

A scale-free Higgs potential with discrete renormalization:

$$\kappa_\lambda = 5/3 \text{ and } m_H = 2v/\sqrt{\pi \ln(1/\alpha)}$$

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Abstract

The LHC has established that the couplings of the 125 GeV boson follow the Standard-Model pattern at the 5–10% level, but the shape of the scalar potential itself — and in particular the trilinear self-coupling — remains untested. We revisit the radiatively generated, scale-invariant potential $V(\phi) = B \phi^4 [\ln(\phi/v) - \frac{1}{4}]$ (Coleman–Weinberg/Gildener–Weinberg form), whose shape fixes $\kappa_\lambda = 5/3$ and $\kappa_4 = 11/3$ independently of normalization, and propose a new normalization principle to replace the loop sum that historically ruled this potential out (the top-quark loop makes the Standard-Model sum negative). If the effective theory’s scales are generated multiplicatively in discrete steps of ratio α — so that the quartic running accrues in rungs of length $\ln(1/\alpha)$ — and the per-rung increment is fixed by the two-channel braid structure of an elementary exchange (a unique Temperley–Lieb contact channel costing one closure loop, metered at the unit angular rate over the half-turn), then $B = 1/[\pi \ln(1/\alpha)]$ and

$$m_H = \frac{2v}{\sqrt{\pi \ln(1/\alpha)}} = 125.25 \text{ GeV},$$

against the measured 125.20 ± 0.11 GeV (+0.04%, 0.5σ) — equivalently $\lambda = 2/[\pi \ln(1/\alpha)] = 0.12939$ versus 0.12928 ± 0.00023 . We report the look-elsewhere analysis honestly: in a scan of 1134 simple closed forms, three land within 2σ , and the selection of $1/[\pi \ln(1/\alpha)]$ is structural (the $1/\ln(1/\alpha)$ slot is forced by the discrete-rung hypothesis), not purely numerical. The mass relation is a postdiction; the falsifiable content is $\kappa_\lambda = 5/3$ (with $\sigma_{HH} \approx 0.55 \sigma_{HH}^{\text{SM}}$, testable at the HL-LHC and decisively at future colliders), $\kappa_4 = 11/3$, tree couplings $\kappa = 1$ everywhere with no invisible width, and the linear sharpening of the m_H world average, which tests the normalization directly.

1 Introduction

Ten years of LHC measurements have established that the couplings of the scalar boson at 125 GeV to fermions and gauge bosons are proportional to mass with Standard-Model (SM) normalization at the 5–10% level [1, 2]. What they have *not* established is the shape of the scalar potential. In the SM the potential

$$V_{\text{SM}}(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2, \quad \lambda = \frac{m_H^2}{2v^2}, \quad (1)$$

is postulated; its two parameters are fixed by the measured v (from G_F) and m_H , and it predicts neither. The trilinear self-coupling, which probes the potential’s third derivative, is only bounded: the ATLAS single-plus-double-Higgs combination gives $-0.4 < \kappa_\lambda < 6.3$ at 95% CL [3], the CMS combination $-1.2 < \kappa_\lambda < 7.5$ [4], and the PDG quotes $\kappa_\lambda = 3.8_{-4.0}^{+2.1}$ [9].

An old and economical alternative is the *radiatively generated*, classically scale-invariant potential of Coleman–Weinberg (CW) [6] and Gildener–Weinberg (GW) [7],

$$V(\phi) = B\phi^4 \left[\ln \frac{\phi}{v} - \frac{1}{4} \right], \quad (2)$$

in which the electroweak scale arises by dimensional transmutation rather than by a negative mass term. Its *shape* is rigid: as recalled in Sec. 2, it fixes the trilinear and quartic self-couplings at

$$\kappa_\lambda = \frac{5}{3}, \quad \kappa_4 = \frac{11}{3}, \quad (3)$$

independently of the normalization B — the well-known hallmark of radiative symmetry breaking [7, 8]. What killed this potential *within* the SM is the normalization: at one loop B is a sum over the states coupled to the scalar, and the top quark enters with a negative (fermion-loop) sign that overwhelms the bosonic contributions, $\sum_i n_i (m_i/v)^4 = -2.56$ (Sec. 3); historically this is why the CW Higgs was predicted light and is excluded.

This paper proposes a different normalization principle, tests it against data, and reports the result with its look-elsewhere analysis. The principle has two parts:

H1. Discrete renormalization. The scales of the effective theory are generated multiplicatively in discrete steps of ratio α (the fine-structure constant), so that the renormalization-group flow of the quartic coupling accrues in *rungs* of length $L \equiv \ln(1/\alpha) = 4.92$ e-folds rather than continuously. The β -function is then $\beta_\lambda = \Delta\lambda_{\text{rung}}/L$, and the normalization inherits a structural $1/L$.

H2. Unit-rate angular measure. The per-rung increment is fixed geometrically by the two-channel braid structure of an elementary exchange: the exchange is a half-turn (π radians), its unique Temperley–Lieb contact channel costs one closure loop, and the loop is metered at the unit angular rate — one unit per π radians, i.e. $BL = 1/\pi$ (equivalently $\Delta\lambda_{\text{rung}} = 4/\pi$).

Together with the exact relation $B = \beta_\lambda/4$ (Sec. 3), these give

$$\boxed{B = \frac{1}{\pi \ln(1/\alpha)}} \quad \Rightarrow \quad \boxed{m_H = \frac{2v}{\sqrt{\pi \ln(1/\alpha)}} = 125.25 \text{ GeV}}, \quad (4)$$

against the world average 125.20 ± 0.11 GeV [9] ($+0.04\%$, 0.5σ). Equivalently, the quartic coupling is the pure number $\lambda = 2/[\pi \ln(1/\alpha)] = 0.12939$, versus 0.12928 ± 0.00023 from data.

Because m_H is measured, Eq. (4) is a *postdiction*, and we treat it as such: Sec. 4 reports the null tests (the scan of competing closed forms and the structural-selection argument) so the reader can weigh the look-elsewhere effect. The *predictions* of the framework are collected in Sec. 5: $\kappa_\lambda = 5/3$ with a di-Higgs rate about half the SM expectation, $\kappa_4 = 11/3$, exactly SM-like tree couplings with vanishing invisible width, and a sharp linear test of the normalization as the m_H world average improves.

A companion framework [11] motivates both hypotheses — there, the electroweak scale is observed to satisfy $v \simeq \sqrt{2\pi} \alpha^8 M_{\text{Pl}} (1 + \alpha/4\pi)$ to 3×10^{-5} relative accuracy, exhibiting the multiplicative α -ladder of H1, and the angular measure of H2 recurs in several of its derivations — but the present paper is self-contained: H1–H2 are stated as hypotheses and confronted with data on their own terms.

2 The shape: $\kappa_\lambda = 5/3$ without normalization

Expanding Eq. (2) about its minimum $\phi = v$ gives

$$V''(v) = 4Bv^2 \equiv m_H^2, \quad V'''(v) = 20Bv = \frac{5m_H^2}{v}, \quad V''''(v) = 44B = \frac{11m_H^2}{v^2}, \quad (5)$$

while the SM potential (1) gives $V_{\text{SM}}'''(v) = 3m_H^2/v$ and $V_{\text{SM}}''''(v) = 3m_H^2/v^2$. Hence, independently of B ,

$$\kappa_\lambda \equiv \frac{V_{\text{SM}}'''(v)}{V_{\text{SM}}''''(v)} = \frac{5}{3}, \quad \kappa_4 \equiv \frac{V_{\text{SM}}''''(v)}{V_{\text{SM}}''''(v)} = \frac{11}{3}. \quad (6)$$

This is the standard result of radiative breaking [7, 8]; we recall it because everything that follows hangs on the shape being (2) rather than (1).

The shape is also robust against the next order of the flow. Writing $V = B\phi^4[\ln(\phi/v_0) - \frac{1}{4} + \varepsilon \ln^2(\phi/v_0)]$, the minimum remains exactly at v_0 and an elementary expansion gives

$$\kappa_\lambda(\varepsilon) = \frac{20 + 18\varepsilon}{12 + 6\varepsilon} = \frac{5}{3} + \frac{2}{3}\varepsilon + O(\varepsilon^2). \quad (7)$$

Hypothesis H1 sets $\varepsilon = 0$ *exactly* at this order (uniform rungs), so the question is the size of the residual. Two sources, and both are computable rather than estimated.

(a) *Drift of the rung increment.* If $\Delta\lambda_{\text{rung}}$ itself drifts by a relative $O(\alpha)$ from rung to rung — the natural next step of a ladder in α — then $\beta_\lambda(L) = 4B(1 + \alpha L/L_{\text{rung}})$ with $L_{\text{rung}} = \ln(1/\alpha)$, and integrating gives

$$\varepsilon = \frac{\alpha}{2\ln(1/\alpha)} = 7.4 \times 10^{-4}. \quad (8)$$

The conversion matters: a relative correction of $O(\alpha)$ *per rung* is not an ε of $O(\alpha)$ *per e-fold*; the two differ by $2\ln(1/\alpha) \simeq 9.8$. Hence $\delta\kappa_\lambda/\kappa_\lambda = \frac{2}{5}\varepsilon = 0.03\%$.

(b) *The mode's own loop.* The intrinsic term $3\lambda^2/16\pi^2$ is *quadratic* in λ , and λ is linear in L ; it therefore generates an L^3 term, not an L^2 one, with $c_3 = B/4\pi^2 = 1.6 \times 10^{-3}$. An L^3 term shifts κ_λ by $c_3/2$ (+0.05%) and leaves m_H *exactly* unchanged: $V''(v) = 4Bv^2$ identically, for any c_3 . The mass is protected against the leading intrinsic correction; only the cubic coupling feels it.

Adding the two in quadrature, $\kappa_\lambda = 5/3$ is robust at the 0.06% level, and the theory uncertainty on the mass is

$$\delta m_H^{\text{th}} = \frac{1}{4}\varepsilon m_H = 0.023 \text{ GeV}, \quad (9)$$

five times below the current experimental error. All the σ figures quoted in this paper are therefore experiment-dominated; where they matter we quote the combination, 0.112 GeV.

Four shapes, four cubics: for orientation, a quadratic double well gives $\kappa_\lambda = 1$; a periodic (cosine) potential gives $\kappa_\lambda = 0$ at its symmetric minimum; a potential with an amplitude floor, $V \propto (\phi^2 - v^2)^2/\phi^2$, gives $\kappa_\lambda = -1$. The self-coupling is therefore a genuine *shape discriminant*, not a fine detail: the four readings differ at $O(1)$.

3 The normalization: discrete rungs and the $1/\pi$

3.1 B is a β -function

Writing $V = \frac{\lambda(\phi)}{4}\phi^4$ with $\lambda(\phi) = 4B[\ln(\phi/v) - \frac{1}{4}]$, one reads off

$$\frac{d\lambda}{d\ln\phi} = 4B \equiv \beta_\lambda. \quad (10)$$

This is exact and is the GW structure: a scale-free potential has no coupling constant, it has a *drift*. Any derivation of B is a derivation of β_λ .

3.2 Why the loop sum fails, and what replaces it

In the SM the one-loop β_λ (equivalently the CW B) is dominated by the states coupled to the scalar, $B \propto \sum_i n_i (m_i/v)^4$ with fermions entering negatively. Numerically, with $(m_W, m_Z, m_H, m_t) = (80.4, 91.2, 125.2, 172.6)$ GeV and $(n_W, n_Z, n_H, n_t) = (6, 3, 1, -12)$:

$$\sum_i n_i \left(\frac{m_i}{v} \right)^4 = -2.56, \quad (11)$$

the top alone contributing -2.75 . A matter-loop normalization is excluded — this is the historical reason the CW Higgs is dead in the SM, and we do not resurrect it. Hypotheses H1–H2 replace the loop sum by a property of the flow itself.

H1 (discrete rungs). If the theory's scales are generated multiplicatively with ratio α per step, the flow of λ accrues at the rungs, of uniform spacing $L = \ln(1/\alpha)$ in $\ln \mu$. A constant per-rung increment $\Delta\lambda_{\text{rung}}$ then yields a constant coarse-grained β -function

$$\beta_\lambda = \frac{\Delta\lambda_{\text{rung}}}{L}, \quad (12)$$

which is precisely what the pure-log shape (2) requires (a ϕ -dependent β would generate the $\varepsilon \ln^2$ term; uniform rungs give $\varepsilon = 0$ at this order).

Three quantitative checks support H1 beyond its role as a postulate. (i) *Matter decoupling is selected by the data.* The obvious objection is that in the SM effective theory the top loop runs the quartic ($\beta_\lambda^{\text{SM}} \simeq -0.023$ per e-fold at $\mu \sim v$, top-dominated). If that continuum contribution fed the present flow, the normalization would shift to $B = (\beta_{\text{discrete}} + \beta_{\text{SM}})/4$ and $m_H = 119.6$ GeV — excluded at 51σ ; even a 10% contamination gives -4.6σ . Within this scheme, matter loops renormalize the *matter* operators (the standard running of masses and couplings, which we use as usual), not the medium's quartic stiffness: the sector split is not assumed, it is what the measured m_H selects. (ii) *The mode's own loop is computable and negligible.* The only continuum contribution intrinsic to the scalar mode itself, $3\lambda^2/16\pi^2$, evaluates at the minimum ($\lambda(v) = -B$) to 8×10^{-5} per e-fold (3×10^{-4} of β_λ), inducing $\varepsilon = -1.2 \times 10^{-3}$, i.e. a -0.05% shift of κ_λ and none of m_H : the pure log is exact at the level of the observables. (The expansion is local — $|\ln(\phi/v)|$ of a few rungs, which is all that V'' , V''' , V'''' use; the far-UV continuation of the ladder is beyond this paper's scope.) (iii) *The step ratio is observed.* In the companion hierarchy relations the per-step ratio reads α at the -5 ppm level per step over the 8 steps to the electroweak anchor, and at 3×10^{-4} over the 18 steps to the nucleon hierarchy [11].

H2 (unit-rate angular measure). The per-rung increment is fixed geometrically, and the two-strand braid structure does most of the work. The elementary scale step is an exchange of two strands — geometrically a *half-turn*: two lines twisted by a relative angle θ cross exactly $\lfloor \theta/\pi \rfloor$ times, and at $\theta = \pi$ their ends are exchanged. The four-point space of such an exchange is two-dimensional in the Temperley–Lieb algebra TL_2 [5]: the through-channel 1 (parallel strands) and the cup-cap e ($e^2 = de$). Three exact facts follow: (i) *the contact channel is unique* — e is the only four-point contact the exchange can generate (a quartic; no quadratic term arises, since the through-channel receives a pure phase), so a crossing feeds exactly the quartic flow and nothing else; (ii) *the contact costs one closure loop* — the Markov closure of 1 contains two loops (d^2) and that of e one loop (d); (iii) *the channel-splitting rate is unity* — the exchange $\sigma = A \cdot 1 + A^{-1}e$ has eigenvalues $\{A, -A^{-3}\}$, whose relative phase across the half-turn is $\pi(1 - 2/(k+2)) \rightarrow \pi$ for large level k : one radian of phase per radian of turn. Metering one loop (= one half-turn, π radians) at this unit rate gives the per-rung contact weight

$$B \cdot L = \frac{1 \text{ unit per loop}}{\pi \text{ radians per loop}} = \frac{1}{\pi}, \quad \text{i.e.} \quad B = \frac{1}{\pi \ln(1/\alpha)} = 0.064694, \quad (13)$$

equivalently $\Delta\lambda_{\text{rung}} = 4BL = 4/\pi$. The normalization is a pure angular ratio. The internal alternatives are excluded by the data themselves: metering with the finite- k channel phase

($k = 135$, i.e. $(1 - 2/137)/\pi$) gives $m_H = 124.3$ GeV (7.8σ away); weighting loops by their quantum dimension ($1/\pi d$ or $1/\pi d^2$) gives 88.6 or 62.6 GeV ($\sim 300\sigma$): the flow *counts* loops, it does not evaluate them.

3.3 Confrontation

With $v = (\sqrt{2}G_F)^{-1/2} = 246.2197$ GeV and $\alpha^{-1} = 137.036$:

$$m_H^{\text{pred}} = 2v\sqrt{B} = \frac{2v}{\sqrt{\pi \ln(1/\alpha)}} = 125.252 \text{ GeV}, \quad (14)$$

$$m_H^{\text{exp}} = 125.20 \pm 0.11 \text{ GeV} (S = 1.4) [9] \quad (+0.04\%, 0.5\sigma). \quad (15)$$

Equivalently, the measured combination $c \equiv BL = (m_H^2/4v^2) \ln(1/\alpha) = 0.31805 \pm 0.00056$ against $1/\pi = 0.31831$. In the quartic-coupling language, $\lambda = 2/[\pi \ln(1/\alpha)] = 0.12939$ versus 0.12928 ± 0.00023 .

Two remarks on the choice of α . (i) The result is insensitive to the difference between the measured $\alpha^{-1} = 137.036$ and the integer 137 ($\Delta m_H = +0.003\%$). (ii) It is *not* insensitive to the renormalization point of α itself: using the effective coupling at the electroweak scale, $\alpha^{-1}(m_Z) \simeq 127.95$, gives $m_H = 126.13$ GeV, excluded at 8.5σ . The data select the vacuum (Thomson-limit) value — consistent with H1, where the α -ladder is a property of the vacuum, not of the dressed coupling at any particular scale.

4 Null tests and look-elsewhere

Because Eq. (4) is a postdiction of a measured quantity, the selection of the closed form must be examined for look-elsewhere effects. We report two scans.

In-class scan. Within the structural class $B = c/L$ forced by H1, we compare $c = 0.31805 \pm 0.00056$ with simple constants: $1/\pi$ (+0.08%), $8/25$ (+0.6%), $\pi/10$ (−1.2%), $5/16$ (−1.7%), $1/3$ (+4.8%), $1/e$ (+16%), $1/2\pi$ (−50%), $2/\pi$ (+100%). Only $1/\pi$ falls within 1σ ; the nearest alternatives with any claim to simplicity sit 1–3 σ away (rational mimics of π such as $7/22$ are excluded on principle). The angular-measure alternatives are strongly excluded by the mass itself: reading the cycle as a full turn (2π) gives $m_H = 88.6$ GeV; as the 4π spinor cycle, 62.6 GeV; an aperture of $\pi/3$ gives 128.2 GeV (+27 σ); the planar average $\langle |\cos| \rangle = 2/\pi$ per leg gives 177 GeV. The pair (unit rate, half-turn) is the only survivor.

Out-of-class scan. Scanning 1134 distinct forms $q\pi^p L^r \alpha^s$ (rationals $q = n/d$ with $n, d \leq 10$; $p, r \in \{-1, 0, 1\}$; $s \in \{0, 1\}$) against the 2σ band of B , three forms land inside: $1/(\pi L)$ (+0.08%), $\frac{4}{7}\pi L \alpha$ (−0.28%), and $\frac{9}{5}L \alpha$ (−0.02%). The numerical scan alone therefore does *not* single out our form; the selection is structural: H1 forces the L^{-1} slot (a drift *per rung*), and forms growing as L^{+1} have no mechanism slot at all. Within the structural class the scan returns only $1/\pi$. We state this plainly so the reader can weigh it: the numerical agreement is 0.5σ ; the burden carried by structure (H1–H2) is what distinguishes Eq. (13) from numerology, and H1–H2 are falsifiable through their other consequences (Sec. 5).

5 Predictions and falsification

1. **Trilinear self-coupling** [PREDICTION]: $\kappa_\lambda = 5/3$, robust at the 0.06% level against the next order of the flow (Eqs. 8–9). This sits on the destructive side of the triangle–box interference in $gg \rightarrow HH$: a crude quadratic interpolation through the standard anchor points gives $\sigma_{HH} \approx 0.55 \sigma_{HH}^{\text{SM}}$ at 13–14 TeV, compatible with the current limit $\sigma_{HH} < 2.4 \sigma_{HH}^{\text{SM}}$ [9]. The current κ_λ intervals [3, 4] do not test it; the HL-LHC (sensitivity $\sim \text{SM}$) will graze it; an FCC measures it decisively [10].

2. **Quartic self-coupling** [PREDICTION]: $\kappa_4 = 11/3$ — beyond any planned collider’s direct reach, recorded for completeness.
3. **Tree couplings** [PREDICTION]: exactly SM-like ($\kappa_f = \kappa_V = 1$), with vanishing invisible width. Current global values: $\kappa_V = 1.023 \pm 0.026$, $\kappa_F = 0.94 \pm 0.05$, $\text{BR}(\text{invisible}) < 10.7\%$ [9] — all consistent. The joint signature — $\kappa = 1$ everywhere, $\kappa_\lambda = 5/3$, di-Higgs at about half the SM rate — is reproduced neither by the SM ($\kappa_\lambda = 1$) nor by typical extended sectors (which shift the tree couplings along with the self-coupling).
4. **The mass relation** [POSTDICTION] $\rightarrow$ [PREDICTION]: as the m_H world average sharpens, it tests $c = 1/\pi$ linearly. A world average converging on 125.11 GeV (the current ATLAS-only value) would put Eq. (4) at $+1.3\sigma$ against the combined error (0.112 GeV, experiment-dominated); 125.00 GeV would put it at 2.2σ , and the relation fails at the 3σ level below 124.91 GeV. We state the threshold with its error budget attached: a falsification point quoted against the experimental error alone would not be one.

6 Status of the ingredients

ingredient	status
shape (2) $\Rightarrow \kappa_\lambda = 5/3, \kappa_4 = 11/3$	[DERIVED] from the shape; shape itself [CONDITIONAL] on H1
$B = \beta_\lambda/4$	[DERIVED] (exact)
loop-sum normalization excluded (top sign)	[DERIVED] (numerical)
H1: discrete rungs of length $\ln(1/\alpha)$	[CONDITIONAL] (SM-loop feed excluded 51σ ; self-loop 3×10^{-4} ; step ratio at ppm)
H2: $B L = 1/\pi$	[CONDITIONAL] (TL_2 channel structure exact; alternatives excluded $7.8\sigma/300\sigma$)
$m_H = 2v/\sqrt{\pi \ln(1/\alpha)}$	[POSTDICTION] (0.5σ ; linear future test)
$v \simeq \sqrt{2\pi} \alpha^8 M_{\text{Pl}}(1 + \alpha/4\pi)$	[OBSERVED] (3×10^{-5} ; companion framework [11])

If the last line is taken at face value, the two relations compose into a closed form for the Higgs mass in terms of α and the Planck mass alone,

$$m_H = \frac{2\sqrt{2} \alpha^8 M_{\text{Pl}} (1 + \alpha/4\pi)}{\sqrt{\ln(1/\alpha)}} = 125.26 \text{ GeV}, \quad (16)$$

which we record as a remark rather than a claim: it inherits the conditional status of both inputs.

7 Conclusion

The scale-invariant potential (2) has always had the right rigidity — its shape fixes the self-couplings with no freedom — and the wrong normalization within the SM. We have proposed a normalization that does not proceed through matter loops: a discrete multiplicative flow (α per step) with a unit-rate angular increment ($1/\pi$ per leg and per rung). It passes its one available numerical test at 0.5σ (m_H , equivalently $\lambda = 2/[\pi \ln(1/\alpha)]$), survives the in-class null scan, and converts the outstanding experimental question about the scalar sector into a sharp fork:

$\kappa_\lambda = 1$ (postulated polynomial potential) versus $\kappa_\lambda = 5/3$ (scale-free potential, this work),

with the di-Higgs rate at about half the SM expectation in the second case. The HL-LHC will approach this fork; a future collider decides it.

Declarations

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Availability of data and material. The analysis scripts (potential bench, normalization scan, angular-measure discriminant, Temperley–Lieb closure, discrete-flow tests) are archived with a timestamped preprint on Zenodo (doi:10.5281/zenodo.22127667) and reproduce every number in the paper; all experimental inputs are from Ref. [9] and the cited LHC combinations.

Competing interests. The author declares no competing interests.

Authors' contributions. Not applicable (single-authored work).

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