radial integration (C.4) around

any point of the tube gives a **net resultant that vanishes by topological symmetry**:

$$n = 1 \implies \mathbf{F}^{\text{rad}} = \mathbf{0} \implies \mathbf{s}^* = \mathbf{1}. \tag{C.5}$$

The isolated knot is therefore reference: it undergoes no plasticity. Any phenomenological difference between elementary fermions (e.g., u vs d vs e^-) pertains *to a mechanism other* than P23: for u, d specifically, the reference stoikhion mass is the knot mass m_ℓ of the σ_χ catalogue (the former P23′ Track 2 extraction is archived, §C.2); the heterogeneity of the $\overline{\text{MS}}$ current quark mass is not a TQNT observable (§9.4).

Non-trivial plasticity for $n \geq 2$. For a link with $n \geq 2$ components, the material point of a component K_i is immersed in the superposition of the field B_i of K_i *and* the fields $\{B_j\}_{j \neq i}$ of the other components. This superposition **breaks the topological symmetry** that cancelled the integral (C.4) in the isolated case. The net radial resultant is nonzero; it deforms each component toward the dissipative attractor $\mathbf{s}^*$ determined by equation (C.1).

Origin of the formation: imprint of the EM field. Plasticity is not an effect of the *current* EM field at the time of measurement, but an *imprint* left by the formation conditions:

- A particle is produced via a topological process (parent decay, high-energy collision, P22 nucleation at boundaries). This process involves extreme local EM fields (up to $E \sim E_{\text{Schwinger}}$).
- These fields plastically deform each component; after the process, the components do not return to their reference catalogue shape but retain the state reached at dissipative equilibrium.
- The attractor being determined by $\sigma(L), \gamma(L)$ and the topology alone (not by the details of the path), all particles of the same topology converge to the same state; the PDG masses are therefore universal, consistent with observation.

Material analogy

P23 is the topological analogue of **metallurgical plasticity**: a rolled steel retains the deformation imposed by the rolling, independently of the particular furnace in which it was heated. The final shape is an attractor of the protocol, not a memory of the path.

C.1.3 Potential consequences (status: hypotheses)

Warning. The consequences listed below *would be* the manifestations of P23 *if* the mechanism were quantitatively validated. The validation test (assessment archived outside the text) showed that in its current state, P23 does not provide a quantitative prediction better than strict Ω_{ts}. Consequences C1–C4 must therefore be read as *ambitions of the theoretical framework*, not as established results.

(C1) Scope restricted to links $n \geq 2$. P23 *does not apply* to single components. Elementary fermions represented as isolated knots (e^-, as well as the 3_1 topology shared by u and d, §9.4) have **a radial plasticity that is identically zero** by topological symmetry. For u and d specifically, the reference stoikhion mass is the knot mass m_ℓ of the σ_χ catalogue (P23′ Track 2 extraction archived, §C.2); the old V4 heuristic of distinct geometric embeddings (R, a, b) was a calibration and not a prediction (numerical test `simulation/experiments/test_p23_quarks_ud.py`: ± 50 to 400 %, i.e. two orders of magnitude beyond what P23 would predict — confirmation that these embeddings are outside its scope).

(C2) μ^- residual as plastic attractor. For $L = 3_1 + 6_1$ formed in the decay $\pi \to \mu\nu$, P23 predicts:

$$s_z(6_1)^* = 1 + \frac{(-1)\cdot\gamma(L)}{k_p}\cdot(\mathrm{cr}_{6_1} - \overline{\mathrm{cr}}) \approx 1.044$$

which reproduced the μ^- residual *of* Ω_{ts} (analysis archived — residual nonexistent under the reference formula).

(C3) Standard QM corrections as first-order P23. The η-η' mixing, the χ_{cJ} and χ_{bJ} spin-orbit, the Coulomb and isospin differences (phenomenological concepts of the Standard Model) become *particular manifestations* of the plastic deformation $\mathbf{s}^*$ for each family: eventually, their five calibrated coefficients ($M_{12}, \alpha_{SO,c}, \alpha_{SO,b}, k_{EM}, k_{\mathrm{iso}}$) are computable consequences of the single global parameter of P23, C_{plast} (equivalent to the ratio C_p/k_p of the internal constants).

(C4) Universality of the PDG masses. The attractor being independent of the formation path, all particles of a given type have the same measured mass. P23 thus explains why $m_{\mu^-}^{\mathrm{PDG}}$ is universal to within $\sim 10^{-8}$ across all experiments.

C.1.4 Predictive status and remaining work

Identified and validated.

- General form of the dynamics (C.1).
- Sign selector $\sigma(L) = +1$ pseudoscalar / -1 others (empirical agreement 12/12 on a test sample).
- Amplitude $\gamma(L) \propto 1/\sum U_\kappa U_{\tau_\perp}$ (Pearson correlation $r = 0.97$).
- Distribution by complexity gradient.

To be refined.

- Saturating form of the force (tanh, logistic) to prevent the pathological far attractors observed on the lightest pseudoscalars.
- Derivation of C_{plast} (equivalent of the ratio C_p/k_p of the internal constants) from a first principle (e.g., Schwinger energy as natural scale).
- Rigorous test on the reference table of the main chapter, with assignments validated by quantum numbers (review archived outside the text).

Testable conjectures.

- An ultra-cold trap mass measurement (at $B \to 0$) of a *multi-component* particle normally produced by collision could reveal a systematic drift toward the *pre-plastic* mass (Ω_{ts} with $\mathbf{s} = \mathbf{1}$).
- The radial integral (C.4) computed numerically on the links $\mu^-, \tau^-, \eta, \phi, p, n, \ldots$ must reproduce the $\sigma(L)$ and $\gamma(L)$ extracted empirically.
- An isolated knot (e^-, ν) must show no sensitivity to a change of production protocol (consistent with observation: the PDG gives m_e identical by all methods).

C.1.5 Experimental alteration of measurements (archived)

The hypothesis "the PDG masses are *post-plastic* masses altered by the experimental protocol" (catalogue `TQNT_catalog_plas_complete.xlsx`, $C_{\mathrm{plast}} = -2.92$, 8 reassignments, proposed trap/flight tests) belongs to the Ω_{ts} era and is **archived** (`Archives_recherches/archive_p23prime_cascade.tex`, item 1): under the reference formula $m = \sqrt{\sigma_\chi}\,\mathcal{S}$, the scoring (median 0.13 %, 89 states) treats the PDG masses as reference observables, and C_{plast} is archived (§27.2). What survives is a qualitative reading grid: the separation of instrumental/theoretical regimes (trap $|\Delta| \sim 0.01\,\%$ / decay products 0.5–3 % / broad resonances $\sim \Gamma/m$).

Conclusion of Postulate P23 (measured). P23 proposes a physical framework for the plastic deformation of the components of a link under the effect of the formation EM field: Biot-Savart radial integral, sign selector $\sigma(L)$ identified, amplitude $\gamma(L)$ correlated with $1/\sum U_\kappa U_{\tau_\perp}$, geometric constant ξ_{BS} computable analytically. *As a theoretical framework, P23 is coherent and well formulated.*

However, the quantitative validation test (assessment archived outside the text) showed that in its state at the time, P23 provided **no** measurable RMS improvement beyond strict Ω_{ts} with the right assignments. The residual $\mu^- = -2.27\,\%$ *of* Ω_{ts} remained the only persistent anomaly, describable by a deformation $s_z(6_1) = 1.044$ but not predicted by P23 universally. *Obsolete under the reference formula: this residual does not exist (C_{plast} archived).*

C.2 Extension P23′: assessment and archiving (2026-07-15)

What it was. P23′ conjectured that the effective mass of a knot *confined* in a host link is amplified by the intrinsic fields of its companions ("quark = confined knot"). Two tracks were explored (June 2026): **Track 1** (exponential amplifier $\Pi_C = e^{\alpha_C \xi_C}$, one global parameter) — **negative** (0.7–1.2 decade of discrepancy, the dynamics of ξ_C does not cover the four decades of masses); **Track 2** (hadronic differential $m_{\text{eff}}(q)$ extracted from hadron masses) — *apparent* encouraging results (5-flavour constituents at $\sim 6\,\%$, then a cascade of extensions: light mesons, exotics, heavy baryons, nuclei, neutron stars, black holes).

Why the cascade is archived (2026-07-15)

1. **Abandoned basis**: all the numbers started from the Ω_{ts} hadronic masses ($m_p = 938.54$, $\Lambda = 1\,139.22$, …) and the assignments of the V5 xlsx catalogue — the operator is archived and the reference scoring is $m = \sqrt{\sigma_\chi}\,\mathcal{S}$.
2. **Assignment conflict**: the table $s \to 5_1$, $c \to 7_1$, $b \to 9_1$ contradicts the main document (torus knots are the *leptons*) and the pinning of m_s (2026-07-15: 5_2 pinned, 5_1 excluded by the torus discriminant).
3. **Double truth**: the Track 2 constituents ($m_s = 513$, $m_c = 1\,358$–$1\,602$, $m_b = 4\,700$ MeV) are incompatible with the reference knot masses of the σ_χ catalogue ($m_\ell = 310$ calibrated, $m_s = 494.8$ (calibrated ; demoted 2026-07-22), $m_c = 1\,557$, $m_b = 4\,952$ MeV) — two sets cannot coexist.
4. **Internal inconsistency**: the "positive validation" boxes contradicted the V6 reading ("negative result, owned").

Full text: `Archives_recherches/archive_p23prime_cascade.tex` (item 2). The G^{TQNT}, gravitational-wave and Bekenstein-Hawking entropy tests that closed the cascade re-tested results that live in the main text (§17.1, V5.8–V5.10); nothing is lost.

What survives.

- **The scale insight**: TQNT captures the quarks at the *constituent* scale ($\mu \sim \Lambda_{\text{QCD}}$); the $\overline{\text{MS}}$ "current quark mass" is scheme-dependent and is *not* a TQNT observable (§9.4). This conceptual gain is now carried by the knot masses of the σ_χ catalogue.
- **The qualitative reading** "quark = mode of a confined knot" (P7ter), without the Π_C machinery.
- **The requalified predictions of the black-hole sector** (2026-07-15, kept here because reference): *(i)* horizon structure $\Omega^2(r)$ without a naked singularity — structural, *with no deadline* (the κ_g skin is not discernible, following the logic of the NICER audit); *(ii)* **absence** of post-merger dark matter (P22-WKB null + Vassiliev suppression α^c, DM entirely primordial P18): a residual topological signal would refute; *(iii)* Lk-spin correlation: conjecture to be operationalized (no measurement of Lk_{tot} independent of the spin); *(iv)*

long-term Hawking radiation *coherent*, not thermal (deterministic P22 nucleation, regime $\xi \gg 1$). Tidal $\Lambda \approx 350$–600: indicative only (ρ_0/volume open issue).

C.3 Chapter synthesis

Measured assessment of P23/P23′

1. **P23 framework (radial plasticity)**: clean mathematical form (Biot-Savart integral, analytical ξ_{BS}); null for $n = 1$ by symmetry; *negative quantitative validation* in the Ω_{ts} era; its historical target (μ^- residual) **does not exist** under the reference formula — C_{plast} archived, P23 reformulated *without any calibrated constant* (embedding plasticity + magnetism of materials, §20.17).
2. **P23′ extension**: Track 1 negative; Track 2 cascade archived (§C.2); constituent-scale insight retained.
3. **Methodological stratification** (lasting gain): separate pure topology / phenomenological corrections to be derived / post-formation plasticity / instrumental resolution — independent of numerical refinements.

Appendix D

Structure and hyperfine splitting of composites

Chapter contents

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This chapter gathers two **reference** results arising from the analysis of composites (historically explored in the P23′ thread, now independent): the *taxonomy* of composites along the complexity axis (three knot roles + one residual sector; the meson as a $K+\bar{K}$ construction) and the *unified hyperfine* mechanism $\tau_\perp$ (ρ-π and N-Δ, a single mechanism).

D.1 Structure of composites: knot roles and residual sector

The consistency analysis of the assignments (archived outside the text) and the exhaustive scan lead to a structural clarification of the particle taxonomy, organized along the *complexity axis* (crossing number c). At the *base* of the axis ($c = 0$, no knot) sit the *free* χ *waves* (photon, neutrino); at *low* complexity ($c \lesssim 8$) lives the *visible* sector, where one distinguishes the *fundamental knot roles* from the *residual constructions*; at *high* complexity ($c > 18$) lives the *dark* sector (dark matter, chap. 24). Six categories in all, from the massless photon to the dark knots.

D.1.1 Base of the axis: the knotless particles ($c = 0$)

At the bottom of the complexity axis sit the excitations of the χ field that carry *no* topological charge: the *free waves*, of zero crossing number. Two members:

- the **photon**: quantum of the χ gauge field (U(1), $\alpha_{\mathrm{EM}} = 1/(k+2) = 1/137$), *exactly massless*, knotless;
- the **neutrinos**: *self-rotating* free χ waves, with no topology (§28.6.3); their mass comes from no knot invariant but from the *self-rotation curvature* $m_\nu = \hbar c/2R$. Dirac fermions, $0\nu\beta\beta = 0$ exactly, $\Sigma m_\nu \approx 66$ meV.

The knotless particles

Photon and neutrinos are not knots: they are *waves of the* χ *field* of zero topological charge. The photon is its massless gauge quantum; the neutrino, a self-rotating wave whose mass emerges from its *curvature*, not from a topology. They form the base ($c = 0$) of the complexity axis, below the knotted knots of matter.

D.1.2 Visible sector: three knot roles and a residual sector

The assignment procedure — type by core multipole, core knot by multiplet, state index, stability filter, propagation through decays — pins a unique core knot for three families, and *fails cleanly* for a fourth:

Family	Topological definition	Nature	Core
Leptons	3_1, $\mathbb{Z}_3$ modes	knot role	3_1
Baryons	spin $\frac{1}{2}$ (odd braid), $B \neq 0$	knot role	$7_2/5_2$
EW sector	core 8_3, $f = 1$ (unconfined)	knot role	8_3
Mesons	integer spin, $B = 0$, knot+antiknot	**residual sector**	none

The EW sector illustrates that *spin is not the category criterion: the core is.* Four particles share the 8_3 core ($f = 1$, unconfined), their spin varying as a mere state index: W (7_1+7_2+8_3) and Z (7_2+8_3+9_1) are vectors (spin 1), the Higgs (7_1+8_3+10_1) is scalar (spin 0), the top (8_2+8_3+10_1) is a fermion (spin $\frac{1}{2}$). Although totalling ~ 22 crossings, they are *links of low-c knots* (each ≤ 10), which interact through their components: they are *visible* (produced at the LHC) and unstable, not dark — dark matter requires an *isolated simple* knot of $c > 18$.

For the mesons, the "type", "core" and "propagation" steps designate no unique knot: the `omega_unified` scan produces 4 to 19 $K + \bar{K}$ candidates per meson (at $\pm 5\,\%$), and only the "J index" (0 pseudoscalar / 1 vector) and "stability" (all unstable) steps apply. This is not a failure of the procedure: it is the correct detection that **no meson knot exists**.

Mesons are the residual sector

There is no *meson knot* but a *meson construction*: $K + \bar{K}$ (knot + its mirror), applicable to *any* knot. Mesons are the closure of the set of knots under mirror pairing: the residual bosonic sector ($B = 0$, matter+antimatter). The TQNT taxonomy is therefore

$$\underbrace{\text{lepton } 3_1 \;\big|\; \text{baryon } 7_2/5_2 \;\big|\; \text{EW } 8_3}_{\text{three knot roles}} + \underbrace{\text{mesons}}_{\text{residual sector}} ,$$

exactly the logic of the Standard Model ($q\bar{q}$ = "the rest of the hadrons"), but *derived* from the structure of knots rather than postulated.

This reperiphery explains at a stroke all the properties that "resisted":

- **no core** $\Rightarrow$ it is a pair, not a knot;
- **multiplicity** (4–19 candidates) $\Rightarrow$ one meson per knot $\times$ values of J, not a knot to be found;
- **all unstable** $\Rightarrow$ matter+antimatter $\rightarrow$ P15 reconnection;
- $B = 0 \Rightarrow$ matter/antimatter cancellation;
- **integer spin** $\Rightarrow$ two half-integer braids add up.

Caveats

This result is *structural*, not quantitative: it explains *why* the mass of a given meson remains underdetermined (4–19 candidates), it does not predict it. The residual freedom is now *expected*, not an anomaly. Additional caveat: the pion (anomalously light Goldstone) requires a triplet $3_1 + 3_1 + 5_1$ (0 pairs, 2 compatible triplets) — even "$K + \bar{K}$" is not rigid: the construction is the right conceptual framework, not a closed formula.

Categorization of mesons and reduction of the underdetermination

The residual sector is not amorphous: it categorizes on two levels, one topological (the content), the other by excitation factors.

Level 1 — topological categories. The mirror relation between the two knots partitions the mesons: *diagonal* ($K + \bar{K}$, component = its own mirror ⇒ self-conjugate: π^0, η, ρ^0, ω, ϕ, quarkonia) versus *off-diagonal* ($K_1 + \bar{K}_2$, $K_1 \neq K_2$ ⇒ particle–antiparticle pair: $\pi^\pm$, K, D, B). A second axis counts the components: *binary* (2 knots, ordinary) versus *multi-knot* (light Goldstones, exotics/tetraknots — "tetraquarks", 4 knots).

Level 2 — categories by factors $(L, S) \to J^{PC}$. The relative spin $S \in \{0, 1\}$ and the orbital momentum L of the pair generate the spectroscopic categories, with $P = -(-1)^L$ (intrinsic mirror $\times$ orbital) and $C = (-1)^{L+S}$ (diagonal mesons):

L	S	J	P	C	J^{PC}	category
0	0	0	−	+	0^{-+}	pseudoscalar (π, K, η)
0	1	1	−	−	1^{--}	vector $(\rho, \omega, K^*, \phi)$
1	0	1	+	−	1^{+-}	axial (b_1, h_1)
1	1	0	+	+	0^{++}	scalar (a_0, f_0)
1	1	1	+	+	1^{++}	axial (a_1, f_1)
1	1	2	+	+	2^{++}	tensor (a_2, f_2)

Parity thus distinguishes the pseudoscalar (0^{-+}, L=0) from the scalar (0^{++}, L=1): same content, different orbital excitation.

Reduction of the underdetermination. These two levels *constrain* the assignment, where the mass criterion alone left 4 to 52 candidates. Two cumulative levers:

1. **factor (spin average)**: Ω_{ts} being independent of J, it does not match π or ρ but their average $M_{\text{avg}} = (3M_V + M_P)/4$ (hyperfine-free baseline, $\tau_\perp$ then separating); this *divides by* ~ 2 the number of candidates;
2. **topological category (diagonal)**: a self-conjugate meson *is* $K + \bar{K}$ with the same knot, which excludes the off-diagonal pairs.

The combination brings the candidates down to 0–2:

self-conjugate family	mass alone	+ M_{avg}	+ diagonal
ϕ ($s\bar{s}$)	~ 13	10	**2**
J/ψ ($c\bar{c}$)	~ 52	23	**2**
Υ ($b\bar{b}$)	~ 87	43	**0**

Categorization tightens the assignment by an order of magnitude

Crossing the topological category (diagonal) with the factor (spin average M_{avg}) reduces the underdetermination of the self-conjugate mesons from ~ 50 candidates to 0–2 (charmonium: $\{7_4, 7_6\}$). The residual sector is therefore not amorphous: its structure by categories strongly *constrains* the knot content.

Third factor — the radial spectrum lifts the degeneracy. For the heavy quarkonia, the radial tower ($1S$, $2S$, $3S$) settles the remaining $\times 2$ degeneracy. Each candidate fixes an effective mass ($M_{\text{avg}}/2$), hence a Compton scale, hence a Cornell spacing $V(r) = -a/r + \sigma r$. For charmonium:

level	observed	$7_4 + \bar{7}_4$	$7_6 + \bar{7}_6$
$1S$	3069	3129 (+2.0 %)	3288 (+7.1 %)
$2S$	3674	3728 (+1.5 %)	3883 (+5.7 %)
$3S$	4040	4166 (+3.1 %)	4315 (+6.8 %)
RMS over the tower		**2.3 %**	6.6 %

Subtle point: it is *not* the spacing ($\Delta(2S{-}1S) = 600$ vs 595 MeV, nearly degenerate) but the *consistency over the whole tower* that decides — the 7_4 tracks the three levels at $\sim +2\,\%$, the 7_6 *systematically overshoots* by $\sim +6$–$7\,\%$. Three consistent absolute points (7_4) against three systematic overshoots (7_6) fix charmonium at $\mathbf{7_4 + \bar{7}_4}$. The three-factor categorization (M_{avg}+ diagonal + radial tower) thus reduces 52 candidates to **1**.

Caveats

The radial spectrum *lifts* the degeneracy for the heavy quarkonia, but the general closure remains partial:

- the discrimination comes from the **absolute tower** ($1S/2S/3S$ consistent), *not* from the spacing alone (nearly degenerate); it assumes a clean radial spectrum (heavy quarkonia, Cornell);
- the 7_4 remains at 2.3 % RMS (> the 0.8 % of Ω_{ts} on the clean composites): clearly favoured, not perfect;
- for the **light mesons** (ϕ, u/d), the radial spectrum is blurred by flavour mixing (η-η', ω-ϕ) and the Goldstone anomaly; the alternative route (hyperfine splitting $M_V - M_P$ via $\tau_\perp \propto 1/(m_i m_j)$) runs, for its part, into the *constituent mass* — an effective QCD concept with no topological equivalent (the same wall as B, L);
- **catalogue gaps**: the diagonal spectrum jumps from $7_6\bar{7}_6 \simeq 3.3$ GeV to $8_3\bar{8}_3 \simeq 17$ GeV; the bottomonium (9.4 GeV) falls into this hole and requires an intermediate knot (9_1, $10_1\ldots$), hence a *triplet* assignment (the catalogue puts $J/\psi = 4_1 + 5_2 + 9_1$), which reopens some freedom.

The gain remains real: the ×2 degeneracy is lifted for the heavy quarkonia (charmonium $= 7_4 + \bar{7}_4$), the general closure remaining bounded by the light mesons and the catalogue gaps.

Bottomonium: catalogue gap and self-conjugate triplet. No diagonal pair of the loaded catalogue reaches the bottomonium ($M_{\text{avg}} \simeq 9445$ MeV): $10_1\bar{10}_1 = 5074$ MeV ($-46\,\%$), then a jump to $8_3\bar{8}_3 = 17058$ MeV ($+81\,\%$); the 9_1, a *torus* knot $T(2,9)$, stays light ($9_1\bar{9}_1 = 392$ MeV) despite its 9 crossings — the crossing number does not make the mass. The radial spectrum nevertheless remains *correctly predicted* by $\mu = M_{\text{avg}}/4$ (spacings $\Delta(2S{-}1S)$ and $\Delta(3S{-}1S)$ at +3 and +4 %): the *method* is sound, only the *content* is missing. Two self-conjugate triplets $K + \bar{K} + A$ (A amphichiral) fill the gap: $7_4\bar{7}_4 4_1 = 9533$ MeV ($+0.9\,\%$) and $7_6\bar{7}_6 4_1 = 9333$ MeV ($-1.2\,\%$), both tracking the radial tower at $\sim 1\,\%$. The ×2 degeneracy therefore reopens; two *weak* but *convergent* criteria lift it:

1. **extended spectrum (S+P)**: over 5 levels ($1S, 1P, 2S, 2P, 3S$), the 7_6 better matches the *slope* of the tower (its deviations decrease from -1.2 to $-0.8\,\%$; those of the 7_4 grow from $+0.9$ to $+1.2\,\%$) — RMS 0.96 % versus 1.07 %;
2. **inter-family consistency**: charmonium being 7_4, a distinct flavour ($b \neq c$) requires a distinct knot $\Rightarrow 7_6$.

Both point to $\boxed{\text{bottomonium} = 7_6 + \bar{7}_6 + 4_1}$.

Caveats

The bottomonium lifting is *weak* and conditional:

- criterion (a) is *marginal* (0.96 vs 1.07 %, $\sim 10\,\%$ RMS difference): the two triplets bracket the mass, the tie-break rests on the slope of the tower;
- criterion (b) is a *heuristic* (distinct knots for distinct flavours), not derived; their convergence is suggestive, not a proof;
- the 7_6 core (simple 493) is only $\sim 5\,\%$ heavier than 7_4 (469): it *does not reflect* the mass ratio $b/c \sim 3$ — the mass comes from the composition and the 4_1, not from the core (not to be over-interpreted as "7_6 = quark b");
- the triplet (with a "binding" 4_1) remains a less natural representation than a pair for a 2-body system.

Light mesons: relativistic Salpeter tower. The light sector (u/d, s) escapes the *non-relativistic* tower: the light quarks are ultra-relativistic, and the non-relativistic Cornell overestimates the spacings ($\sim 20\,\%$ RMS). The spinless Salpeter equation $H = 2\sqrt{p^2+m^2} - a/r + \sigma r$ restores the agreement:

meson	candidate	tower $1S/2S/3S$	observed	RMS
ϕ ($s\bar{s}$)	$7_5\bar{7}_5$	983/1626/2107	1019/1680/2170	**3.2** %
ρ/ω	$6_2\bar{6}_2$	765/1419/1911	779/1450/1700	7.4 %
ρ/ω	$8_1\bar{8}_1$	771/1425/1917	779/1450/1700	7.5 %

The $\phi = 7_5 + \bar{7}_5$ is *confirmed* (3.2 % over three levels; $\sim 20 \to 7\,\%$ relativistic gain for ρ/ω). For ρ/ω, the tower does not separate 6_2 from 8_1 (nearly identical masses) — but this is not a degeneracy to be lifted: the diagonal catalogue contains *exactly two* knots in the window ($6_2\bar{6}_2 = 765$, $8_1\bar{8}_1 = 771$) for the *two* nearly degenerate physical states $\rho^0(775)$ and $\omega(783)$, with the right gap: 5.9 MeV predicted against 7.4 MeV observed. The ρ-ω near-degeneracy — an isospin coincidence in the Standard Model — becomes here two distinct diagonal knots (consistent with Rule 0: isospin is bookkeeping, the true distinction is topological).

Which knot is ρ^0, which is ω? The **decay width** decides through three concordant criteria. The standard lifetime formula (M_{raw}, n) does not separate them (nearly identical masses), but their structure differs radically, and the P15 reconnection rate varies as $\exp(-\lambda\, c)\, M_{\text{raw}}^{-\nu}$:

	crossings c	U_κ (loops)	U_τ (torsion)	$\to$
6_2	6	42.6	16.8	ρ^0 (loopy, **broad**)
8_1	8	21.7	38.2	ω (twisted, **narrow**)

Three topological handles concur: (i) 6_2 has *fewer* crossings ($6 < 8$) $\Rightarrow$ easier to unknot $\Rightarrow$ broad; (ii) 6_2 is "loopy" ($U_\kappa \gg U_\tau$, the loops ease reconnection) while 8_1 is "twisted" ($U_\tau \gg U_\kappa$, the torsion resists); (iii) the mass ordering ($6_2 < 8_1$ like $\rho^0 < \omega$). The three give $\boxed{\rho^0 = 6_2 + \bar{6}_2, \quad \omega = 8_1 + \bar{8}_1}$.

Caveats — light sector

- the width *assigns* ρ^0 vs ω (direction + ordering, triple concordance) but does not reproduce the *magnitude*: the topological handles give $\Gamma(\rho)/\Gamma(\omega) \sim 4.5$–7, against 17 observed — the discrepancy is the selection rule $\omega \to \pi\pi$ *forbidden* (G-parity; ω entirely pushed into the suppressed 3π channel), topological but not derived here; the mass gap 5.9 vs 7.4 MeV is a small-numbers test ($\sim 20\,\%$);
- the **pion** (and η) remains *outside the tower*: its anomalous lightness (140 against

$M_{\text{avg}} = 616$) is chiral symmetry breaking (Goldstone), a distinct effect that the vector tower does not capture;
- the $3S$ overshoots (+12 % for ρ/ω): the $\rho(1700)$ is probably a D wave or a mixed state, not a pure $3S$;
- standard Cornell parameters ($a = 0.52$, $\sigma = 0.18$), not derived here.

Light pseudoscalars: chiral breaking and mirror Goldstone. The pion and the η escape the tower because they are *not* normal knots: they are the *Goldstone bosons* of chiral symmetry breaking. In TQNT, chiral symmetry *is* the *mirror* symmetry of the χ field, spontaneously broken by knot condensation (each knot chooses a chirality); the pion is the Goldstone *phase mode*, a collective $K \leftrightarrow \bar{K}$ rotation. Hence its *lightness* (zero mass in the chiral limit) and its *pseudoscalar* character (the mirror rotation is odd under parity). The vector ($\rho/\omega/\phi$) stays at the *normal* scale (Ω_{ts} gives it); the pseudoscalar is *pulled below* that scale by Goldstone ($m_\pi/m_\rho = 0.18$), with the signature $m^2 \propto m_q$ (GMOR, in $\sqrt{m_q}$, non-linear). The Gell-Mann–Okubo relation $4m_K^2 = 3m_\eta^2 + m_\pi^2$ holds at +6.4 %, and $m_s/\hat{m} = 25$ (known $\sim$ 24–27).

The η': mass from the topological susceptibility of the χ field

The η' is not a Goldstone: the $\mathrm{U}(1)_A$ anomaly gives it its mass. Through the Witten–Veneziano relation, this mass measures the *topological susceptibility* of the χ vacuum (density of knotting fluctuations):

$$m_{\eta'}^2 + m_\eta^2 - 2m_K^2 = \frac{2N_f}{f_\pi^2}\,\chi_{\text{top}} \quad \Longrightarrow \quad \chi_{\text{top}}^{1/4} = 179 \text{ MeV},$$

i.e. exactly the known value ($\sim$ 180 MeV) and the same order as the TQNT scale $\Lambda_\chi = m_p(1 - f(3)) \approx 216$ MeV. A topological and TQNT-native result: the mass of the η' measures the density of knotting fluctuations of the χ vacuum.

Caveats — chiral sector

- GMOR, Gell-Mann–Okubo and Witten–Veneziano are the *standard* chiral relations: TQNT supplies the *mechanism* (mirror Goldstone) and the *topological interpretation* of χ_{top} (winding of the χ), not a new derivation of the relations;
- the *absolute scales* (B, f_π, absolute χ_{top}) require the density of the χ condensate, *not derived*: $\chi_{\text{top}}^{1/4} \sim 179$ is matched to $\Lambda_\chi \sim 216$ at $\sim$ 17 %, not derived first-hand; the absolute pion mass (B, f_π) remains underived;
- the *chiral* condensate ($\langle\bar{q}q\rangle \sim$ mirror-asymmetric structure of the χ) is distinct from the electroweak VEV, which TQNT does not have (consistent with "χ without a Higgs-like VEV").

D.1.3 Dark sector: high complexity

The four preceding families all live at *low complexity* ($c \lesssim 8$ for the cores). A *fifth* sector occupies the other end of the axis: the knots of *high* complexity ($c > 18$), which constitute dark matter (chap. 24). The Vassiliev suppression renders their cross section $\sigma_{\text{DM-}N} \approx \sigma_{\text{EM}}\,\alpha_{\text{EM}}^{c-18}$ *invisible* (for $c > 18$, undetectable), and the Chern–Simons level $k = 135$ renders them *stable* (barrier forbidding decay, $\tau > 10^{347}$ s).

The dark sector is defined by complexity, not by a role

Dark matter is neither a knot role (no core fixed by a multiplet) nor a residual construction (no pairing): it is the sector of knots *too complex to interact*. Its **dominant reservoir** consists of *isolated prime knots* ($n = 1$, $c > 18$: families $T(2, q \geq 19)$, $T(3, q \geq 10)$), stable through the Chern–Simons barrier.

Caveats — dark links are secondary

Dark-matter *links* ($n \geq 2$) form a *secondary* population, not the dominant reservoir: an instability theorem renders them generically unstable on the cosmological scale ($\tau \ll t_U$), the only exception being the 6 *strict* amphichiral links ($6_3 + 6_3$, $4_1 + 8_3$, $8_3 + 8_3$, ...), intrinsically neutral, 1.07–1.64 GeV reference — rederived 2026-07-15, ACPR 22/07; the Ω_{ts} values obsolete), a secondary stable subpopulation. The 111 "realizable" links (charge cancelled by an integer Lk) are transient or contextual (P22). The dominant reservoir remains the *isolated* knots (P18, Big Bang condensation kinetics).

D.1.4 The desert theorem: over-generation resolved by stability

The preceding taxonomy must answer an objection raised by the combinatorics itself: the catalogue generates **1539** possible links (up to $n = 3$ components, $c \leq 30$ crossings), whereas the observed spectrum counts only about thirty realized visible states (plus the 117 dark candidates). Why do we not see fifteen hundred particles? This is the *over-generation problem*, and its resolution is a stability theorem.

The mechanism: the P15 cascade. A link with several components ($n \geq 2$) can *reconnect internally* (P15, strand exchange at a crossing, $\Delta\mathrm{Lk} = \pm 1$): it sheds a component as soon as the balance $Q = m(\text{parent}) - m(\text{daughter}) - m(\text{shed knot}) > 0$ — a generic condition, since link bindings are of order $\mathcal{O}(100\ \text{MeV})$, small compared with the component masses. No conservation law protects an *arbitrary* combination of knots: the reconnection therefore proceeds on the strong/electromagnetic timescale, and the link *tumbles* from configuration to configuration towards the lighter states — the *P15 cascade*, in $\sim 10^{-18}$ s. Such a state is a transient, not an observable particle.

The quantitative verification. The scan of the "desert" — the mass window $[12, 80]$ GeV, between the known hadronic spectrum ($\Upsilon \sim 10$ GeV) and the electroweak quartet ($W \sim 80$ GeV), where the Standard Model correctly has *nothing* — counts 617 links of the catalogue. The stability filter yields **617/617 unstable** (cascade lifetimes $\sim 10^{-18}$ s, median 7×10^{-19} s): the desert is *emptied*, without exception. The corresponding widths ($\Gamma \sim \hbar/\tau \sim$ hundreds of MeV) forbid any narrow resonance: detection is impossible in principle, not for lack of luminosity.

The survivors of the cascade (exhaustive list). What structures the observed spectrum is precisely what *escapes* the cascade:

- the **conservation floors**: the electron (leptonic floor) and the proton (baryonic floor) — the cascade stops there for lack of an accessible lower state;
- the **simple knots** ($n = 1$): nothing to shed — undoing itself would require changing knot type through the Chern–Simons barrier. All live below ~ 4 GeV (maximum 8_3): none in the desert — the cores of the taxonomy (3_1, 5_2, 7_2, 8_3) are exactly the objects that pass the filter, which makes the categorization by core *self-consistent*;
- the **dark sector**: the isolated knots $c \geq 23$ (CS barrier, $\tau > 10^{347}$ s) and the 6 strict amphichirals ($Q = 0$ exact);

- the **assigned composites** (nucleon, mesons, hyperons, EW quartet), which live out their Fermi/Sargent channels (§22.4.4): there the cascade is braked by the sector protections, not by the absence of a channel.

Desert theorem

The combinatorics over-generates (1539 links), *stability* prunes: every unprotected $n \geq 2$ link cascades by P15 reconnection towards the floors in $\sim 10^{-18}$ s. Verification: 617/617 links of the $[12, 80]$ GeV window unstable (median 7×10^{-19} s). The observed emptiness between the Υ and the W is not an embarrassment: it is *predicted* — and the ATLAS heavy-proton search (null, clean) confirms it. Attached falsifiable prediction: **no narrow non-EW resonance in** $[12, 80]$ **GeV**, at any future luminosity.

This theorem is the *visible* counterpart of the dark forbidden band $c = 19$–22 (a charged knot at $c > 18$ would be invisible-yet-charged, a Vassiliev contradiction; a knot at $c < 23$ does not survive cosmologically, §24.4): two deserts, two mechanisms, and *every* void of the spectrum is a prediction.

Caveats

(i) The *energy* filter alone is too aggressive — it would mark the proton unstable ($3_1{+}5_2{+}5_2$ is lighter in its $\Sigma\chi$ class): the actual stability of the floors requires the conservation laws B/L, which TQNT *imposes* without deriving them from the topology (free integer Lk); for the desert, no protection exists — there the verdict is unconditional. (ii) The window count (617 states in $[12, 80]$ GeV) comes from the historical scan under the Ω_{ts} scale (archived): the *verdict* is structural (the shedding channels remain exothermic under any additive law with small bindings), but the per-window count is to be reread under the sector laws — under the référence, the conclusion is even strengthened: no sector law *populates* $[12, 80]$ GeV (stoikhions $\lesssim 11$ GeV for $b\bar{b}$, EW quartet at 80–170 GeV).

D.1.5 Free knots: masses and lifetimes of the realizable cores

The desert theorem leaves one category without numbers: the simple knots ($n = 1$) below the dark threshold, "realizable cores (contextual, P22)". A category without numbers is not accountable: for the *absence* of free 4_1, 5_2, 6_1 to be a prediction — and for a P22 detection to be one too — they must be given a mass and a lifetime (`predictions_noeuds_libres.py`). Three canonical pieces suffice, with *zero new constants*:

1. **mass**: the geometric law of the light plateau, $M_{\text{topo}}(K) = m_\ell\, \ell(K)/\ell(3_1)$ (ACPR rope-lengths, $m_\ell = 310$ MeV);
2. **channel**: the weak P15 *unknotting* — an $n = 1$ knot has nothing to shed (the desert cascade does not apply); its only channel is the crossing change, and the unknotting number is $u = 1$ for 4_1, 5_2, 6_1, 6_2, 6_3, 7_2: *one* reconnection unknots everything, the mass goes into χ radiation ($+\, e^{\pm}$ if charged); partial channels are Sargent-crushed ($(Q_{\text{part}}/Q_{\text{tot}})^5 \sim 4{\times}10^{-3}$);
3. **rate**: the derived Fermi structure (§22.4.4), $\Gamma = NQ^5/(384\pi^3 v^4)$, $g_{\text{topo}} = 1$, $N = 1$ (minimal channel: τ = upper bound). **Control**: the same formula on the muon yields τ_μ at $-0.4\,\%$ of the PDG.

K	canonical role	m [MeV]	τ [s]	$c\tau$ [m]	Γ [GeV]
4_1	meson core / strict DM	398	2.9×10^{-9}	0.86	2×10^{-16}
5_2	strange knot	468 [468–495]	1.3×10^{-9}	0.38	5×10^{-16}
6_1	orphan	537	6.5×10^{-10}	0.19	1×10^{-15}
6_2	orphan	540	6.3×10^{-10}	0.19	1×10^{-15}
6_3	orphan / strict DM	548	5.9×10^{-10}	0.18	1×10^{-15}
7_2	nucleon companion	605	3.6×10^{-10}	0.11	2×10^{-15}
8_3 (u=2)	EW core	674	4.2×10^{-10}	0.12	2×10^{-15}

The widths $\Gamma \sim 10^{-15}$ GeV say the essential: these are *ultra-narrow* states — the exact opposite of the link desert (widths of hundreds of MeV). Non-detection therefore does not come from the width but from *production*: entry is by pair (entry theorem, §3.4); at a collider the pair is born at fm separation and *binds* (meson construction: immediate hadronization). A *free* knot exists only if a critical field separates the pair before binding — exactly Schwinger: the P22 regimes at $\xi \gg 1$ (horizons, critical fields). The astrophysical flux is not computed (named target).

The realizable cores are accountable

Free 4_1: 398 MeV, 2.9×10^{-9} s; free 5_2: 468 MeV, 1.3×10^{-9} s; free 6_1: 537 MeV, 6.5×10^{-10} s — muon control at $-0.4\,\%$ under the same formula. Signature (charged variant): anomalous-mass track 0.40–0.67 GeV, *hadronic* interaction (χ tube) but decay to $e^{\pm}$+ missing energy, displaced vertex $c\tau \sim 0.2$–0.9 m — no SM state has this combination. Refutations: a *stable* free state at these masses (contradicts P15 unknotting); a singleton at a collider (contradicts the pair gate); a τ very different from the table (contradicts $g_{\text{topo}} = 1$ or $N = 1$ up to a factor).

Caveats

m_ℓ calibrated (± 4–$7\,\%$ anchor $\Rightarrow \pm 5\,\%$ on m, $\times 1.3$ on τ); $N = 1$ declared (a channel count would shorten τ, never beyond the table); 3-body m^5 measure declared (multi-body χ emission assumed of the same measure); the $+5.6\,\%$ dressing of 5_2 is measured *in composites*: its absence in the free state is itself a sub-prediction (declared band). Scope: 3_1 is *excluded* from the table (its free state **is** the charged lepton — $\mathbb{Z}_3$ modes + condensate projection, not $M_{\text{topo}} = 310$); the torus knots $5_1/7_1$ (lepton class) have no established free-state law: M_{topo} is only a scale there (open).

D.2 Unified hyperfine splitting of composites: the $\tau_\perp$ mechanism

The *intra-multiplet* mass splittings (vector–pseudoscalar for the mesons, decuplet–octet for the baryons) arise from a *single* mechanism: the spin-spin coupling of the knot's intrinsic magnetic field, $\tau_\perp$ (strong-force sector, and *not* the V5p27 mutual EM which is an oscillation effect, 14 orders of magnitude smaller).

Under the De Rújula–Georgi–Glashow contact form $\Delta E = A\,(\mathbf{S}_i \cdot \mathbf{S}_j)$ with $A \propto |\psi(0)|^2/(m_i m_j)$, the baryonic splitting relates to the mesonic splitting through

$$\Delta_{N\text{-}\Delta} = \underbrace{\tfrac{3}{2}}_{\text{spin}} \times \underbrace{\tfrac{1}{2}}_{\text{junction}} \times \underbrace{636}_{\rho\text{-}\pi} \times \underbrace{0.61}_{\text{contact}} = 293 \text{ MeV} \quad \text{(observed 293)}.$$

D.2.1 Derived factors

- **Spin** $\frac{3}{2}$: exact, by group theory ($\Delta\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle = 1$ for the vector–pseudoscalar meson, $\frac{3}{2}$ for the decuplet–octet baryon).
- **Junction** $\frac{1}{2}$: derived from the *geometry of the flux junction*. The meson is a straight tube (pair at 180°); the baryon is a *Y-junction* of three equal-tension tubes balancing at 120° (Steiner point). The $\tau_\perp$ coupling varying as cos(angle):

$$\text{junction} = \frac{|\cos 120°|}{|\cos 180°|} = \frac{1/2}{1} = \frac{1}{2}.$$

  This is *not* the QCD colour factor: it is the geometry of the baryonic junction, consistent with Rule 0 (no colour).

D.2.2 Contact factor

The contact factor — ratio of coincidence densities $|\psi(0)|^2_{\text{baryon}}/|\psi(0)|^2_{\text{meson}} \approx 0.61$ — is derived down to ~ 0.665 from the structure of the flux tube:

- **geometry** (linear tube + equal-tension Steiner junction): the Y-tree is longer than a straight tube, the pair is more spread out ⇒ ratio ≈ 0.79 (variational calculation);
- **wave tail + 6D depletion**: the exact ground state of a linear confinement is Airy-like (more spread out than a Gaussian), and the baryonic configuration space has dimension 6 (two Jacobi vectors); the exact solution in dimension D gives $\kappa_6/\kappa_3 = 0.841 \Rightarrow 0.665$.

Unified meson–baryon hyperfine splitting

A *single* $\tau_\perp$ mechanism governs ρ-π (meson) and N-Δ (baryon). The spin factor ($\frac{3}{2}$) is exact by group theory; the junction factor ($\frac{1}{2}$) is derived from the Steiner junction at 120°; the contact factor is derived down to ~ 0.665 from the physics of the χ flux tube (linear confinement, Airy tail, 6D depletion), in agreement with the observed value 0.61 to $\sim 8\,\%$.

Caveats

The $\sim 8\,\%$ residual on the contact factor (0.665 against 0.61) *is not* a missing physical mechanism. Three candidates were tested and discarded: (i) relativistic correction — net 0.79, by near-cancellation between the kinetic delocalization and the Breit factor $1/E$; (ii) rotation mode of the junction — the P-wave is exactly zero by the $1 \leftrightarrow 2$ exchange symmetry of the Steiner potential, the D-wave amounts to $\sim 0.1\,\%$; (iii) quantum gravitational field — the metric distortion at the hadronic core is $\sim 10^{-39}$ ($\alpha_G = Gm^2/\hbar c$), i.e. 37 decades too small (the *strong* χ field is already counted in the Steiner geometry and the Airy tail). The residual is the *modelling uncertainty* of the confinement (Gaussian ansatz versus exact Airy, $D = 6$ approximation) and, above all, the constituent mass, which remains an effective QCD concept with no topological equivalent — the same wall as B, L and the constituent masses.

Bibliography

[1] E. Witten, *Quantum field theory and the Jones polynomial*, Commun. Math. Phys. **121**, 351–399 (1989).

[2] V. F. R. Jones, *A polynomial invariant for knots via von Neumann algebras*, Bull. Amer. Math. Soc. **12**, 103–111 (1985).

[3] S.-S. Chern and J. Simons, *Characteristic forms and geometric invariants*, Ann. Math. **99**, 48–69 (1974).

[4] N. Reshetikhin and V. G. Turaev, *Invariants of 3-manifolds via link polynomials and quantum groups*, Invent. Math. **103**, 547–597 (1991).

[5] S. Deser, R. Jackiw, and S. Templeton, *Topologically massive gauge theories*, Ann. Phys. **140**, 372–411 (1982).

[6] L. H. Kauffman, *State models and the Jones polynomial*, Topology **26**, 395–407 (1987).

[7] C. McA. Gordon and J. Luecke, *Knots are determined by their complements*, J. Amer. Math. Soc. **2**, 371–415 (1989).

[8] J. W. Alexander, *Topological invariants of knots and links*, Trans. Amer. Math. Soc. **30**, 275–306 (1928).

[9] K. Reidemeister, *Elementare Begründung der Knotentheorie*, Abh. Math. Sem. Univ. Hamburg **5**, 24–32 (1927).

[10] J. S. Birman, *Braids, Links, and Mapping Class Groups*, Ann. Math. Studies **82**, Princeton University Press (1974).

[11] J. Hoste, M. Thistlethwaite, and J. Weeks, *The first 1,701,936 knots*, Math. Intelligencer **20**, 33–48 (1998).

[12] Planck Collaboration (N. Aghanim et al.), *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. **641**, A6 (2020). `arXiv:1807.06209`.

[13] R. L. Workman et al. (Particle Data Group), *Review of Particle Physics*, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022, updated 2024).

[14] BICEP/Keck Collaboration, *Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season*, Phys. Rev. Lett. **127**, 151301 (2021).

[15] M. Atiyah, *The Geometry and Physics of Knots*, Cambridge University Press (1990).

[16] S. Gukov, *Three-dimensional quantum gravity, Chern-Simons theory, and the A-polynomial*, Commun. Math. Phys. **255**, 577–627 (2005). `arXiv:hep-th/0306165`.

[17] S. Garoufalidis, *On the characteristic and deformation varieties of a knot*, Geom. Topol. Monogr. **7**, 291–309 (2004). `arXiv:math/0306230`.

[18] J. Cheeger, *Analytic torsion and the heat equation*, Ann. Math. **109**, 259–322 (1979). — W. Müller, *Analytic torsion and R-torsion of Riemannian manifolds*, Adv. Math. **28**, 233–305 (1978).

[19] S. W. Hawking, *Particle creation by black holes*, Commun. Math. Phys. **43**, 199–220 (1975).

[20] S. Chandrasekhar, *Dynamical friction. I. General considerations*, Astrophys. J. **97**, 255–262 (1943).

[21] A. D. Sakharov, *Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe*, JETP Lett. **5**, 24–27 (1967).

[22] A. S. Eddington, *Space, Time and Gravitation*, Cambridge University Press (1920).

[23] C. W. F. Everitt et al., *Gravity Probe B: Final Results of a Space Experiment to Test General Relativity*, Phys. Rev. Lett. **106**, 221101 (2011).

[24] B. P. Abbott et al. (LIGO/Virgo), *GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral*, Phys. Rev. Lett. **119**, 161101 (2017).

[25] Event Horizon Telescope Collaboration, *First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole*, Astrophys. J. Lett. **875**, L1 (2019).

[26] Event Horizon Telescope Collaboration, *First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole in the Center of the Milky Way*, Astrophys. J. Lett. **930**, L12 (2022).

[27] M. C. Miller et al. (NICER), *The Radius of PSR J0740+6620 from NICER and XMM-Newton Data*, Astrophys. J. Lett. **918**, L28 (2021).

[28] B. P. Abbott et al. (LIGO/Virgo), *Observation of Gravitational Waves from a Binary Black Hole Merger*, Phys. Rev. Lett. **116**, 061102 (2016).

[29] R. Abbott et al. (LIGO/Virgo), *GW190521: A Binary Black Hole Merger with a Total Mass of 150 $M_\odot$*, Phys. Rev. Lett. **125**, 101102 (2020).

[30] LHCb Collaboration (R. Aaij et al.), *Observation of an exotic narrow doubly charmed tetraquark*, Nature Physics **18**, 751–754 (2022).

[31] LHCb Collaboration, *Observation of structure in the J/ψ-pair mass spectrum*, Science Bulletin **65**, 1983–1993 (2020).

[32] LHCb Collaboration, *Evidence of a $J/\psi\Lambda$ structure and observation of excited Ξ^- states in the $\Xi_b^- \to J/\psi\Lambda K^-$ decay*, Science Bulletin **66**, 1278–1287 (2021).

[33] LHCb Collaboration (R. Aaij et al.), *Observation of a Narrow Pentaquark State, $P_c(4312)^+$, and of the Two-Peak Structure of the $P_c(4450)^+$*, Phys. Rev. Lett. **122**, 222001 (2019).

[34] LHCb Collaboration (R. Aaij et al.), *Observation of the Doubly Charmed Baryon Ξ_{cc}^{++}*, Phys. Rev. Lett. **119**, 112001 (2017).

[35] LHCb Collaboration, *A model-independent study of resonant structure in $B^+ \to D^+D^-K^+$ decays*, Phys. Rev. Lett. **125**, 242001 (2020).

[36] LHCb Collaboration, *Observation of a $J/\psi\Lambda$ resonance consistent with a strange pentaquark candidate in $B^- \to J/\psi\Lambda\bar{p}$ decays*, Phys. Rev. Lett. **131**, 031901 (2023).

[37] BESIII Collaboration (M. Ablikim et al.), *Observation of a Near-Threshold Structure in the K^+ Recoil-Mass Spectra in $e^+e^- \to K^+(D_s^- D^{*0} + D_s^{*-} D^0)$*, Phys. Rev. Lett. **126**, 102001 (2021).

[38] Yu. Ts. Oganessian et al., *Synthesis of the isotopes of elements 118 and 116 in the ^{249}Cf and ^{245}Cm+^{48}Ca fusion reactions*, Phys. Rev. C **74**, 044602 (2006).

[39] S. Hofmann and G. Münzenberg, *The discovery of the heaviest elements*, Rev. Mod. Phys. **72**, 733–767 (2000).

[40] M. Wang, W. J. Huang, F. G. Kondev, G. Audi, and S. Naimi, *The AME 2020 atomic mass evaluation*, Chin. Phys. C **45**, 030003 (2021).

[41] F. G. Kondev, M. Wang, W. J. Huang, S. Naimi, and G. Audi, *The NUBASE2020 evaluation of nuclear physics properties*, Chin. Phys. C **45**, 030001 (2021).

[42] I. Tanihata et al., *Measurements of interaction cross sections and nuclear radii in the light p-shell region*, Phys. Rev. Lett. **55**, 2676–2679 (1985).

[43] A. Gal, E. V. Hungerford, D. J. Millener, *Strangeness in nuclear physics*, Rev. Mod. Phys. **88**, 035004 (2016).

[44] H. Takahashi et al., *Observation of a ${}^{6}_{\Lambda\Lambda}He$ Double Hypernucleus*, Phys. Rev. Lett. **87**, 212502 (2001).

[45] H. A. Bethe and R. F. Bacher, *Nuclear Physics A. Stationary States of Nuclei*, Rev. Mod. Phys. **8**, 82–229 (1936).

[46] C. F. v. Weizsäcker, *Zur Theorie der Kernmassen*, Z. Physik **96**, 431–458 (1935).

[47] R. C. Tolman, *Static Solutions of Einstein's Field Equations for Spheres of Fluid*, Phys. Rev. **55**, 364–373 (1939).

[48] J. R. Oppenheimer and G. M. Volkoff, *On Massive Neutron Cores*, Phys. Rev. **55**, 374–381 (1939).

[49] J. D. Bekenstein, *Black Holes and Entropy*, Phys. Rev. D **7**, 2333–2346 (1973).

[50] R. K. Bhaduri, *Models of the Nucleon: From Quarks to Soliton*, Addison-Wesley (1988) ; voir aussi J. J. Aubert et al. (EMC), Phys. Lett. B **123**, 275–278 (1983) pour le modèle constituent.

[51] S. Durr et al., *Ab initio determination of light hadron masses*, Science **322**, 1224–1227 (2008).

[52] R. A. Briceno, H.-W. Lin, D. R. Bolton, *Charmed-baryon spectroscopy from lattice QCD with $N_f = 2 + 1 + 1$ flavors*, Phys. Rev. D **86**, 094502 (2012).

[53] A. G. Abac et al. (LIGO/Virgo/KAGRA), *Observation of Gravitational Waves from the Coalescence of a 2.5–4.5 $M_\odot$ Compact Object and a Neutron Star*, Astrophys. J. Lett. **970**, L34 (2024).

[54] R. W. Romani et al., *PSR J0952-0607: The Fastest and Heaviest Known Galactic Neutron Star*, Astrophys. J. Lett. **934**, L17 (2022).

[55] LHCb Collaboration, *Observation of new exotic resonances decaying into J/ψ-pair final states*, preliminary (ICHEP 2024).